Priors for the long run

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- Propose a class of prior distributions for VARs that discipline the long-run implications of the model

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Priors for the long run

- Properties
  - Based on macroeconomic theory
  - Conjugate → Easy to implement and combine with existing priors

- Perform well in applications
  - Good (long-run) forecasting performance

Outline

- A specific pathology of (flat-prior) VARs
  - Too much explanatory power of initial conditions and deterministic trends
  - Sims (1996 and 2000)

- Priors for the long run
  - Intuition
  - Specification and implementation

- Alternative interpretations and relation with the literature

- Application: macroeconomic forecasting

Simple example

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- If ***ρ = 1***, DC is a simple linear trend: $$DC = y_0 + c \cdot t$$

- Otherwise more complex: $$DC = \frac{c}{1-\rho} + \rho^t \left(y_0 - \frac{c}{1-\rho} \right)$$

Pathology of (flat-prior) VARs (Sims, 1996 and 2000)

- OLS/MLE has a tendency to “use” the complexity of deterministic components to fit the low frequency variation in the data
- Possible because inference is typically conditional on $\boldsymbol{y_0}$
  - No penalization for parameter estimates of implying steady states or trends far away from initial conditions

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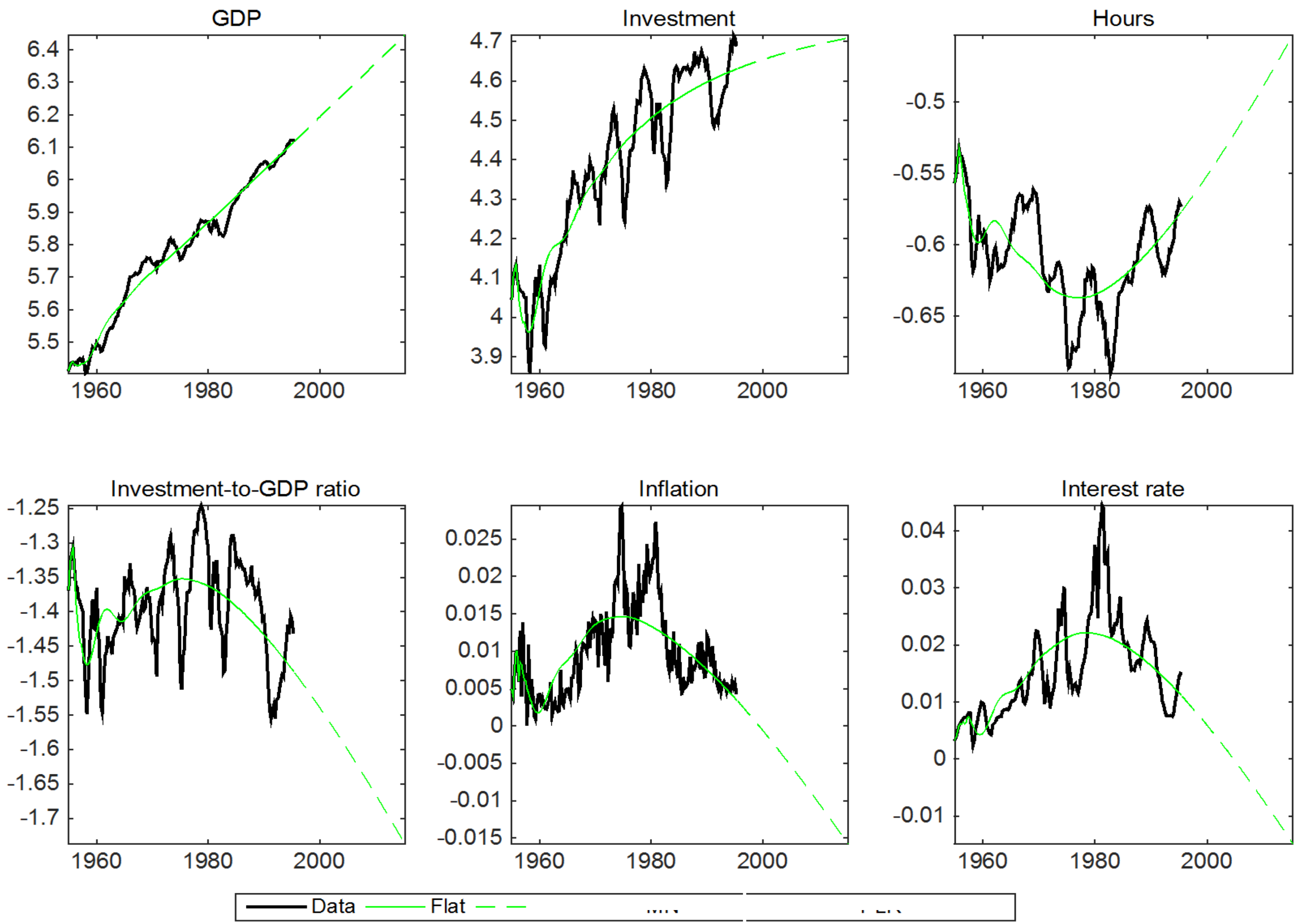
- Example: 7-variable VAR(5) with quarterly data on
  - GDP
  - Consumption
  - Investment
  - Real Wages
  - Hours
  - Inflation
  - Federal funds rate

- Sample: 1955:I – 1994:IV

- Flat or Minnesota prior

"Over-fitting" of deterministic components in VARs



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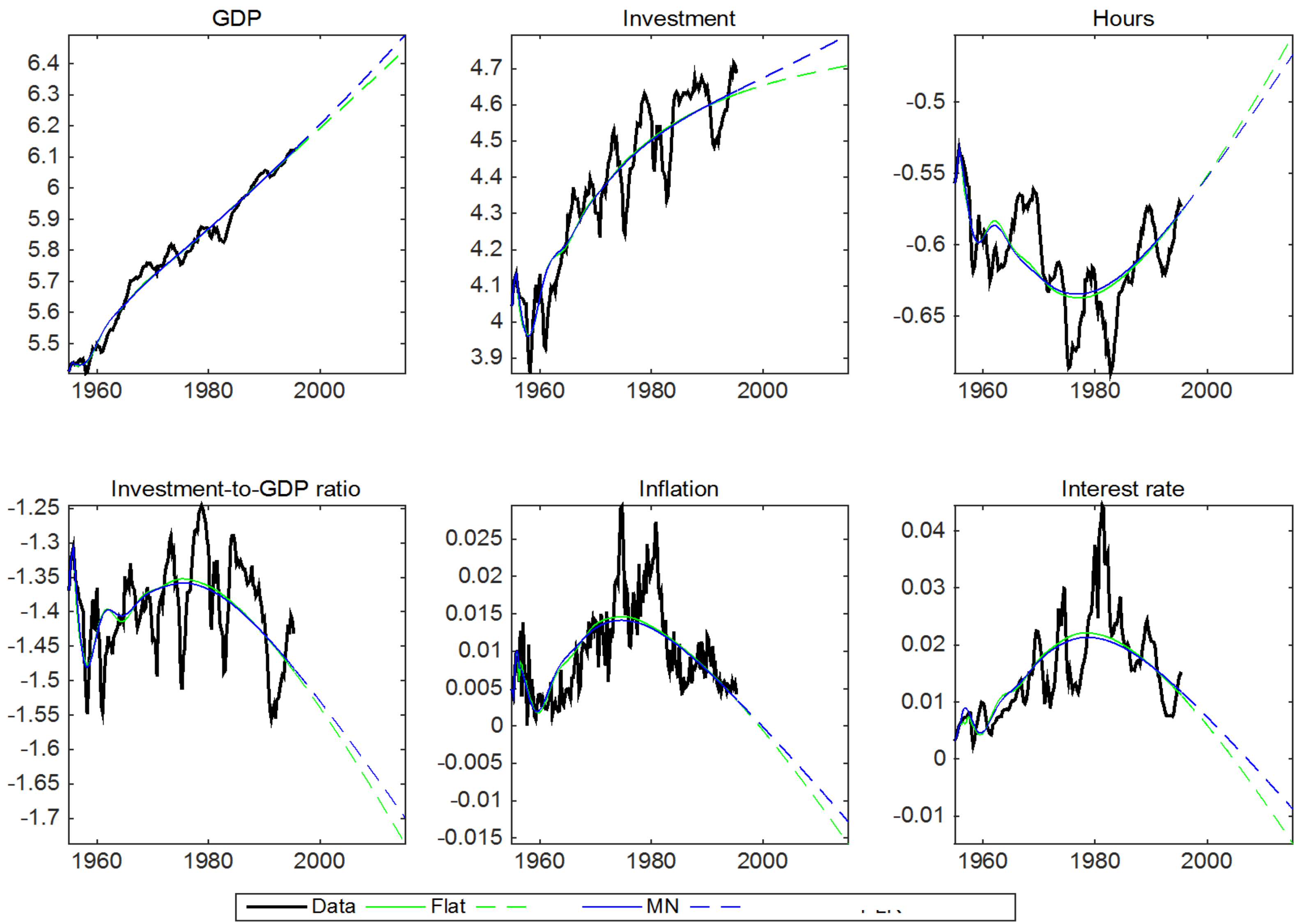

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- Need a prior that downplays excessive explanatory power of initial conditions and deterministic component

- One solution: center prior on "non-stationarity"

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- Standard approach (DLS, SZ, and many followers)
  - Push coefficients towards all variables being independent random walks

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- Loadings associated with these combinations are less(more) likely to be 0

Example: 3-variable VAR of KPSW

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$$\tilde{y}_{t-1} = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{pmatrix} \text{Output} \\ \text{Consumption} \\ \text{Investment} \end{pmatrix}$$

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  - Can be easily combined with existing priors
  - Can compute the ML in closed form
    - Useful for hierarchical modeling and setting of hyperparameters ϕ (GLP, 2013)

Empirical results

- Deterministic component in 7-variable VAR

- Forecasting
  - 3-variable VAR
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\\
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<u>Interpretation of ***H y***</u>

- Real trend
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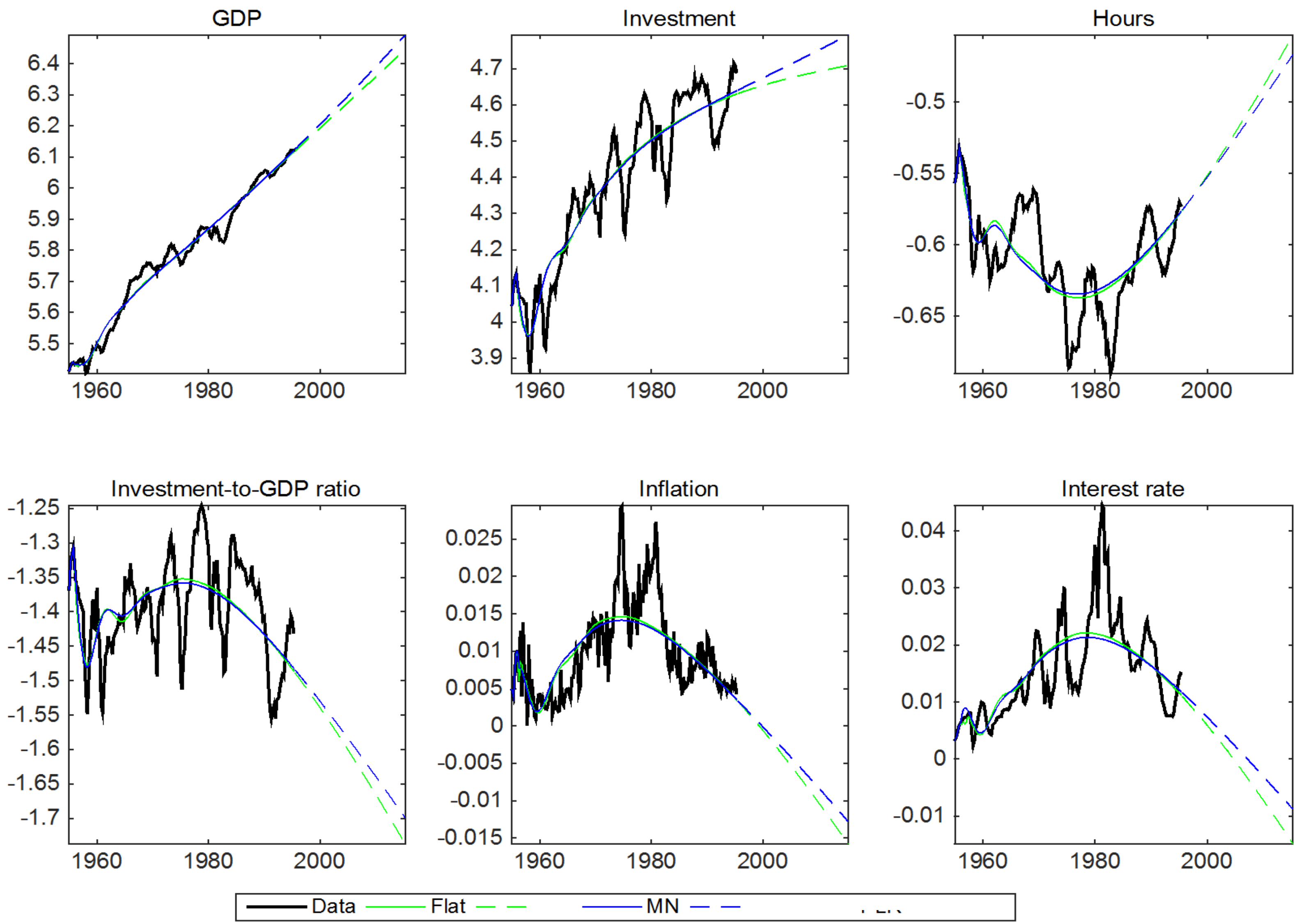
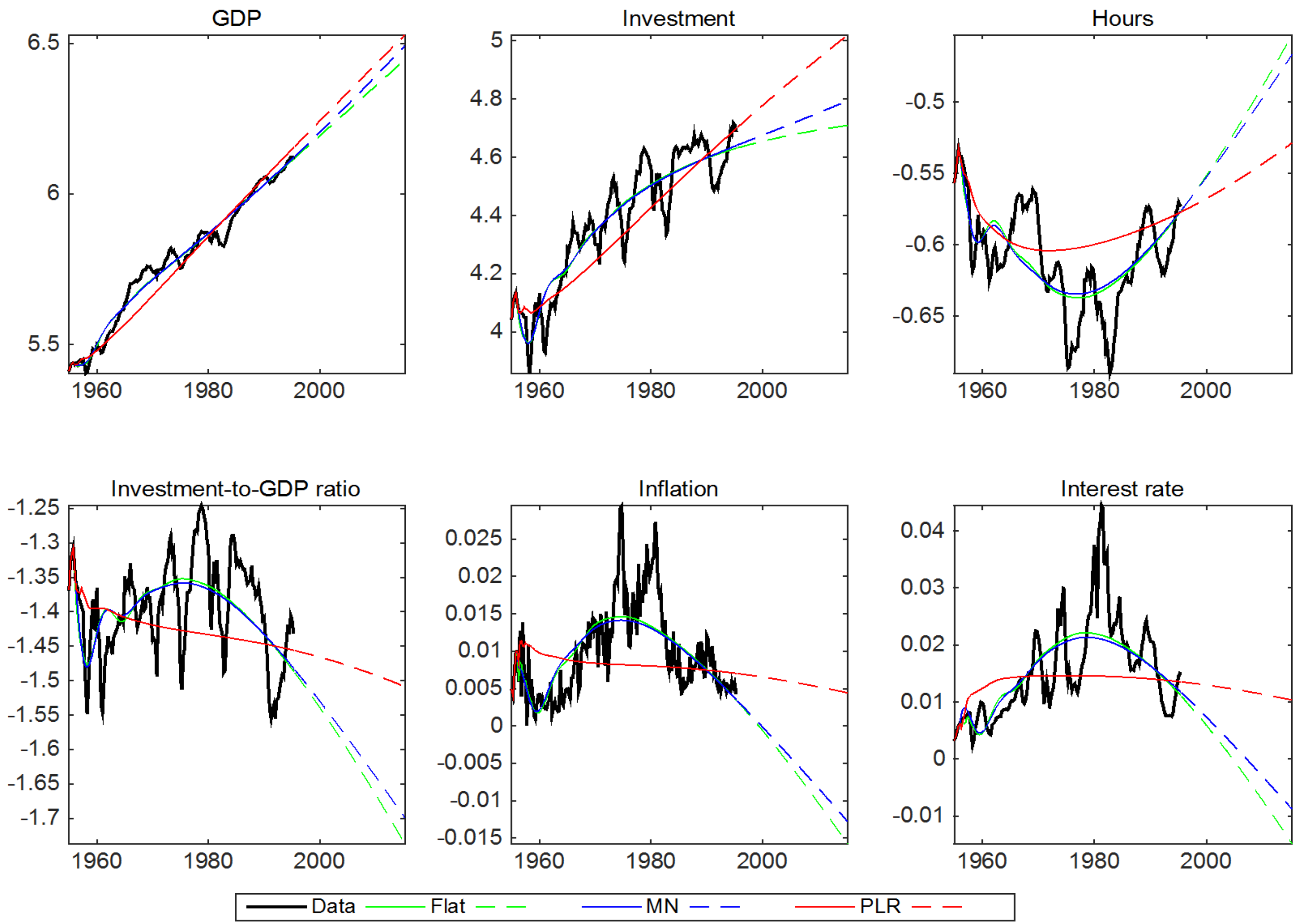


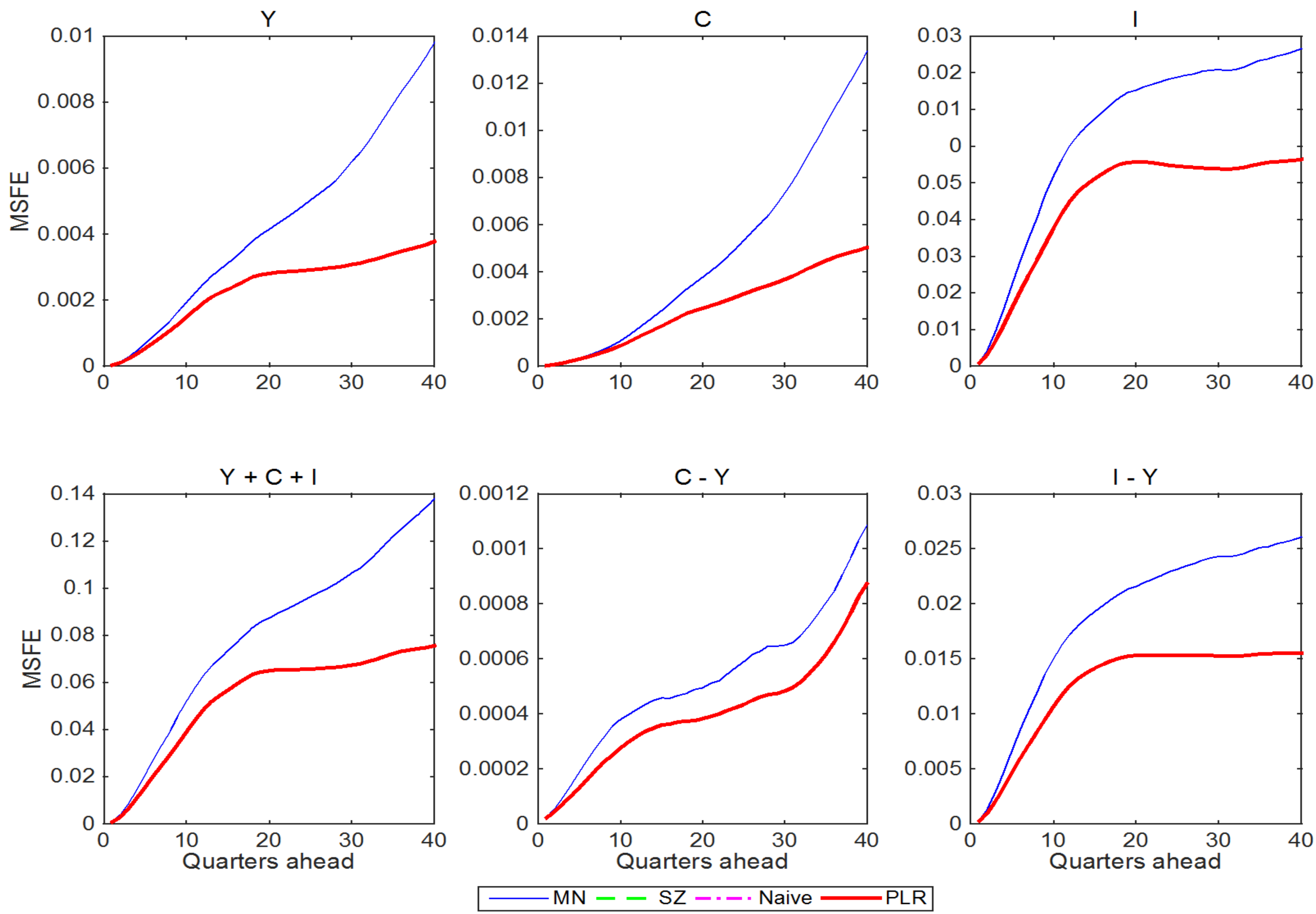
Deterministic components in VARs with Prior for the Long Run



Forecasting results with 3-, 5- and 7-variable VARs

- Recursive estimation starts in 1955:I
- Forecast-evaluation sample: 1985:I – 2013:I

3-variable VAR: MSFE (1985-2013)



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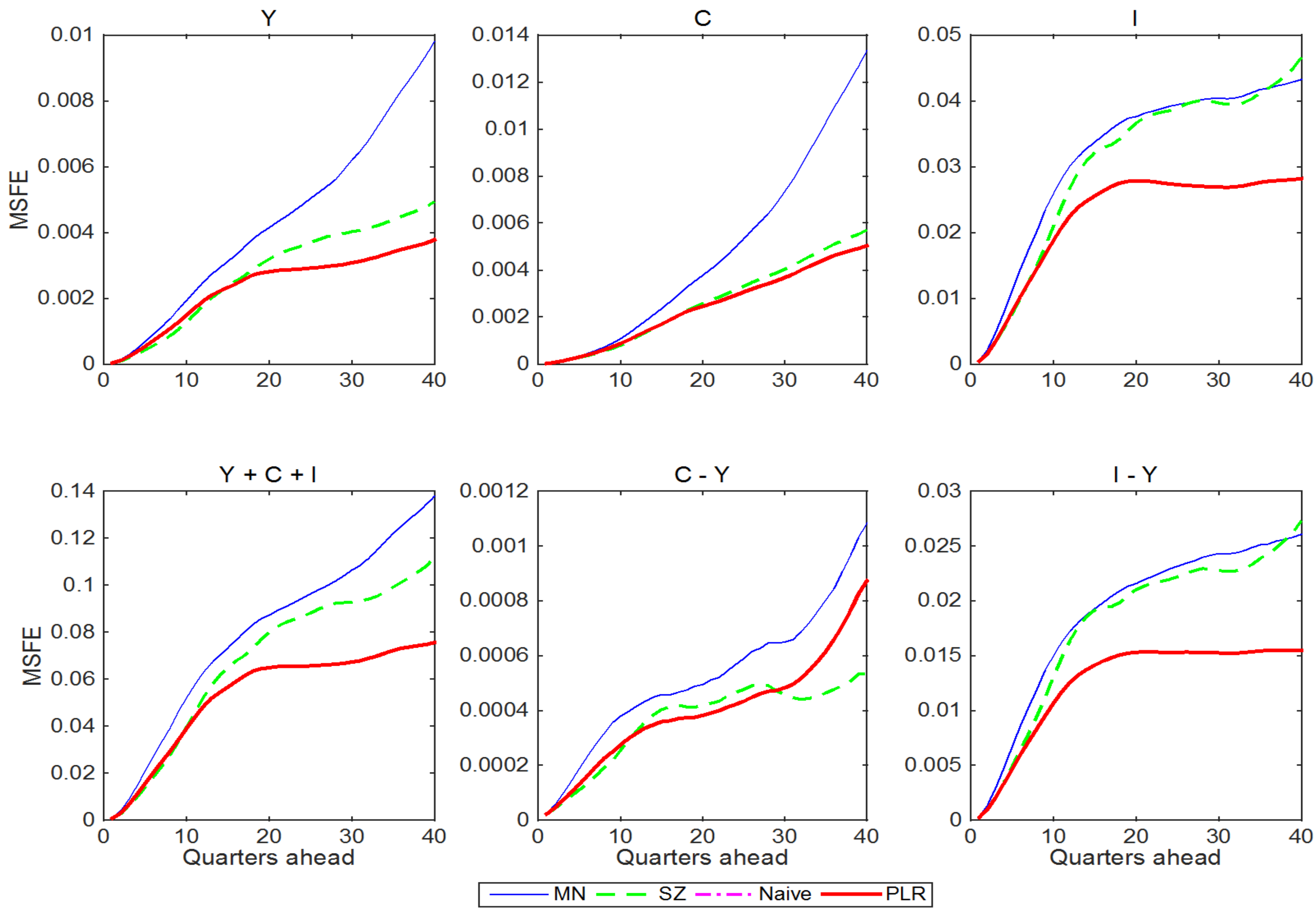
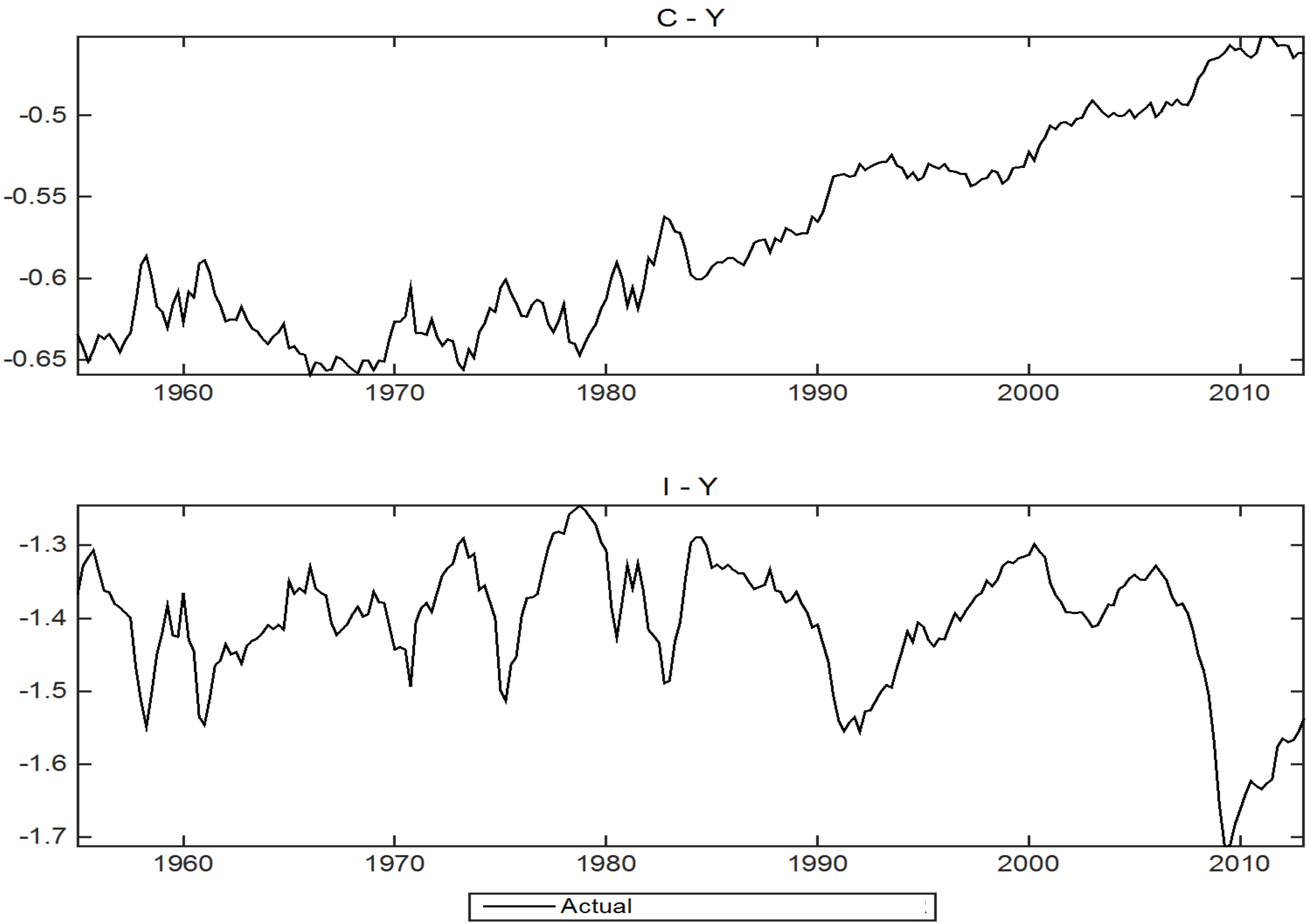


Consumption- and Investment-to-GDP ratios



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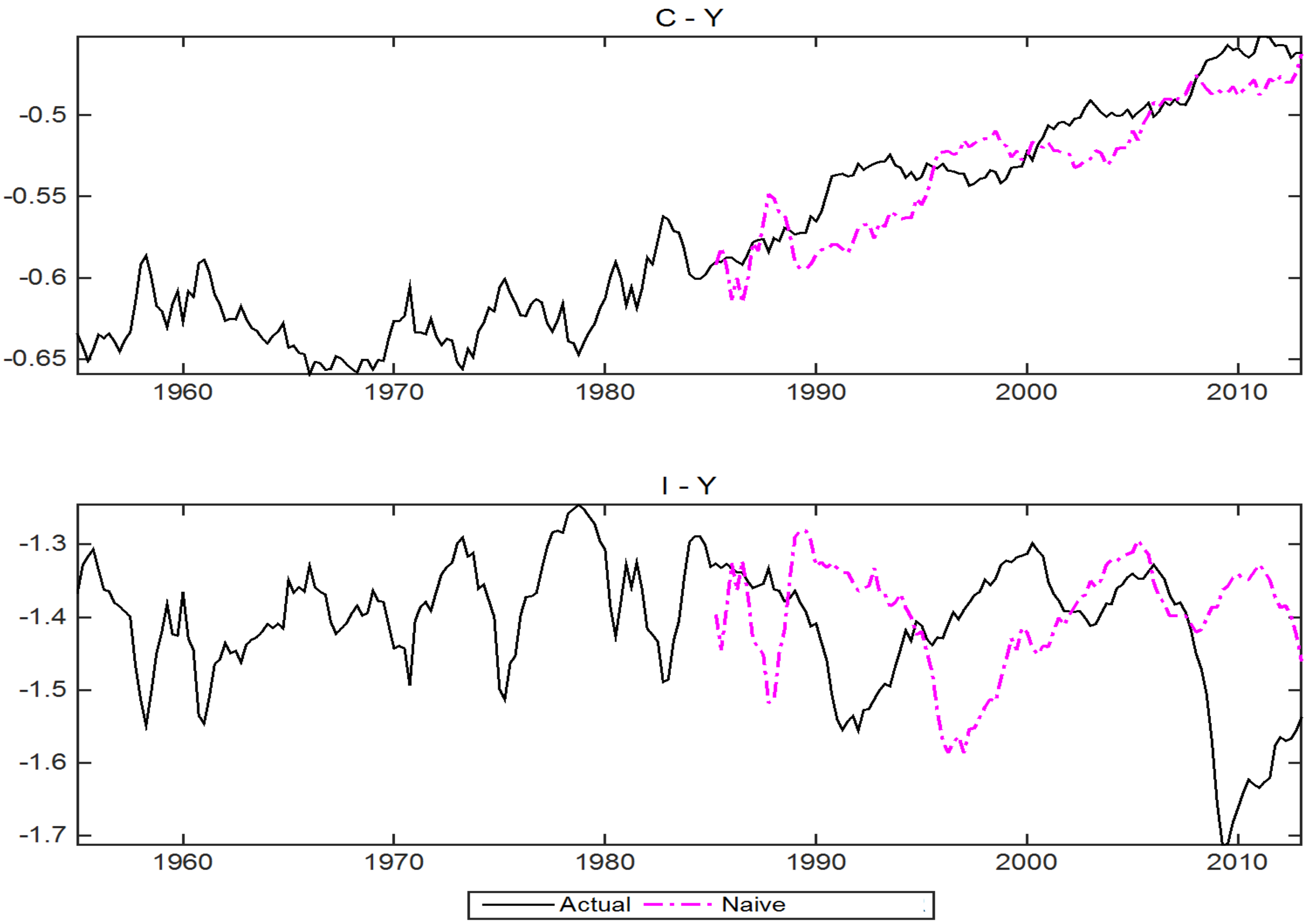

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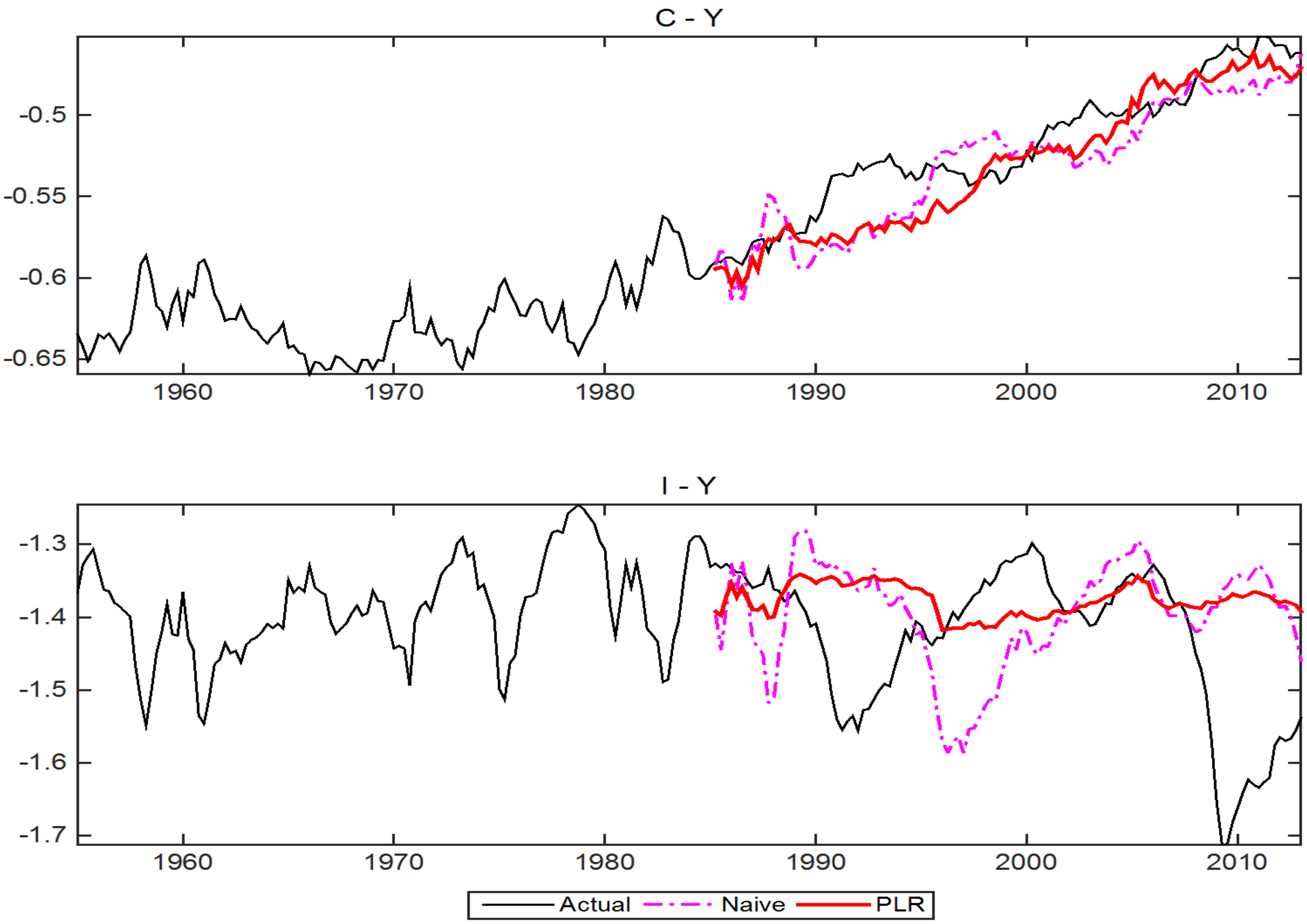
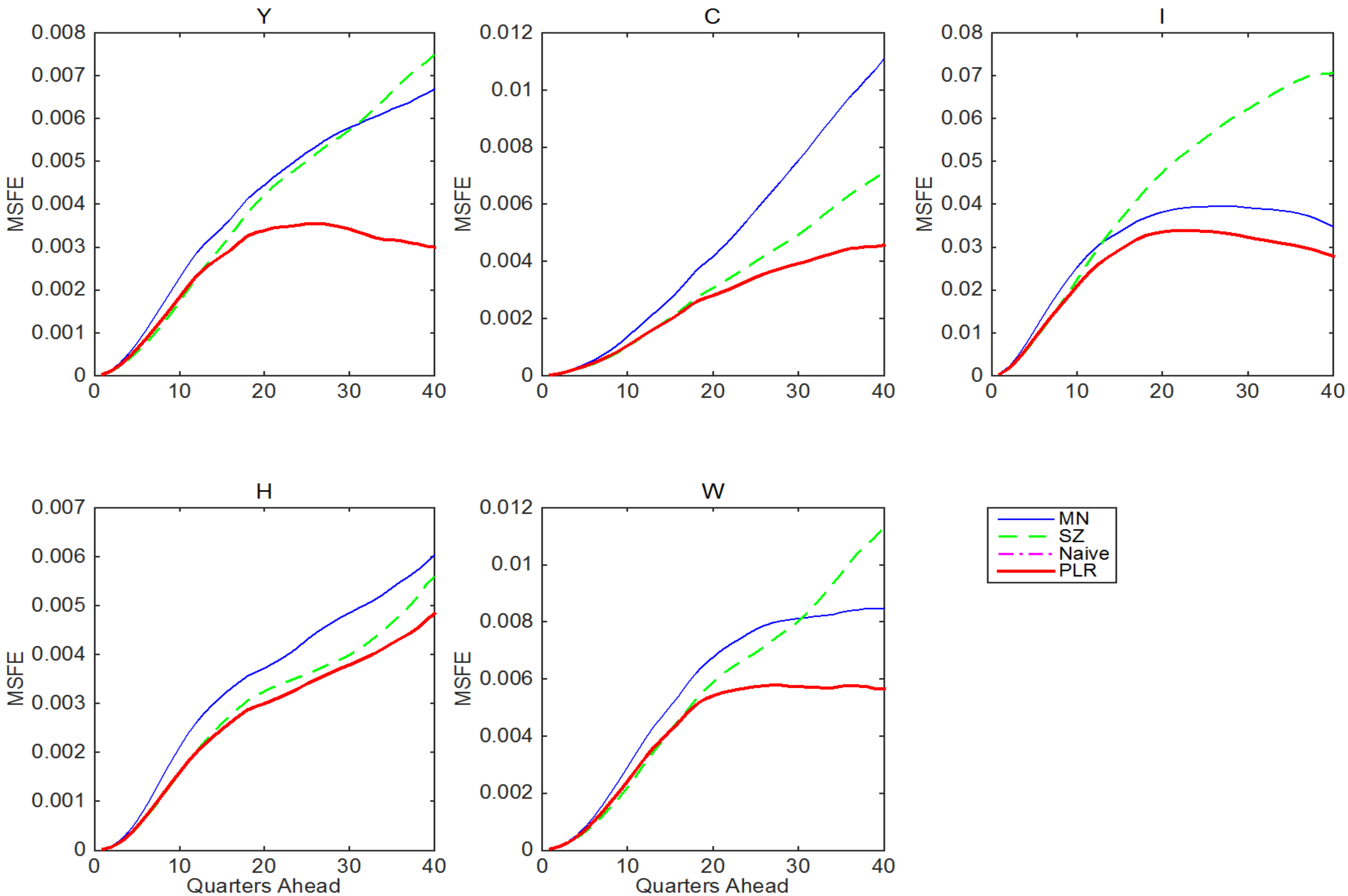


5-variable VAR: MSFE (1985-2013)



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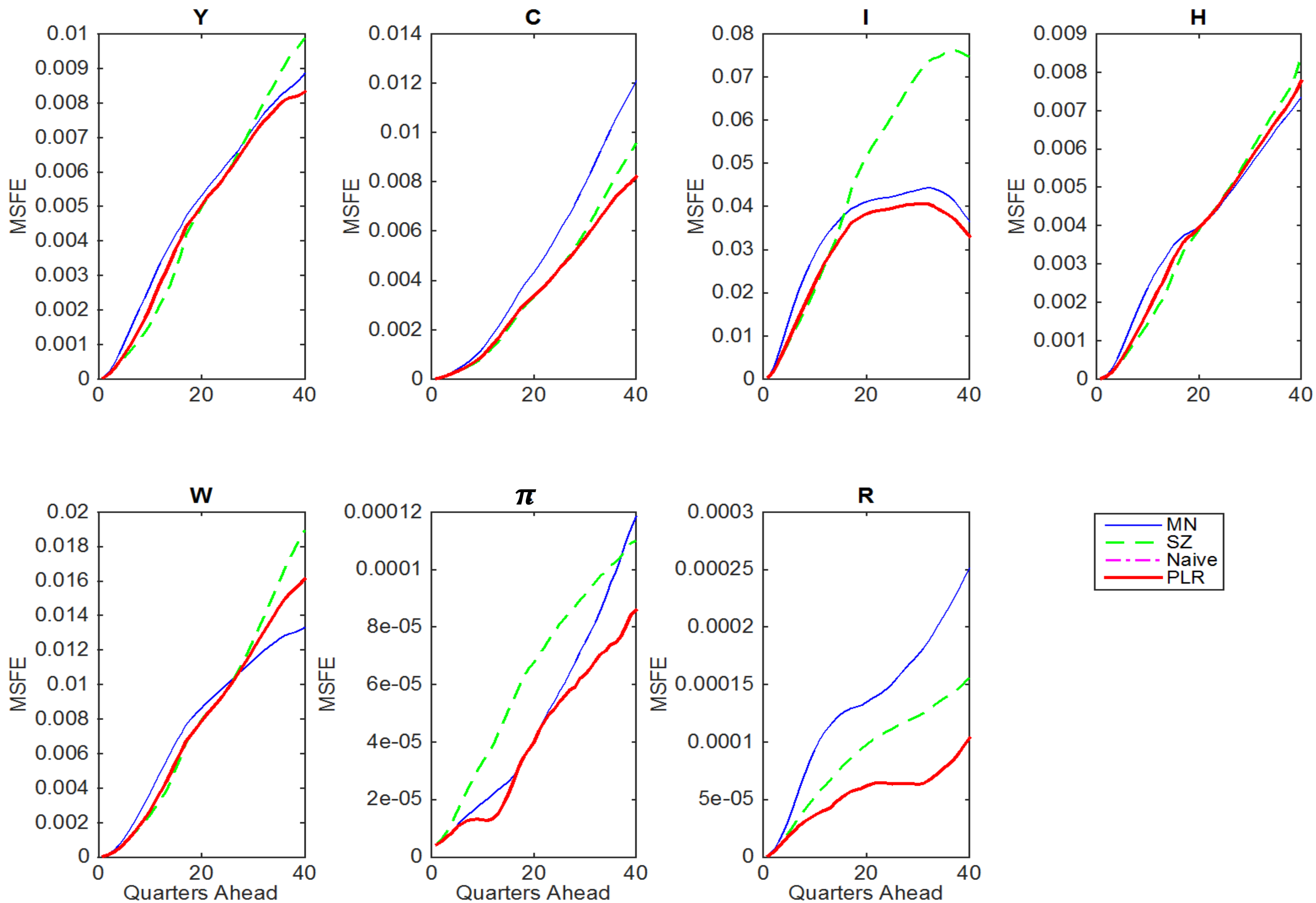

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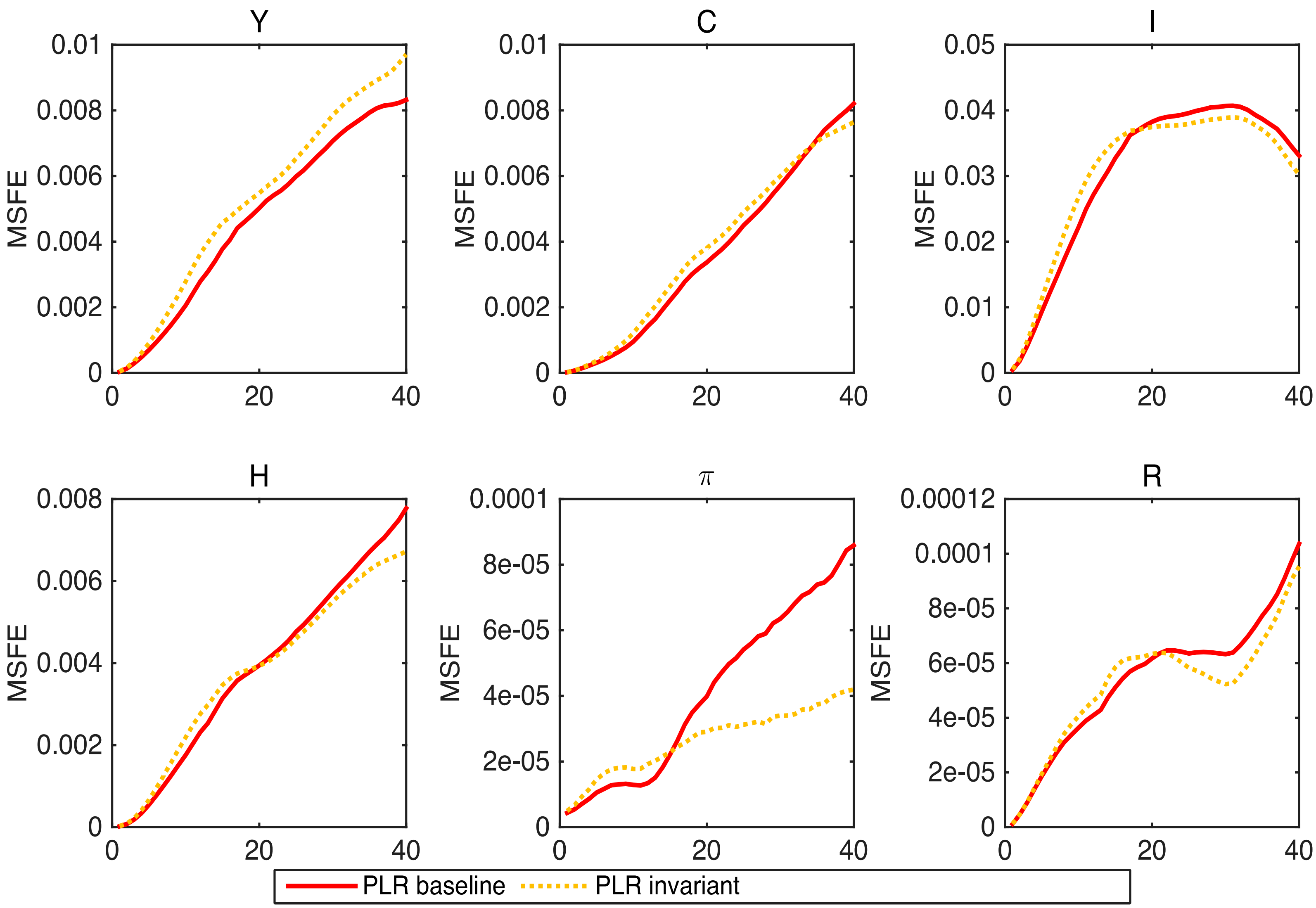
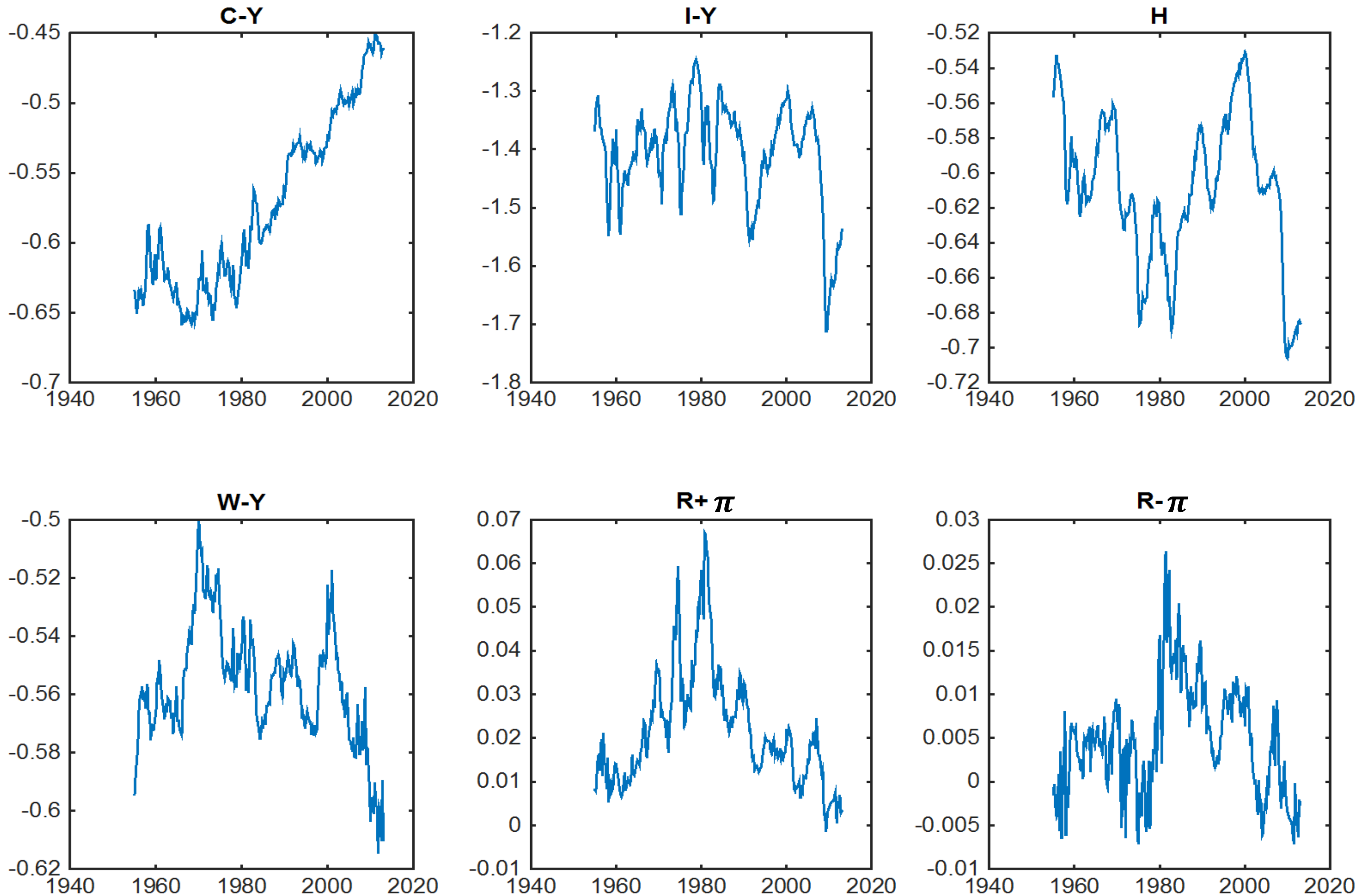
$$\begin{cases} \Lambda_{\cdot i} \cdot (H_{i\cdot}\bar{y}_0) | H, \Sigma \sim N\left(0, \phi_i^2 \Sigma\right), \quad i = 1, \dots, n-r \\ \sum_{i=n-r+1}^{n} \Lambda_{\cdot i} \cdot (H_{i\cdot}\bar{y}_0) | H, \Sigma \sim N(0, \phi_{n-r+1}^2 \Sigma) \end{cases}$$

7-variable VAR: Forecasting results with "invariant" PLR



$H\,y$ in the data



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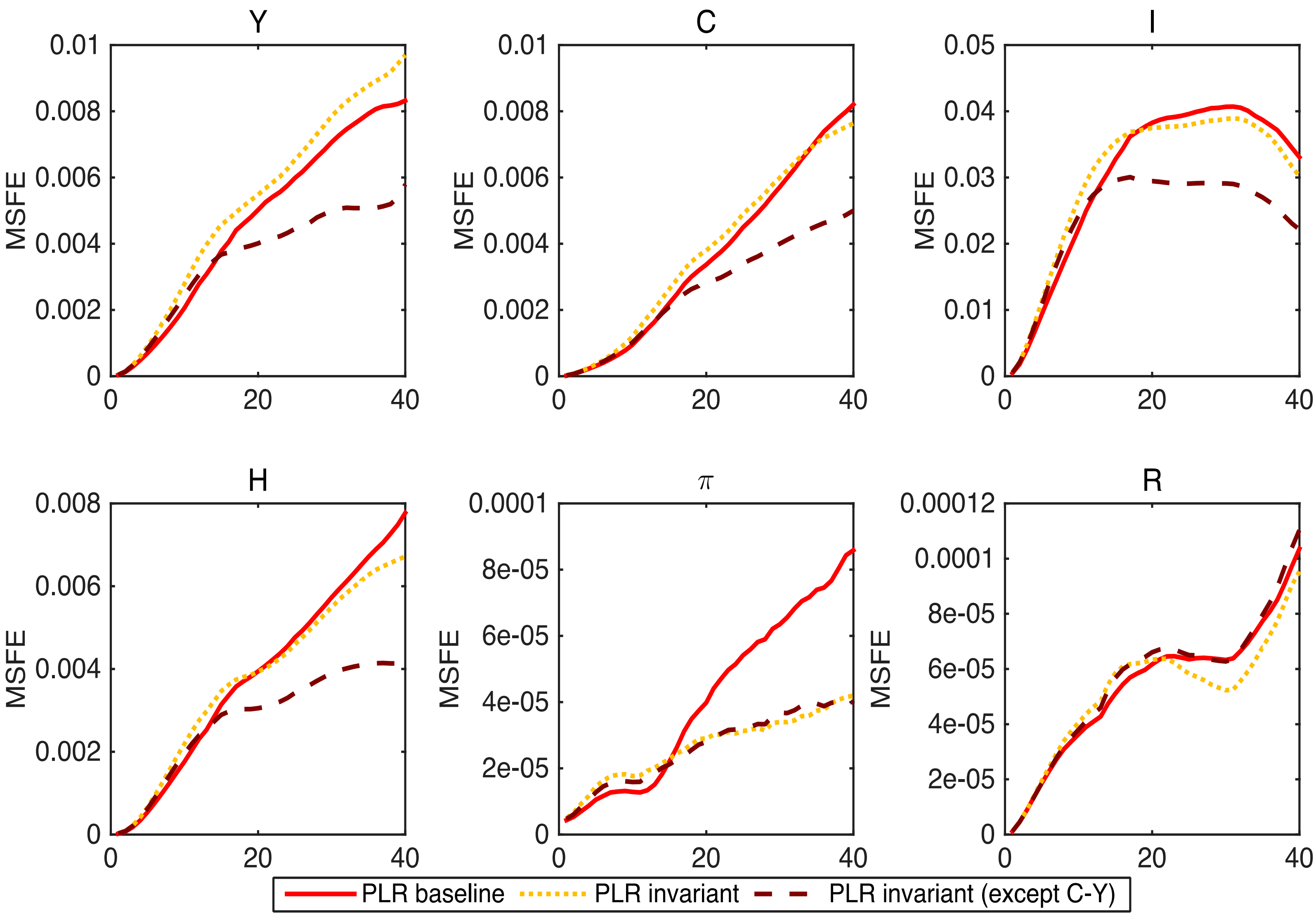

Strengths and weaknesses

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  - Imposes discipline on long-run behavior of the model
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- “Weak” points
  - Non-automatic procedure → need to think about it
  - Might prove difficult to set up in large-scale models → might require too much thinking

Connections and extreme cases

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- Sum-of-coefficients prior (DLS, SZ)
  - $[\beta' \;\; \beta']' = H = I$
  - shrink Λ_1 and Λ_2 to 0

3-var VAR: Mean Squared Forecast Errors (1985-2013)