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* Views expressed are those of the authors and do not necessarily reflect official positions of De Nederlandsche Bank.

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Quantifying Systemic Risk in the Presence of Unlisted Banks: Application to the European Banking Sector*

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Abstract

We propose a credit portfolio approach using CDS prices to evaluate systemic risk and to attribute it across institutions. This allows the application of approaches that rely on information from public equity markets to privately held institutions and coöperative banks also. We extend our approach further to account for fat tails and skewness of asset returns. Application to a sample of European banks shows that the buffers for large banks are relatively low compared to their contributions to systemic risk and highlights consistent differences in capital buffers between different countries, demonstrating the benefits of a consistent systemic approach to all European banks simultaneously.

JEL codes: G01, G20, G18, G38

Keywords: systemic risk, CDS rates, implied market measures, financial institutions, fat tails, O-SII buffers

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1 Introduction

The canonical approach to measuring various aspects of systemic risk in the financial sector relies on equity return correlations to assess interdependencies between banks' losses above Value at Risk (Adrian and Brunnermeier (2016),Acharya et al. (2017)). But in many countries, this approach is thwarted by the presence of state-owned and/or coöperative banks. To circumvent this problem we go beyond the equity based measures used for Adrian-Brunnermeier's CoVaR and Acharya et al. (2017)'s Marginal Expected Shortfall (MES) by relying on CDS contracts rather than equity returns to extract the required information on covariance structure.

We use data on 27 European banks, a large subsection of which are not publicly traded, and develop a valuation-of-loans approach to measure systemic risk and to identify and rank the systemic players. Our approach is appropriate whenever some of the potentially systemic institutions are not publicly traded on the equity market. The analysis confirms that financial institutions need to be monitored in the context of other financial institutions.

Systemic risk measurement is directly relevant for setting the portion of capital buffers designed to mitigate the risks that a bank's distress may pose on the financial system or the wider economy. In a policy context, the minimum capital requirements are intended to reflect and manage the risks that a bank's operations pose on its own distress. The macroprudential buffers, on the other hand, aim to improve the resilience of the financial system by internalizing the systemic risks generated by an institution or the financial sector as a whole.[1] In the current paper we build the foundations for measuring and attributing systemic risk based on co-movements in observed CDS prices; this allows

[1]Within the European Union, banks which are found to be systemically significant for the national economies are subject to the Other Systemically Important Institutions (O-SII) regulatory framework. National authorities, under guidance from the European Banking Authority (EBA), are required to measure the systemic risk contributions of banks and to determine the size of the macroprudential surcharges for these institutions. In addition, banks that are found to be Globally Systemically Important (G-SII) are also required to keep extra macroprudential buffers. If an institution is charged under both the G-SII and the O-SII frameworks, the higher of the two scores applies. For details, see BCBS (2010); FSB (2020); EBA (2020).

us to include banks that are not publicly traded in our sample. In a companion paper to this one, Dimitrov and van Wijnbergen (2023) develop the methodology further and explore its potential to guide the calibration of the macroprudential capital buffers in the Eurozone.

Systemic linkages arise naturally through various channels. One direct source of systemic fragility stems from the structure of the networks through which banks operate on the interbank market.[2] Systemic dependencies may also arise indirectly, due to common exposure of the institutions to the same risk sources - either on the liability side, when funding sources are similar or on the asset side, when the institutions hold similar or correlated asset portfolios.[3] We present a framework that does not require a particular view of what is causing systemic losses. Instead, we identify the potential for high joint distress based on observed dependencies between traded credit protection on the market.

The use of market data for the identification of systemic banks has been underutilized in the regulatory process, presumably at least partially because of difficulties in doing so for banks that are not publicly traded. Busch et al. (2021) compare the assigned G-SIB scores to the corresponding $\Delta CoVaR$ measure as a way to provide stronger empirical foundation for the construction of the empirical scores, but this is obviously not possible when banks are not publicly traded since $\Delta CoVaR$ is based on the distribution of equity prices. Engle et al. (2023) present an approach to including non-traded banks into measures of systemic risk by estimating the relation between accounting data and equity based market data for listed banks in their sample and then applying the same relation to unlisted firms. A problem with this approach might be that that relation may well change in crisis times; CDS contracts are more likely to pick up changes in market sentiment as accounting data unavoidably lag behind and are based on data of the bank itself only. We compare the two approaches (ours based on CDS prices and the Engle approach extending the link between accounting and equity based information to unlisted banks) in Section 5.3. We expect that the CDS based is more sensitive to a bank's tail

[2]Cf. Bräuning and Fecht (2017); Langfield et al. (2014); van Lelyveld et al. (2014); Georg (2013).
[3]Cf. Kosenko and Michelson (2022); Siedlarek and Fritsch (2019).

losses. The two approaches can in fact complement each other: in particular if there are banks in the sample for which there is not only no publicly traded equity but for whom no traded CDS contracts exist either; in that case there is no alternative to the Engle et al. (2023), Engle and Jung (2023) approach.

We use our CDS based approach first of all to show that monitoring the financial risk of an institution in isolation from the risks of its counterparties and from the system as a whole may offer a misleading ranking between Systemically Important Financial Institutions (SIFIs). Second, we illustrate that high-frequency data from the credit default swap (CDS) market can be used to monitor *ex-ante* the build-up of systemic risk and systemic dependencies. This is particularly valuable in the context of the European financial sector, where key institutions are often privately held, and market data on their equity value is not available. Third, we link systemic risk to the potential for joint distress between institutions by evaluating the tail dependencies in their potential losses if a default of one institution were to occur. Fourth, we illustrate how the potential for extreme losses can be incorporated explicitly into the framework through the addition of higher-order common factors which capture tail interdependencies in banks' asset returns.

We define systemic risk both through the prospect that several key institutions become distressed at the same time and through the prospect that the common losses they generate may have a large social impact. To quantify such risk, our model relies on several building blocks.

First, we use a contingent claims approach on a bank's balance sheet (Merton, 1974) and define distress as the situation in which the market value of a firm's assets falls below a default barrier. The observed CDS spreads allow us to estimate the probability of such distress occurring. Second, we assume that one or possibly more latent factors drive common changes in the asset values of banks. Our approach is more general than the one underlying the well-known Vasicek credit model (Vasicek, 1987) which assumes a single correlation parameter driving the dependencies in the whole portfolio. We allow for factor exposure heterogeneity across banks fitted on the CDS sample. To do so we build on the portfolio-of-loans approach suggested by Huang et al. (2009, 2012) and extend it by explicitly focusing on tail risk and by modeling tail dependencies in distress.

We know through Merton (1974) that the market value of a company's assets is related both to the market value of its equity and of its liabilities. The level of the firm's CDS spread at any particular instance relates to the chance that the value of its assets may drop and that it may experience distress in the form of a credit event captured by the CDS contract. Importantly, co-movements in default probabilities can provide information on the tendency of the institutions to become distressed at the same time. Tarashev and Zhu (2006) also follow this line of reasoning in the context of pricing a basket of CDS swaps. Rather than estimating the unobservable asset values, as is done for example in Duan (1994, 2000) and Lehar (2005), we model directly the potential for joint default and correlated losses. This allows us to quantify the distribution of the potential systemic losses and the contribution of individual losses to this potential.

We add to the existing literature on systemic risk because our approach not only quantifies the dependencies between between distress occurrences, but also between the probabilities of distress and the potential default losses given a distress. Such dependencies are important for a number of reasons. First, there is sound empirical evidence that realized losses in default tend to rise in periods when risk probabilities also increase (Altman, 1989; Altman et al., 2004). Second, the potential default of a SIFI by definition will have a strong impact on other players in the industry, and not only by increasing their default risk. Since industry-wide distress often triggers fire sales, the value of the assets backing up banks' liabilities will then also be negatively affected and pushed below fair value. We, therefore, argue in this paper that reliable systemic risk estimation should also take into account the potential for Loss Given Default (LGD) dependencies.

Eventually, systemic risk will be driven by two related components: first, there is a possibility that several companies realize a credit event at the same time; and second, the magnitude of the losses of SIFIs once defaults occur are likely to be correlated too. The reasoning is that both are likely to be driven by deterioration in market conditions causing asset values to decline, and consequently default probabilities to increase while potential recovery rates to decline.

We apply the model to three distinct problems. First, we highlight potential strong interlinkages in the European banking sector by comparing the implied probabilities of joint distress between pairs of institutions. Second, we quantitatively identify systemic banks by attributing overall risk to the institutions in the regulatory portfolio and ranking them. This highlights a discrepancy in Europe between the higher required capital buffers for smaller banks than for larger banks compared to their relative contributions to systemic risk. The discrepancies correlate with similar discrepancies across different countries and regulators in the euro area. Third, we evaluate the risk of the regulatory portfolio over time and find it (in retrospect) be sensitive to the evolution of systemic risks in Europe.

The current paper continues as follows. In Section 2 we review the relevant literature. Section 3 describes the structural credit model we use to describe co-dependencies between institutions in the system. In this section, we also discuss briefly the single-name CDS contract and its potential for implying risk views on key institutions, and discuss further the credit approach to quantifying the sensitivity and the contribution of each institution to systemic risk. Section 4 reviews the empirical data including the structure of the data and the implications of the model for measuring and attributing systemic risk. In Section 5 we test the robustness of our results to various extensions and modeling changes and discuss the robustness of the results after implementing alternative specifications. In this section, we also relax the assumption of asset return normality used earlier. Finally we discuss the policy relevance of the results for the assessment of banks' capital adequacy in the context of their systemic risk contributions within the Eurozone. Finally, Section 6 concludes.

2 Related Literature

Our paper is part of a wider literature using high-frequency asset prices to inform central bank policies. Examples are Hattori et al. (2016); Olijslagers et al. (2019) who use option-implied asset volatility and risk-neutral distributions to evaluate the effectiveness of central bank stabilization policies.

More specifically, we relate closely to the literature that builds on estimated equity return correlations between financial institutions to construct measures of their contributions to systemic risk (Benoit et al., 2017). However, especially in Europe extracting information from market data in this way is not possible because some of the key players in the financial sector are not publicly traded. Approaches that rely on equity price co-movements (like Adrian and Brunnermeier (2016)) then cannot encompass the full system, cannot be used to track the systemic impact of those institutions, and may in fact not be usable at all if too few of the quantitatively important institutions have an equity market listing. For these reasons, we develop a structural approach that utilizes information from the CDS market.

Regarding the use of CDS prices, a large part of the literature relies on reduced-form statistical modeling to link spread changes to bank fragility. Avino et al. (2019), for example, look at the spreads of single-name CDS contracts for European and US banks and evaluate the propensity of spread changes to predict bank distress in the form of recapitalization or nationalization. One standard deviation increase in the CDS spread change of a bank is estimated to correspond to a 7% to 14% increase in the (physical) probability of bank distress. Annaert et al. (2013) look at the determinants of CDS spread changes for a universe of European banks and separate them into a firm-specific credit risk component, a trading liquidity component, and a business cycle components capturing common variation linked to the business environment. These studies provide an initial perspective into the usefulness of CDS rates for predicting bank failure. We take the approach further by developing a framework that extracts the probabilities of failure and the failure correlations from observed CDS spreads and feeds them into a credit portfolio model. This allows us to examine the contribution that individual banks make to total systemic risk, rather than on the isolated risk of single bank failure.

On the methodological front, Oh and Patton (2018) link bank distress to large upticks in the CDS prices of the reference banks, and measure the probability of joint distress through a dynamic factor copula dependency model. Billio et al. (2012) offer an early econometric model which quantifies interconnectedness through principal components and implied networks based on Granger-causality tests. Bräuning and Koopman (2016) extend the idea with time-varying heterogeneity in the link formation between banks using CDS spreads of US and European institutions, thus aiming to capture the dynamic formation of potential core-periphery clusters, which are natural for the financial sector. Moratis and Sakellaris (2021) on the other hand use a panel VAR model to decompose the transmission of systemic shocks across a universe of global banks. These studies offer preliminary evidence that CDS fluctuations can serve as an early warning signal of bank risk, supplementing data from the stock market, credit rating agencies, and accounting data. Our contribution to this literature is to embed CDS spreads into a structured credit portfolio model, which naturally blends the key aspects of systemic importance as size, risk, and distress dependency.

Acharya et al. (2014) use co-movements in CDS rates of sovereigns and local banks during the Euro sovereign debt crisis to show how a *doom-loop* channel evolves, in which a bail-out of a local systemic bank in trouble leads to a deterioration in the creditworthiness of the government, which in turn further depresses the credit-worthiness of the bailed-out bank due to its large exposure to local sovereign bonds.

An earlier branch of the empirical literature also uses structural firm models to imply bank fragility (Gropp et al., 2006; Chan-Lau and Gravelle, 2005; Bharath and Shumway, 2008), in particular, the distance-to-default (DD) measure introduced in (Merton, 1974; Crosbie and Bohn, 2002)), which compares the current market value of assets to the default barrier of the firm.While the foundation in our study is similar, we aim to evaluate cross-linkages and the impact each bank has on the system as a whole. We thus go a step beyond the approaches that are interested in evaluating the individual default risk of a bank in isolation.

Most of all, we relate to the broader literature on measuring systemic risk through asset price co-movements (Lehar, 2005; Segoviano and Goodhart, 2009; Zhou, 2010; Huang et al., 2012; Adrian and Brunnermeier, 2016; Brownlees and Engle, 2017; Acharya et al., 2017; Engle, 2018).

An earlier strand of the systemic literature, most notably Lehar (2005), relies on Merton's theory stating that firm equity can be viewed as a contingent claim on its assets. Merton's model is used to imply the market value of bank assets and the correlations between institutions as a measure of systemic risk. In contrast, our approach does not focus on extracting the value of assets themselves. Instead, we directly focus on extracting and modeling default correlations through common variations in the CDS prices.

The more recent approaches developed in that area can be seen as largely model-free since they do not rely on particular capital structure assumptions of the individual banks. The CoVaR approach of Adrian and Brunnermeier (2016) for example, along with an earlier study by Baur and Schulze (2009), relies on a quantile regression on equity prices to determine tail co-dependencies and risk contributions. Numerous further modifications have been provided to improve the estimation of CoVaR and to make the model more flexible towards nonlinearities in the tail dependency structure of asset returns: Girardi and Ergün (2013) suggest a multivariate GARCH approach; Reboredo and Ugolini (2015) use Copula dependency; Wang (2021) embed a neural network approach.

The CoVaR intoduced by Adrian and Brunnermeier (2016) and the MES introduced by Acharya et al. (2017) are conceptually similar in that both draw on measures used in risk management to quantify the tail dependencies between the losses of an asset and the portfolio of which it is a part. CoVaR quantifies the tail boundary for a portfolio, given that one asset is also at the boundary of its loss distribution, where the boundary is defined by a tail quantile. The quantile is known as the Value at Risk (VaR) in risk management. The MES on the other hand looks at the average loss of the asset, given that the portfolio is in its tail with potential losses above its VaR.

Two properties of the MES make it a more appealing choice for the attribution of risk across assets in a portfolio compared to CoVaR. The MES is in essence an expectation operator. Also, in evaluating MES for different assets in a portfolio, all are conditioned on the same event - the portfolio being in its tail. This allows us to show that the MESs of all assets in the portfolio, once they are weighted, add up to the portfolio's tail risk. In our interpretation, the portfolio will stand for the financial system, and each asset will represent a banking institution that is part of it.

We also relate to the literature that applies concepts from extreme value theory. An example is Zhou (2010) who computes the expected proportion of institutions in distress given a failure and uses multi-variate extreme value theory to evaluate a systemic risk ranking between banks. Using copula default dependencies, Segoviano and Goodhart (2009) define the probability of at least one more bank defaulting given a default in a particular bank (PAO). In a similar approach Bochmann et al. (2022) use the joint probability of default (JPD) between banks allowing it to vary with the financial cycle, as a measure of systemic contagion.

We view the regulatory space as a portfolio of risky loans, similar to Chan-Lau and Gravelle (2005); Huang et al. (2009, 2012); Puzanova and Düllmann (2013); Kaserer and Klein (2019). In that approach, systemic losses arise when an institution defaults and cannot cover the value of its liabilities. The tendency of particular institutions to drive systemic losses will result in a higher contribution to systemic risk.

From this perspective, the modeling tools developed by the securitization literature, typically used to value n-th to default derivatives on loan portfolios, can be applied (Hull and White, 2004; Tarashev and Zhu, 2006).[4]In particular, Tarashev and Zhu (2006) link the correlation structure embedded in CDS prices to the correlation between asset values in the Merton capital structure framework. A latent factor model driving the asset return variations can then be used to connect the default probabilities of the different institutions.

Our innovation is to also embed a model of correlated losses between the institutions. Earlier studies typically assume a fixed LGD (Puzanova and Düllmann, 2013) or assume that Recovery Rates (RR) are random but sampled independently from each other (Huang et al., 2012; Kaserer and Klein, 2019). In a tail scenario, a SIFI's default can be expected not only to raise the default risk of other participants in the sector, but also to simultaneously decrease the value of the assets backing up their liabilities. From

[4]For an earlier model of this kind, relying on equity co-movements as an approximation to asset correlations, see also Pascual et al. (2006).

that point of view, our approach of endogenizing the LGDs relates to the literature on fire sales. For example, Shleifer and Vishny (1992) argues that in times of industry-wide distress and increased default rates, assets tend to go to industry outsiders who may lack the necessary skills to manage them and will thus be willing to buy them only at a discount to fair value. As a result, LGDs will tend to rise with the drop in liquidation prices. This has been empirically observed among others by Acharya et al. (2007).[5]

We also relate to studies that compare the policy and the academic approaches for measuring systemic risk. For example, Brogi et al. (2021) compare the G-SII buffer rankings to systemic risk rankings based on Huang et al. (2012) and find significant differences in the two approaches and argue that the regulatory framework would benefit by incorporating also a risk contribution metric into generating systemic rankings. Bianchi and Sorrentino (2021), on the other hand, explore a small sample consisting of the four Italian banks designated as systemically important and largely find consistency in the ranking based on the CoVaR measure and based on the O-SII buffer rates set by the Italian central bank. Yet, having higher frequency data allows them to link systemic risk estimates to the evolution of bank characteristics and conditions.

It needs to be acknowledged that there is currently little theoretical examination on determining the size of the macroprudential capital buffers that institutions need once they are designated as systemic. The policy approach has been to recommend a two-step heuristic, where in the first step institutions are evaluated on a set of criteria associated with systemic importance, and in the second surcharges are set to equalize the impact between a systemic and a non-systemic bank. This holds both for O-SIIs and for G-SIIs. Previous studies have found that the approach is very sensitive both to the ranking and bucketing methodologies used (Brogi et al., 2021). In the methodology that we propose, it is natural to link the size of the capital surcharges directly to the measured systemic contributions. In this paper we compare the current systemic contributions to the current capitalization and O-SII buffers that banks hold. In Dimitrov and van Wijnbergen (2023)

[5]See also IJtsma and Spierdijk (2017) for a discussion of fire sales, endogenous LGDs, and the relation to systemic risk.

we develop the framework further to solve for the optimal macroprudential buffer sizes.

3 The CDS Approach

In this section we provide a modular presentation of our quantitative approach to analyzing systemic risk based on CDS prices. We first go into the structure of CDS contracts in Section 3.1. We then show in Section 3.2 how to derive empirical probabilities of default using the well known pricing model presented by Duffie (1999). Next in Section 3.3 we present a structural model of default that allows us to extract the relevant correlations that play a key role in our analysis of systemic risk from empirical estimates of default probabilities. In 3.4 we set up the basic Gaussian model of interdependencies that drives the propensity of several banks to default at the same time, using a copula-based factor setup. Finally Section 3.5 pulls everything together to derive explicit expressions for measures of systemic risk and each bank's contribution to those measures.

3.1 What is a CDS contract and why we use them

A CDS is an insurance contract, which is traded over-the-counter (OTC), and in which the protection buyer agrees to make regular payments, the CDS spread rate over a notional amount, to the protection seller. In return, the protection seller commits to compensate the buyer in case of default of the contractually referenced institution. There are multiple features of the CDS market that make it an attractive source of information on the risks which are evolving in the financial sector.

First, it is more liquid and has fewer trading frictions compared to credit traded directly through the corporate bonds market. In terms of information transmission, CDS spreads have been shown to lead bond markets, especially in distress periods, and have an edge over credit rating agencies (Bai and Collin-Dufresne, 2019; Avino et al., 2019; Culp et al., 2018; Annaert et al., 2013). This relates to the fact that in contrast to conventional asset markets, the CDS market almost by definition is composed of insiders (Acharya and Johnson, 2005). Furthermore, liquidity and transparency in the market have increased substantially in recent years. After the Financial Crisis of 2008/09, OTC derivatives, and as such also CDS contracts, became subject to increased regulatory scrutiny through the EMIR framework in Europe and the Dodd-Frank Act in the US. To improve financial stability, central clearing was introduced with increased contract standardization, and transparency was improved by introducing reporting mandates for counterparties[6].

Second, CDS prices trade on standardized terms and conditions and do not have be pre-processed through bootstrapping or interpolation as do bond yields. Also, comparison between the underlying institutions is easier, because, unlike corporate fixed-income securities, single-name CDS contracts do not contain additional noise from issue-specific covenants, such as seniority, callability, or coupon structure (Zhang et al., 2009; Culp et al., 2018).

Several general concerns regarding CDS prices need to be mentioned as well, however. First, CDS rates also price in the risk of default of the protection seller and not only the reference entity. The size of this extra premium, however, has been shown empirically to be economically negligible (Arora et al., 2012), and with the recent rise of Central Clearing for OTC derivatives it is likely to have decreased further (Loon and Zhong, 2014, 2016). Second, single-name CDS contracts are not as liquid as public equity and this raises concerns that the spreads could be overstating default risk by confounding it with an illiquidity premium. Even though the argument is valid, it misses two important points. Illiquidity risk tends to be correlated with default risk, as protection dries up at times when it is most needed (Kamga and Wilde, 2013; Augustin and Schnitzler, 2021). Also, strong illiquidity in the CDS contract even in normal times may be indicative of the market's unwillingness to fund a particular financial institution due to fears that a possible future fire sale could push it into insolvency.[7]

Overall, we take the view of Segoviano and Goodhart (2009), which they back up with empirical evidence, that even though in magnitude CDS spreads may be overreacting

[6]For an overview of the market structure, and recent regulatory reforms of the CDS market see Aldasoro and Ehlers (2018) and Paddrik and Tompaidis (2019).

[7]Cf. Diamond and Rajan (2011) and Shleifer and Vishny (1992) for a theoretical underpinning of firesales and bank assets.

to bad news in certain situations, the direction is usually justified by information on the reference institution's creditworthiness. Thus, we use the CDS mid quotes without correcting them further for non-credit related premia.

3.2 Extracting Default Probabilities from CDS Prices

We start with a short discussion on how the observed CDS prices can be used to extract the underlying banks' risk-neutral probabilities of default (PD).

Following Duffie (1999) we assume at this stage that the expected Recovery Rates (RR) are constant and known over the horizon of the contract. The goal at this point is to extract the PDs through a basic and reliable model. For this reason we do not try to capture the evolution of the RR as a separate process and accordingly we do not try to identify it separately from the observed CDS data. There are alternative and more sophisticated approaches in the literature that try to identify separately the RRs and the PDs. Yet, the simplifying assumption we employ in estimation is widely used in the literature and is hard to improve on given the identification problem that exists. At the simulation stage of the model we will relax the assumption of fixed RRs.[8][9]

We denote $CDS_{i,t}$ as the price at initiation date t of the CDS contract written on the debt of bank i. By market convention the spread is set to ensure that the value of the protection leg and the premium leg of the contract are equal, such that the contract has

[8] For example, Pan and Singleton (2008) identify separately the RR and the default intensity of the credit process exploiting the term structure of the CDS curve constructed from contracts with different maturities. Christensen (2006) models jointly the dynamics of the RR, the default intensity, and interest rate by breaking away from the standard Recovery of Market Value (RMV) approach of Duffie and Singleton (1999) according to which at default the bondholder receives a fixed fraction of the prevailing market value of the firm. Under the RMV approach, the default intensity only shows up within a product with the recovery rate, so the two cannot be identified separately. Having one collateral model when assessing LGD correlations and another one when extracting default probabilities from observed CDS spreads comes down to an inconsistency that is well known in the literature (see Tarashev and Zhu (2006)'s discussion of precisely this issue).

[9] Furthermore, we should point out that we are ignoring correlation risk premia. We rely on evidence provided by Tarashev and Zhu (2006) that such premia, if they exist at all, are quantitatively very small in CDS prices.

a zero value at time date t:

$$\underbrace{CDS_{i,t} \int_{t}^{t+T_{CDS}} e^{-r_\tau \tau} \Gamma_{i,\tau} d\tau}_{\text{PV of CDS premia}} = \underbrace{(1 - ERR_{i,t}) \int_{t}^{t+T_{CDS}} e^{-r_\tau \tau} q_{i,\tau} d\tau}_{\text{PV of protection payment}} \tag{1}$$

where T_{CDS} is the term of the contract in years, r_τ is the risk-free rate, $CDS_{i,t}$ is the observed CDS spread for a contract traded on day t with an underlying bank i, $q_{i,\tau}$ is the implied annualized instantaneous risk-neutral default probability for the bank, $\Gamma_{i,\tau} = 1 - \int_t^\tau q_{i,s} ds$ is the risk-neutral survival probability until time τ, and $ERR_{i,t}$ is the expected recovery rate in case of default, assumed to be constant over time.

For simplicity, we assume that the risk-free rate r_τ and the annualized default rate $q_{i,\tau}$ are fixed and known at the initiation of the contract. Then the default probability at time t follows from equation (1):

$$q_{i,t} = \frac{aCDS_{i,t}}{a(1 - ERR_{i,t}) + bCDS_{i,t}} \tag{2}$$

with $a = \int_t^{t+T_{CDS}} e^{-r\tau} d\tau$ and $b = \int_t^{t+T_{CDS}} \tau e^{-r\tau} d\tau$. Setting $T_{CDS} = 5$ to capture 5-year CDS contracts, we can imply the annualized default probabilities.[10]

3.3 A Structural Model of Default

Next, we define the structural credit risk model behind the occurrence of systemic losses. Key here will be the assumption driving asset value correlations, as it will effectively determine the correlations in the default probabilities of banks, and in their default losses.

We start from Merton (1974) and describe the evolution of the value of assets of each bank $i = 1, ..., n$ under the risk-neutral measure through a standard GBM process with

[10] In credit risk (and more generally in survival analysis), the variable q relates to the *hazard rate*, the constant arrival rate (in a Poisson sense) of a credit event. At any instant, given that default has not yet occurred, the time until it does is exponentially distributed with parameter q. For a small Δt and small q, the probability of default is then $\Delta t \cdot q$. See Duffie (1999) for details.

drift:

$$d \ln V_{i,t} = rdt + \sigma_i dW_{i,t} \tag{3}$$

where r is the risk-free rate, σ_i is the variance of asset returns, and W_t is a Brownian Motion.

In Merton's setting, default occurs at maturity $(t+T)$ when a firm's assets fall below the face value of its debt such that:

$$\begin{aligned} PD_{i,t} &= \mathbb{P}(V_{i,t+T} \leq D_i) \\ &= \mathbb{P}\left(V_{i,t} \exp\left((r - \frac{\sigma_i^2}{2})T + \sigma_i W_{i,t+T}\right) \leq D_i\right) \end{aligned}$$

Consider next the Distance-to-Default (DD) measure:

$$DD_{i,t} = \frac{\ln \frac{V_{i,t}}{D_i} + \left(r - \frac{\sigma_i^2}{2}\right) T}{\sigma_i \sqrt{T}} \tag{4}$$

which allows us to rewrite the expression for the probability of default as:

$$PD_{i,t} = \mathbb{P}\left(\underbrace{\frac{W_{i,t+T}}{\sqrt{T}}}_{\equiv U_i} \leq \underbrace{-DD_{i,t}}_{\equiv X_{i,t}}\right)$$

The relationship above provides a bridge between Merton's structural default model to the class of latent-variable default threshold models used in securitization.[11] The random variable U_i can be interpreted as a the standardized asset return over the coming one-year period, and $X_{i,t}$ as the standardized asset loss threshold below which the firm would default. In our baseline model, U_i follows a standard normal distribution in line with the Merton model assumptions in (3). In Section 5.2 we explore alternative distributional specifications for this variable, deviating from the original Merton specification.

[11] Cf. Bolder (2018) and McNeil and Embrechts (2005).

Assume going forward that the maturity of the firm's debt is one year from the current date so that $T = 1$. Then, we can write the one-year ahead probability of default as:

$$PD_{i,t} = \mathbb{P}(U_i < -DD_{i,t}) = \Phi(-DD_{i,t}) \tag{5}$$

where $\Phi(.)$ is the cumulative standard normal distribution. Note that by inverting this relation, we can infer from observing the CDS spread for the day (correspondingly the default probability) the default barrier

$$X_{i,t} \equiv -DD_{i,t} = \Phi^{-1}(PD_{i,t}) \tag{6}$$

Any realization of the variable U_i below the threshold $X_{i,t}$ would indicate a default of bank i.

Furthermore, we can then relate the default under the Merton model to the PD implied from the CDS spreads in Equation (1) by setting $PD_i \equiv q_{i,t}$ from Equation (2). This essentially implies that we are working under the risk-neutral distribution of default and allows us to determine Merton's DD based on the observed CDS prices.

The next step is to determine how the asset value changes between different institutions correlate. For this purpose, note first that based on (4) we can link changes in a bank's DD between two periods to the log changes in the unobserved bank's market asset values. The discrete first difference of the $DD_{i,t}$ becomes:

$$\Delta DD_{i,t} = \frac{\Delta \ln V_{i,t}}{\sigma_i}$$

The correlation between asset returns can be written as:

$$\begin{aligned} \rho_{i,j} &= \mathbb{Corr}(\Delta \ln V_{i,t}, \Delta \ln V_{j,t}) \\ &= \mathbb{Corr}(\sigma_i \Delta DD_{i,t}, \sigma_j \Delta DD_{j,t}) \end{aligned} \tag{7}$$

Correlations are invariant to linear transformation, so we can drop the σ terms. Then after substituting in the inverted relationship (5), the asset correlations can be implied from the correlations between the transformed probabilities of default:

$$\rho_{i,j} = \mathbb{Corr}\left(\Delta\Phi^{-1}(PD_{i,t}), \Delta\Phi^{-1}(PD_{j,t})\right) \tag{8}$$

This equation is of crucial importance. Combining (7) with (8) we relate the co-dependencies in changes in the (transformed) probabilities of default (PDs) to the unobserved asset return correlations of the underlying banks.

This allows us to use PDs that can be derived from observed single-name CDS prices to pinpoint values for the correlations between institutions. In the following section, we discuss in detail how these asset correlations can be used as targets against which to estimate the parameters of a factor model.

Our reliance on the Merton (1974) framework implies that we assume default to occur when a fixed default barrier is crossed at debt maturity. Further refinements have been developed to relax this assumption, of which we mention in particular Leland (1994) who endogenizes the default barrier and defines it as the boundary beyond which equity holders refuse to supply new equity to avoid default.

Even though the Merton framework may be conceptually restrictive, it is widely used as a raw approximation of default. The related Merton-based DD has a wide application to risk management as a predictable indicator of bank fragility (Gropp et al., 2006; Chan-Lau and Sy, 2007), and actual defaults (Bharath and Shumway, 2008). Jessen and Lando (2015) also shows that it has certain robustness against model misspecification. As a result, we do not pursue any of the structural extensions in this study.

3.4 Modeling Interdependencies: the Baseline Gaussian Model

Next, we turn to the model of dependencies in the creditworthiness of individual banks. This in turn will determine the propensity of several banks to default at the same time, thus driving a key component of the systemic risk model. We start with the Normal setting consistent with the Merton specification provided earlier. The following factor setup is known in credit risk analysis as a Gaussian Factor Copula.

Default Dependencies: It is reasonable to assume that part of the bank's asset risk U_i is driven by a set of common factors, and that part of it is entity-specific. The most widely used approach in credit risk analysis is to model default dependency by specifying a Gaussian factor model of the form

$$U_i = A_i M + \sqrt{1 - A_i A_i'} Z_i \tag{9}$$

where $M = [m_1, \ldots, m_f]'$ is the vector of stochastic systematic factors, and Z_i is the firm-specific factor, each of which follows a standard normal distribution. $A_i = [\alpha_{i,1}, ..., \alpha_{i,f}]$ is the vector of factor loadings, such that $A_i A_i' \leq 1$.[12] All factors are assumed to be mutually independent with zero mean and a standard deviation of one. All factors M and Z_i are characterized by standard normal distributions.

We do not provide a concrete interpretation of the factors, even though they can be thought of as economy, industry, or geographically specific risk drivers.[13] Instead, we use a statistical procedure to extract the exposures of the individual banks to the factors by observing the common components in the default probability variation across all banks in our universe.

In a Gaussian framework, the asset return dependencies are linear and can be fully captured by the correlation between the latent variables determining the creditworthiness of two banks. In turn, the correlation can be expressed in terms of the banks' exposures to the common factors

$$\mathbb{Corr}(U_i, U_j) = A_i A_j'$$

Note that if we assume that there is a single risk driver and all banks have the same exposure to that driver we would get as a special case the well-known Vasicek loan

[12] Here we follow the convention from the securitization literature where M is a column vector and A_i is a row vector.

[13] Cf. Pascual et al. (2006) for an attempt at factor identification in a similar credit risk framework.

pricing model (Vasicek, 1987). In the approach used here, however, we allow for exposure heterogeneity.

Loss Dependencies: The next building block of the model is to determine the size of the potential losses if a default were to occur. A common simplifying assumption in the systemic risk literature to which we relate is that the RR is either fixed (Puzanova and Düllmann, 2013) or stochastic but independent across firms and from the realization of default (Huang et al., 2009, 2012; Kaserer and Klein, 2019).Relying on strong assumptions about default losses is inevitable, as bank defaults, and especially defaults of SIFIs, are rarely observed. Yet, there is strong empirical evidence that as default rates in the economy increase, the recovery values on assets decrease (Altman et al., 2004; Acharya et al., 2007). We address this stylized fact by allowing default losses to be dependent on the latent factors driving asset correlations. Accounting for this will inevitably have significant consequences for the quantification of systemic risk which naturally depends on the tail risk dependencies between institutions.

To do so, we follow Frye (2000) and Andersen and Sidenius (2005) and model the RRs based on the value of a stochastic collateral process $C_{i,t}$ per euro of liabilities. The total collateral backs up the bank's liabilities. Dependency between the RR and the PD is then achieved by making the value of the collateral dependent on the same set of factors that drive the asset value processes. In particular, we define the stochastic changes in the collateral value over the coming year as:

$$d \ln C_i = \sigma_c U_i^c \tag{10}$$

where U_i^c is a standard normal variable, and σ_c is a scaling parameter determining consequently the variance of the RR.

We assume that the variation in the value of the collateral is driven by the same common factors defining the asset correlations in (9) with the same factor exposures of the bank estimated from the CDS data. This seems reasonable, as the collateral for the bank's liabilities after all has to correspond to the market value of the bank's assets. Formally, therefore, we have

$$U_i^c = A_i M + \sqrt{1 - A_i A_i'} Z_i^c \tag{11}$$

where Z_i^c defines an independent factor capturing possible firm-specific discrepancies between the underlying assets of the firm and the value of recovered collateral. This discrepancy could be due to a loss in the value of the bank's intangible assets, any other restructuring costs due to liquidation, or legal delays in seizing the collateral.

Finally, we make sure that in case of default, the recovery rate (RR_i) as a proportion of liabilities is never larger than 100% of the recovered liabilities, so we can write the realization of the RR as:

$$\begin{aligned} RR_i &= ERR \min(1, C_i) \\ &= ERR \min\left(1, \exp\{\sigma_c U_i^c\}\right) \end{aligned} \tag{12}$$

where ERR and σ_c are calibrated to match the assumption of the expectation and the variance of the RRs. This is discussed in detail in Section 4.1.

Factor Estimation: The structural model of Section 3.4 allowed us to translate the co-movements in the CDS prices into correlations between banks' asset value changes. As a result, in estimating the latent factor model, it is sensible to pick the factor coefficients which minimize the distance between the implied asset correlations $\rho_{i,j}$ estimated from co-movements in the transformed default probabilities (cf. Equation 7) and the factor model implied correlations $A_i A_j'$. We do this by minimizing the sum of the squared differences between the two:

$$\min_{A_1,\ldots,A_n} \sum_{i=2}^{N} \sum_{j=1}^{N} (\rho_{ij} - A_i A_j')^2 \tag{13}$$

Andersen and Basu (2003) develop an algorithm which solves this minimization problem numerically in an efficient way through an iteration over the asset correlations' principal components. This avoids a costly direct minimization over all factor model parameters. The algorithm is outlined in Annex B.1.[14]

3.5 Measuring Systemic Risk

We now have the machinery in place to start modeling systemic risk. We define systemic risk as the potential for large default losses in the banking system. A single entity's contribution to systemic risk then will be measured as its propensity to increase that potential. To capture these effects, we model the universe of supervised institutions as a structured credit portfolio.

Several elements can thus drive the systemic risk contributions of an institution: first, both increases in the default probability and decreases in the proportion that can be recovered in case of default; second, the size of the institution, measured by its outstanding liabilities relative to the size of others; third, the propensity of the institution to become distressed or to realize large losses whenever other institutions in the portfolio are distressed.

An institution becomes distressed if a credit event occurs in its subordinated debt. From the point of view of a regulator, each institution's total liability amounts to the regulator's Exposure at Default (EAD). A fraction of the EAD is lost whenever an institution defaults and cannot deliver the full promise of its outstanding liabilities to its counterparties. Thus, formally, we define the default loss on an individual bank, scaled by the size of its liabilities, as

$$L_i = \mathbb{1}_i(1 - RR_i) \tag{14}$$

where RR_i is defined in (12) and $\mathbb{1}_i$ is a stochastic default indicator behaving in line with

[14]For an alternative approach using Kalman Filtering techniques see Tarashev and Zhu (2006). As they show, the two extraction methods produce practically the same results.

the Merton model in Section 3.3, such that

$$\mathbb{1}_i = \begin{cases} 1 & \text{if } U_i \leq -DD_i \\ 0 & \text{otherwise} \end{cases} \tag{15}$$

So, overall, the loss will be zero if bank i does not default and will be equal to the random realization of the RR if the bank does default.

With the distribution of losses known, we can evaluate risk through *Expected Shortfall* (ES), which measures the average losses of a bank or portfolio of banks in the worst α-th percentile of its potential loss distribution:

$$ES_i = \mathbb{E}(L_i | L_i \geq VaR_i) \tag{16}$$

where VaR_i stands for the Value-at-Risk of the institution at confidence level $1 - \alpha$:

$$\mathbb{P}(L_i \geq VaR_i) = \alpha$$

Typically, α stands for the tail probability and takes values of e.g. $5\%, 1\%, .01\%$ depending on how far in the tail we want to measure the potential for extreme losses. Then, given the potential loss distribution, we are $(1 - \alpha)\%$ certain that losses will not exceed the corresponding VaR estimate.

The ES thus measures the average loss once the VaR-threshold of an institution has been exceeded.[15] It quantifies the potential losses that could occur if an institution is distressed. However, it does not take into account correlated losses and the fact that

[15] An appealing feature of the ES is that it is coherent, in the sense of Artzner (1999), and thus allows for capturing diversification in an intuitive way when the losses of a portfolio are aggregated. The set of coherent risk measures are defined axiomatically through a number of intuitive properties: (1) *Monotonicity*: comparing several random payoffs, lower losses in all states of nature imply lower risk; (2) *Positivide homogeneity*: scaling a portfolio random payoff by a positive factor also scales its risk by the same factor; (3) *Sub-additivity*: the risk of the portfolio is not greater than the sum of the risks of the assets which comprise it; (4) *Invariance*: adding cash to a portfolio reduces its risk by the amount added. ES covers all of the properties, while VaR fails at sub-additivity. In fact, functionals that satisfy (2) and (3) are convex, a feature that defines mathematically the concept of diversification in modern portfolio theory (Rachev et al., 2008).

distress in one institution may correlate with or even cause the failure of other institutions. From a macroprudential point of view, we thus need to define a measure that does take this co-dependency into account.

For this reason, we define total systemic loss L_{sys} as the weighted sum of the individual losses of each bank:

$$L_{sys} = \sum_{i=1}^{n} w_i L_i \tag{17}$$

where $w_i = \frac{B_i}{\sum_{j=1}^{N} B_j}$ is the relative weight of the institution's liabilities (B_i) in the systemic portfolio.

Then, we follow Acharya et al. (2017) to capture a bank's systemic risk sensitivity through its Marginal Expected Shortfall (MES), which is the average loss of institution i given that the systemic portfolio is in the worst α-th percentile of its distribution of potential losses:

$$MES_i = \mathbb{E}\left(L_i | L_{sys} \geq VaR_{sys}\right) \tag{18}$$

One can easily show that the weighted sum of all MESs in the portfolio provides the ES of the system.[16]This follows from (17) and (16):

$$\begin{aligned} ES_{sys} =& \mathbb{E}\left(\sum_i w_i L_i | L_{sys} \geq VaR_{sys}\right) \\ =& \sum_i w_i \mathbb{E}\left(L_i | L_{sys} \geq VaR_{sys}\right) \\ =& \sum_i w_i MES_i \end{aligned} \tag{19}$$

This additivity property allows us to break down the total ES of the systemic portfolio into shares of the total risk attributable to each bank. We thus define Percentage

Contribution to ES (PCES) as:

$$PCES_i = \frac{w_i MES_i}{ES_{sys}} \tag{20}$$

which will be a useful metric further on in attributing risk across institutions and ranking them by systemic importance.

3.6 Model Simulation

Finally, we assemble all modelling pieces together and perform a Monte Carlo simulation in order to evaluate systemic risk and attribute it across banks. On a step-by-step basis is done as follows:

- We draw 500K independent simulation scenarios for the idiosyncratic and the common factors of Equation (9). Based on the estimated factor exposures we can translate the factor simulations into scenarios of standardized asset value changes over the coming year for all assets. This provides the multivariate distribution of the creditworthiness variables (U_i for all banks) where the common factor exposures will drive co-movements in those variables.

- The default boundary per each bank has been derived by inverting the default probability in Equation 6. In the Monte Carlo approach we can then evaluate in each set of simulated scenarios if any (and potentially how many) banks would default. The average number of joint default scenarios then gives us the overall probability that multiple banks jointly fall into default.

- In addition, based on factor scenarios, we can also simulate the potential correlated losses in case of default for each bank as indicated in Equation 12. The steps so far provide a multivariate distribution of the potential losses across all banks.

[16]Note that (19) implies also that the MES measure can be interpreted as the sensitivity of the system's tail risk to the weight of the institution in the portfolio since $\frac{\partial ES_{sys}}{\partial w_i} = MES_i$.

- By applying liability weights for each bank, we estimate the distribution of potential losses for the regulatory portfolio.

- Based on the distribution of individual bank losses and the portfolio loss, we evaluate MES through Equation 18. Consequently we can evaluate aggregate systemic risk (ESS, cf. Equation (16)) for the portfolio, and attribute it across the institutions in the system via the PCES breakdown (cf. Equation (20)).

4 Empirical Analysis

In this section we show three applications of the model presented so far: first, we use it to evaluate the dependencies of potential distress between banks, especially by examining the estimated factor model exposures of Section 3.4 and the consequent probabilities of joint default; second, we use the model to attribute systemic risk over all banks in the regulatory portfolio in line with Section 3.5; third, we we use the model to provide an aggregated indicator which can track the evolution of systemic risk factors over time. We apply the model to a universe of key banks from the Eurozone. First, we go through a description of the dataset, and then we outline the results.

4.1 Data and Parameter Assumptions

We consider 27 large European institutions for which CDS rates are available (Cf. Table (5) in the Annex for a list of the banks included in the analysis). We use weekly mid prices for ISDA2014-compliant CDS contracts on the subordinated debt of the banks. Five-year CDS rates are used for all banks. The data is collected from Bloomberg.

About a third of the banks included in our universe are not publicly traded.This latter group consists of coöperative banks such as France's *CRMU*, Germany's *DZ, BAY, LBBW, HESL*; Netherland's state-owned *VB*; and private banks such as *RABO* and *INGB*. ABNAMRO's equity has been re-listed in 2015 after earlier government intervention, and only just for a minority share. Using CDS rates allows us to include them in the analysis, and as a result to get a more complete picture of the financial system. Note that we use the CDS rate for ING Bank (*INGB*), a subsidiary of ING Group (which is publicly traded). *INGB* operates mostly in Europe and thus is more relevant for European regulators. The availability of this CDS contract allows us to focus more accurately on the risks embedded in the European operations of the bank.

For five banks, domiciled in Germany and Austria, only CDS rates on their senior debt are available. Senior debt is lower-risk than the subordinated (SUB) debt and is sensitive only to very large shocks. As a result, senior CDS rates are lower and less responsive to news compared to the subordinated CDS rates of the same issuer. To ensure that these five banks are on the same footing as the rest of the universe we add to each of them the median cross-sectional spread between the subordinate and senior CDS prices the period (excluding banks domiciled in Italy and Spain).

Annual balance sheet data is collected from FactSet and from publicly available financial statements of the firms whenever the data provider has a gap. The annual numbers are interpolated to weekly with a cubic spline to avoid jumps at fiscal year-end, driven by accounting conventions.

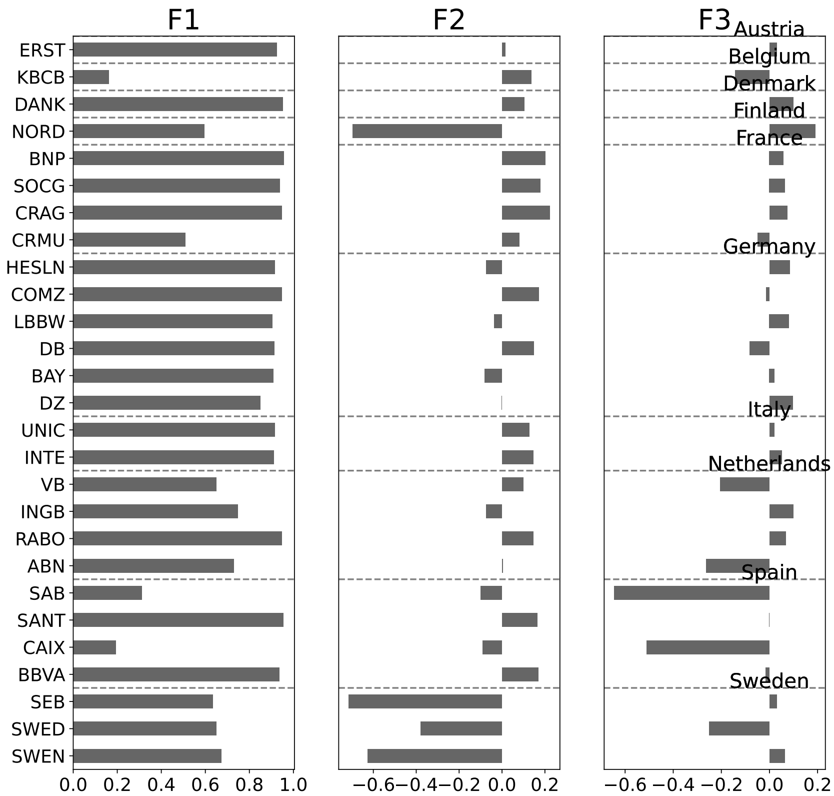
The choice of expected RR is largely arbitrary in the literature. Kaserer and Klein (2019); Huang et al. (2012); Black et al. (2016) calibrate it to survey data from Markit and find that expectations do not vary significantly over time and stay between 30% and 40%.. Puzanova and Düllmann (2013) use a conservative 0% recovery. We use an expected RR assumption (ERR) of 60% for all banks. In any case, the LGD assumption made here affects the *levels* of the estimated PD and the MES numbers. We are interested, however, in *changes over time* of the risk trends, and in the relative risk attribution (the $PCES$) across banks. As a result, the LGD assumption here is immaterial for our results.

Finally, in the RR process defined in (12), we assume a homogeneous $\sigma_c = .5$ for all banks. This produces a roughly 30% standard deviation in the simulated RR, which is in line with related studies (cf. Huang et al. (2012)).

4.2 Assessing Systemic Dependencies

Factor Exposures: The first building block for evaluating the potential systemic losses relies on the estimation of the latent factor model. This is also the first application of the model that we will look into. For the corresponding analysis, we use the period from August 31st, 2019 to August 29th, 2022 to estimate the model.

Figure 1: Factor Loadings



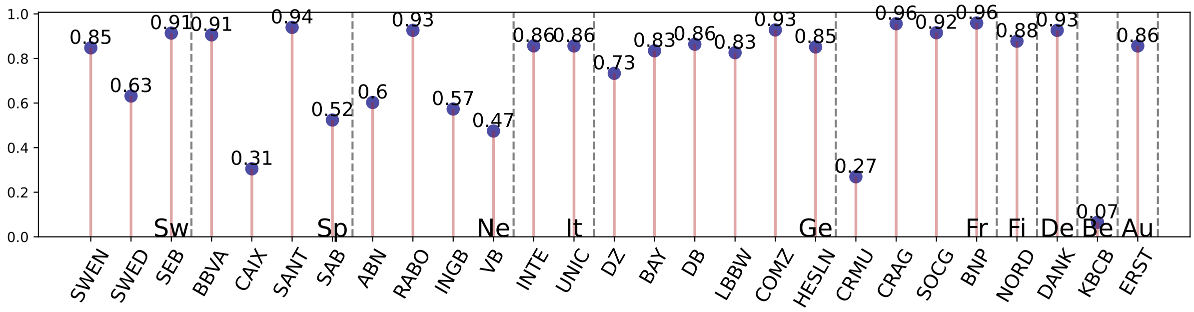
Note. This figure shows the estimated factor loading for the universe of banks, ranked by domicile country.

We use a three-factor specification of equation (9).[17] Figure 1 presents the estimated factor loadings for each bank. The clustering of exposures across banks provides indications on how to interpret the latent factors. The first factor (F1 on Chart 1) accounts for

[17]In determining the number of factors to be included , we are looking for the smallest number that can explain the bulk of the co-variation in the data. To find out we run a Principal Component Analysis (PCA) directly on the universe of CDS rates and observe that the first three principal components (PC) explain cumulatively about 80% of the variance of the weekly CDS prices changes (cf. Figure (11) in Annex A.2). After the third factor, the incremental explained variance from each next factor becomes marginal and less than 1% (the dotted blue curve).

most of the joint co-variation in all banks' asset values. As a result, it can be interpreted as the banks' exposure to an overall market factor. We can see that all factor loadings are positive, and with few exceptions close to the upper bound of one. The second factor allows for co-variation between banks which is distinct from the overall market. Here, there is a notable clustering of similar exposures for banks from Sweden and Finland. Finally, the third factor captures any residual co-variation for banks which are distinct from the first two groups. Here we can see a couple of Spanish banks with common exposure.

Figure 2: Share of Common Factor Risk



Note. This figure shows the share of common factor risk for each bank in the universe, where a share close to one indicates that most of the variance of the bank's assets is factor-driven, rather than idiosyncratic.

Through the factor loadings, we can also evaluate the share of total asset return variation for each bank that is due to common risk vs. the share attributable to idiosyncratic risk. Formally, the inner product of the factor loadings for each bank indicates the proportion of factor risk as:

$$
\begin{aligned}
\frac{\mathbb{V}\mathrm{ar}(\Delta \ln V_i)}{\sigma_i} &= \mathbb{V}\mathrm{ar}(\Delta W_i) \\
&= A_i A_i' \mathbb{V}\mathrm{ar}(M_i) + (1 - A_i A_i') \mathbb{V}\mathrm{ar}(Z_i) \\
&= \underbrace{A_i A_i'}_{\text{Factor Risk Share}} + \underbrace{(1 - A_i A_i')}_{\text{Idiosyncratic Risk Share}} = 1
\end{aligned} \tag{21}
$$

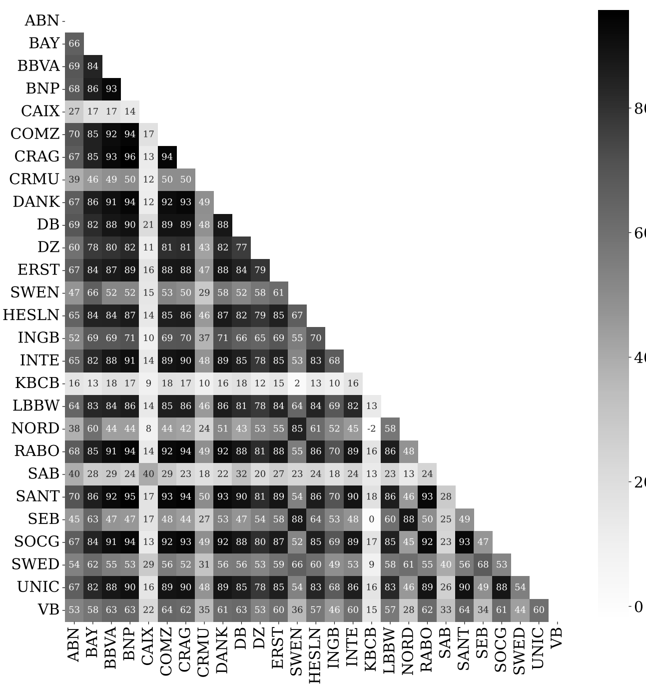
This can be seen as an initial crude estimate of the systemic sensitivity implied in an institution's assets. The higher the share of a bank's factor risk, i.e. the closer it is to one, the more its assets will tend to co-move together with the rest of the universe. The closer the share is to zero, the more the bank's risk is driven by idiosyncratic. Figure 1 shows the estimates for each bank in the sample. Among the most sensitive institutions are *BNP, CRAG, SANT, COMZ* and *RABO*.

Asset Correlations and Probabilities of Joint Defaults Next, we take a closer look at the dependencies that the model implies. Figure 3 shows the implied correlations between banks' assets, based on the transformation implied by the multivariate Merton model in Equation (8).

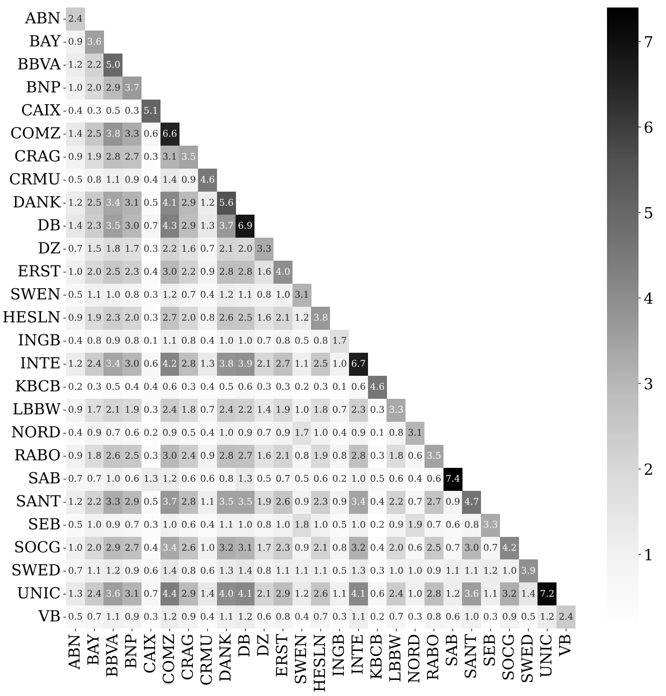
First of all, banks that have high market exposures (i.e. exposure to F1 closer to one in Figure 1) also have high implied correlations. Most notably, this includes the cluster of German, French, Italian, and Dutch banks (*BAY, BBVA, BNP, DANK, DB, DZ, EST, RABO, SANT, UNIC*).

In addition, the multifactor model allows for certain network effects to crystallize. This can be seen again in Figure 3 where clusters of correlations due to exposures to lower order factors show up. For example, *SEB, SWEN, SWED* and *NORD* appear to be highly correlated among each other, while they otherwise have low correlation to all other banks in the universe. This is due to their relatively low correlation to the market factor (F1) and the high exposure to the second factor (F2). Similarly, *CAIX* and *SAB*, the two banks with the highest exposure to factor F3, appear to have a relatively high correlation to each other, even though other correlations are close to zero.

Figure 3: Implied Dependencies(%)



(a) Asset Correlations

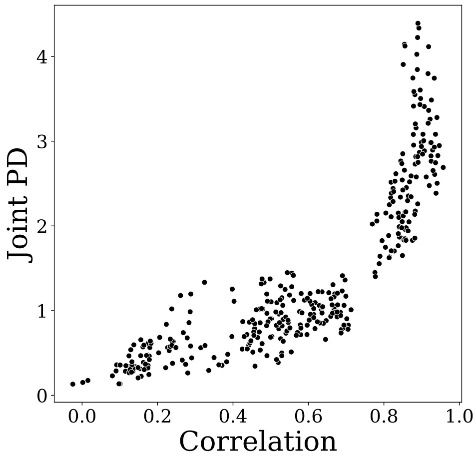


(b) Joint Probability of Default (%)

Next, we can examine the probabilities of joint default (JPD) between banks. In comparison to the asset correlations, now we also factor in the probability that banks may actually become distressed at the same time. The underlying intuition is that high asset correlation by itself is not necessarily an indication of systemic distress, as long as the probability of any of the bank pairs to be distressed is low. In that sense, some clusters of high correlations observed earlier do not lead to significant joint default probability. Figure 3b shows the estimated JPDs as the off-diagonal terms of the plotted matrix. We can see, for example, the relatively high joint distress rate between *COMZB, SANT, UNIC, INTE, DB* and *DANK*. On the other side are *CRAG* and *INGB* which have relatively high correlations to the rest. Yet having relatively low own default rates they do not show up on top of the ranking in terms of joint default potential.

We can translate the joint probabilities into probabilities of one institution's default

Figure 4: Dependency Pairs



Note. This figure shows a scatterplot of each pair of banks in terms of the estimated asset correlation and probability of joint default.

conditional on the default of another institution using the relationship:

$$CPD_{i|j} \equiv \mathbb{P}(\mathbb{1}_{d_i} = 1 | \mathbb{1}_{d_j} = 1) = \frac{\mathbb{P}(\mathbb{1}_{d_i} = 1, \mathbb{1}_{d_j} = 1)}{\mathbb{P}(\mathbb{1}_{d_j} = 1)}$$

Through the definition above we can also see that as long as the probability of bank j to default is low, observing a high probability of joint distress between i and j would indicate high potential for i to go down given that j goes down. Figure (12) in Annex A.3 provides the detailed results.

When considering these results several aspects of the derived distress probabilities should be kept in mind to avoid misinterpretation. First, they are risk-neutral and should not be interpreted as physical probabilities of default. Since asset risk premia are typically positive, the risk-neutral estimates can be expected to be more conservative than real-world default probabilities.[18] Second, the magnitude of the probabilities depends on the size of the expected RR assumption. For a given observed CDS spread a lower RR

[18]The discrepancy between risk-neutral and physical default probabilities has been noted among others in Altman (1989); Hull et al. (2005). Translating from one to the other requires an estimation of the respective risk premia. This is not a trivial task, especially considering the time-varying nature of risk premia. There is ongoing research in this area. See Ross (2015); Bekaert et al. (2022) for some suggested approaches, and Figlewski (2018); Cuesdeanu and Jackwerth (2018) for an overview of the challenges. Heyndericks et al. (2016) compare risk-neutral densities estimated from European CDS contracts to physical densities derived from rating agencies.

assumption in (2) implies a higher default probability.

We are, however, first of all, interested in highlighting dependencies between banks in the context of the systemic. Our focus is on risk attribution, not on measuring actual physical default probabilities nor in predicting the average number of defaults over the coming period. Therefore we can refrain from taking a stance on the formation of risk premia and from attempts to extract the physical default probability from asset prices. Moreover, we are more interested in structural changes in systemic risk over time, rather than in the absolute level of the risk estimate. From that point of view, focusing on the risk-neutral distribution may even be beneficial, as it captures purely structural factors and excludes shifts in the estimates due to for example changes in risk perceptions.

Finally, we highlight a specific aspect of systemic risk: even though the asset correlations and joint default measures largely agree in rankings (cf. Annex A.4 for an evaluation of the rank correlations better pairs), higher asset correlations tend to be associated with exponentially higher probability of joint distress as shown Figure 4. By considering the JPD rather than pure asset correlations, we can capture

4.3 Systemic Risk Attribution and Capital Requirements

In the introduction, we noted that there exists an apparent large disconnect between the academic and regulatory approaches used to measure systemic risk. The academic approach, as we saw in Section 2, favors the use of market data and asset pricing methods. Regulators on the other hand rely on balance sheet and regulatory data. In particular, for European regulators, the general guidance by the EBA is to focus on several criteria of systemic relevance such as size, importance, complexity, and interconnectedness (EBA, 2020). At a national level, a score is provided in each systemic category and the four categories are weighted up to a single O-SII score number.[19]

The mapping from O-SII scores to capital buffers, however, is gives local regulators in Europe significant discretion on modelling choices (Cf. ESRB (2017)).[20] The method-

[19]O-SII stands for "Other systemically important institutions".

[20]The mapping technology may vary significantly: either a direct mapping can be applied with banks

ology developed in this paper allows us to compare banks' capitalization rates across countries to their contributions to systemic risk through an objective approach, independent of regulatory scores and mapping assumptions. This puts countries on the same level playing field, and thus gives us the possibility to focus purely on banks' systemic importance.

First, we proceed to attribute the overall systemic risk to individual banks. Now we will take into account the fact that the default of larger institutions is likely to have a larger impact on systemic losses, first through wider repercussions on the economy, and second by the higher potential bail-out cost for the regulator.

Table 1 shows the details. First, we show the standalone tail risk for each bank, measured by its ES. Second, we also provide each bank's sensitivity to systemic risk, assessed through its MES, following Acharya et al. (2017). The percentage contribution figures (PCES) defined in (20) then evaluate the share of the overall risk that can be attributed to each individual bank in line with Equation (20).

Note that the five largest banks in the sample, *BNP, CRAG, SANT, SOCG* and *DB* account for 55% of the downside risk in the system, measured by $PCES$ (while they account for about 45% of the outstanding liabilities in the sample, measured in Table 1 by w). From a policy point of view, we would like to compare their capitalization to that of banks ranked as less systemic, i.e. with lower PCES.

Figure 5a shows the results: in the figure, we plot the regulatory O-SII buffer rates against the corresponding $PCES$ numbers. The results are striking: although there seems to be a strong positive relationship between the two approaches for most banks, there is a cluster of large banks which does not fit the pattern in that their buffers seem low by comparison to the rest of the sample. The cluster is located on the right side of the chart and consists of the four largest banks in our universe, three of them domiciled in France *SOCG, CRAG, BNP*, and one domiciled in Spain *SANT*. Location on the right side of the

bucketed based on their O-SII score, and with O-SII capital buffer surcharges applied in each bucket accordingly; or an indirect approach may be applied aiming to equalize the score between a systemic and a non-systemic bank weighted by each bank's default probability. And even when the same methodology is applied, the choice of assumptions and parameters may still create a source of discrepancy.

Table 1: Systemic Risk Attribution Ranking (Gaussian Model)

Short Code	w		EL		ES		MES		$PCES$	
BNP	13.24	(1)	1.48	(17)	81.25	(2)	80.12	(1)	16.33	(1)
CRAG	10.51	(2)	1.43	(19)	81.28	(1)	79.85	(3)	12.93	(2)
SANT	7.87	(3)	1.89	(9)	81.24	(3)	79.97	(2)	9.70	(3)
SOCG	7.32	(4)	1.72	(12)	81.19	(6)	78.63	(7)	8.86	(4)
DB	6.64	(5)	2.72	(3)	81.10	(8)	78.25	(9)	8.00	(5)
INTE	5.28	(6)	2.70	(4)	81.10	(7)	78.40	(8)	6.37	(6)
UNIC	4.49	(8)	2.96	(2)	80.94	(12)	78.00	(10)	5.39	(7)
BBVA	3.22	(11)	2.02	(7)	81.23	(4)	79.08	(6)	3.93	(8)
RABO	3.15	(12)	1.43	(18)	81.22	(5)	77.75	(11)	3.77	(9)
DANK	2.66	(15)	2.29	(6)	81.04	(10)	79.74	(4)	3.26	(10)
DZ	3.14	(13)	1.36	(21)	79.34	(19)	64.70	(16)	3.13	(11)
COMZ	2.33	(16)	2.65	(5)	81.07	(9)	79.57	(5)	2.85	(12)
INGB	4.71	(7)	0.68	(27)	72.66	(24)	34.28	(19)	2.49	(13)
ERST	1.51	(21)	1.61	(13)	80.84	(13)	74.81	(12)	1.74	(14)
LBBW	1.41	(22)	1.38	(20)	80.54	(16)	68.50	(15)	1.49	(15)
BAY	1.34	(23)	1.48	(17)	80.54	(17)	70.75	(14)	1.46	(16)
CRMU	4.15	(9)	1.83	(11)	71.54	(26)	22.70	(24)	1.45	(17)
NORD	2.82	(14)	1.21	(24)	80.75	(14)	28.08	(23)	1.22	(18)
HESLN	1.07	(26)	1.52	(14)	80.57	(15)	72.46	(13)	1.19	(19)
ABN	1.99	(17)	0.98	(25)	75.84	(22)	36.84	(17)	1.13	(20)
SWEN	1.61	(19)	1.24	(23)	80.44	(18)	34.77	(18)	0.86	(21)
SEB	1.59	(20)	1.28	(22)	80.99	(11)	31.78	(21)	0.78	(22)
SWED	1.32	(24)	1.49	(15)	78.67	(20)	33.97	(20)	0.69	(23)
CAIX	3.38	(10)	1.98	(8)	72.26	(25)	8.01	(26)	0.42	(24)
SAB	1.25	(25)	2.98	(1)	78.54	(21)	13.99	(25)	0.27	(25)
VB	0.34	(27)	0.90	(26)	73.09	(23)	29.00	(22)	0.15	(26)
KBCB	1.67	(18)	1.89	(10)	64.96	(27)	5.37	(27)	0.14	(27)
System	100		1.79		64.92		64.92		100	

Note. This table shows the liability weight (w), Expected Loss (EL), Expected Shortfall (ES), Marginal Expected Shortfall (MES), and the Percentage Contribution to Expected Shortfall ($PCES$) evaluated at 99% Confidence Level. The numbers in brackets show the ranking based on the corresponding statistic. The statistics are evaluated for August, 29, 2022 using a two-year weekly time window to estimate the factor model loadings.

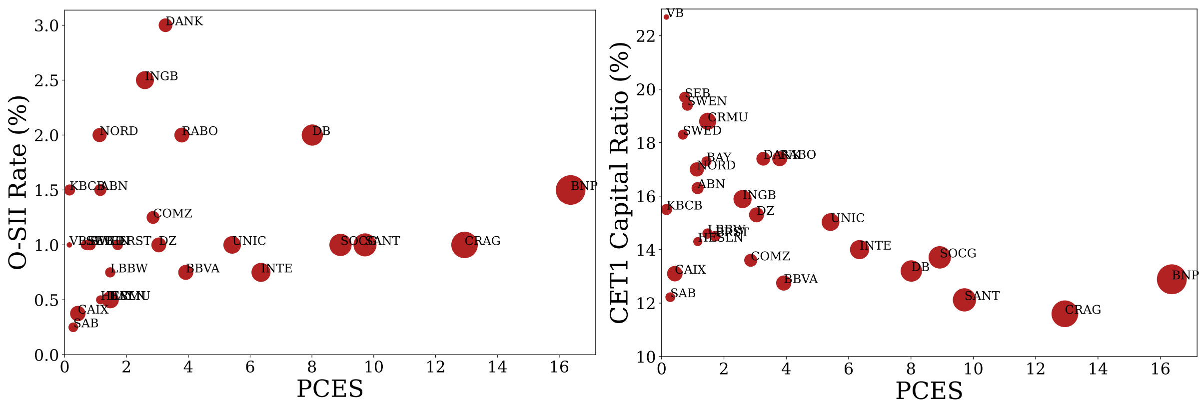
plot implies that their required O-SII buffers are low compared to their contribution to systemic risk: they are required to hold between 1% and 1.5% buffers, even though their respective contributions to systemic risk are several times higher than those of smaller banks with a comparable buffer requirement.

In order to verify if other types of capital buffers compensate for their relatively low O-SII rates, Figure 5b sets off their total CET1 capital ratio against their contribution to systemic risk. Figure 5b tells the same story: it shows again that the largest contributors to systemic risk are undercapitalized relative to their share in total systemic risk when compared to the smaller banks.[21]

So for an important subgroup of large European banks the size of the buffers they are required to hold does not seem to be in proportion to their contribution to systemic risk: the buffers are lower than for other banks with comparable or lower contributions to

[21]The higher of the required G-SII and O-SII buffers applies if a bank is subject to the two. In our sample however, the O-SII requirement was at leas as high as the G-SII one for all banks.

Figure 5: PCES vs. Capitalization



(a) O-SII Buffers

(b) Current Capitalization

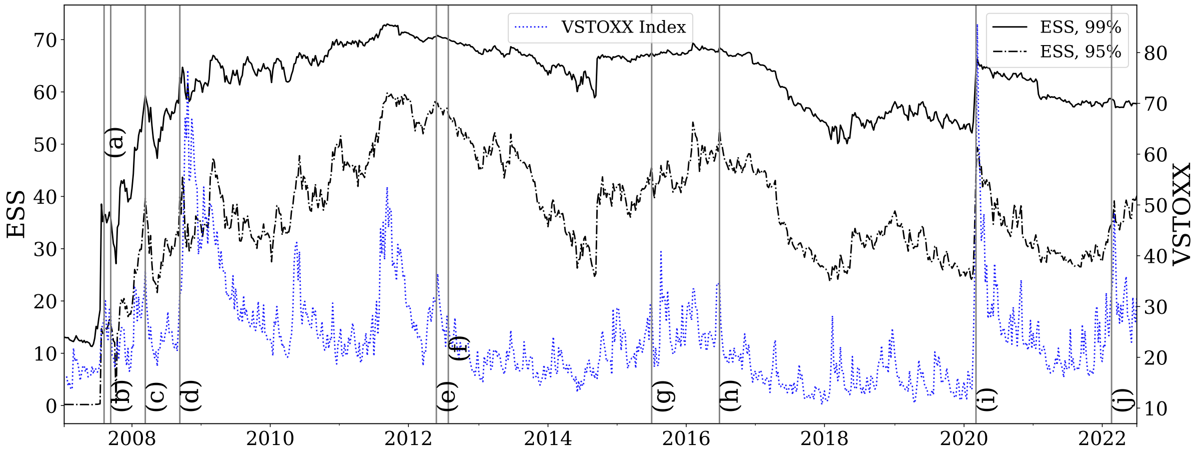
Note. This figure shows our model estimates for systemic risk contribution measured by PCES at 99% confidence level versus (a) the size of the required O-SII buffer rate for 2022, and (b) banks' total CET1 capitalization ratio for 2022. The size of each dot corresponds to the relative liability weight of the bank in the regulatory portfolio.

systemic risk. This leads to two questions. First of all, how appropriate are the current buffer rates when risk spillovers outside of the national economy and within the Eurozone are taken into account? And second: how consistent are the approaches of translating systemic importance into buffer requirements between European countries? In Dimitrov and van Wijnbergen (2023) we explore the calibration of macroprudential capital buffers based on systemic risk using the methodology developed in this paper and address the capitalization discrepancies observed here.

4.4 Measuring Systemic Risk Over Time

So far, we have discussed the use of the shortfall measures as a way to attribute total systemic risk across banks and to rank institutions by their contribution to systemic risk. Next, we show how systemic risk itself can be quantified by calculating the *Expected Systemic Shortfall* ESS, measured as the expected shortfall of the total portfolio of banks (cf. Equation (16)). We then evaluate the potential of the ESS measure as an early indicator of financial distress. For this purpose, we evaluate its evolution over the period from January, 2007 up to August, 2022. Following Lehar (2005), we use a rolling-window approach, in which a time window of 2 years of weekly observations is used to estimate the parameters of the model.[22] After the risk number for the portfolio is then calculated, the window is shifted forward by a week and the model is re-evaluated. This produces a series of out-of-sample risk metrics. Figure 6 below plots the ESS over time for respectively the 95% and 99% confidence levels, on such a rolling-window basis. We also include as a model-free indicator the VSTOXX (on the right axis).

Figure 6: Expected Systemic Shortfall vs. VSTOXX



Note. This plot shows the tail risk of the systemic portfolio quantified by the *ESS* at a confidence level of 95% and 99%. The dotted line represents the VSTOXX index. The vertical lines indicate the dates for (a) BNP announces CDO related losses; (b) Northern Rock seeks liquidity support by BoE; (c) Bear Stearn downgraded; (d) Lehman files for bankruptcy; (e) Mario Draghi's "courageous leap" speech to save the Euro; (f) Draghi's “whatever it takes speech”; (g) Greece misses and IMF payment; (h) Brexit referendum; (i) the first Covid lock-downs in Europe (in Italy); (j) the Russian invasion in Ukraine.

We place on the chart several key events for the development of systemic risks. First, we can see in retrospect that the ESS measures, both at 95% abd 99% start increasing after the initial signs of the looming Financial Crisis (vertical lines (a), (b) and (c) on the chart). As a result, it peaks signigicantly earlier than the VSOTXX's reaction to the Lehman collapse (vertical line (d)). Consequently, the ESS at 95% declines gradually after Draghi's "courageous leap" speech (f), and "whatever it takes" speeches in 2012, indicating that the measure was able to capture the subsequent decline in systemic risk from the Euro debt crisis. Then we can see the sudden spike in risk with the first Covid lockdowns in Europe in January 2020 and the subsequent decline after the ECB's

[22]Shorter and expanding period is used for the first few windows, as data for most series starts in 2006.

involvement to secure liquidity in the market. The Russian invasion of Ukraine in 2022 had a much smaller impact on the ESS than on VXTOXX, possibly because the ESS is strictly focused on the banking sector, while VSTOXX covers the direct effects of the war on all sectors.

We see that the 95 % ESS estimates are responsive in the short term to fluctuations in asset prices compared to the 99% based ESS. The ESS at 99% on the other hand tends to move with the overall trend and catches the major events but seems less responsive (than ESS at 95%) to period-by-period changes, unless an unexpected tail event hits, as the first Covid lockwons in Europe.

Figure 6 also compares the constructed ESS measures of systemic risk with the value of the VSTOXX index, which is often used to track risk appetite and market panic. VSTOXX, like its US counterpart VIX, measures the implied volatility derived from near-term exchange-traded options on the Euro Stoxx 50 equity index. The options are widely used by investors for hedging purposes. As a result, the index indicates informally the current price investors are willing to pay in order to hedge extreme risks and can serve as a measure of investors' view on the potential for systemic events to realize.[23] It is interesting to observe that in its overall trend since 2016 our constructed risk measure, especially the one based on a 95% confidence interval, matches very closely the VSTOXX index. The match becomes particularly notable from 2018 onwards. A benefit of the constructed ES measure relative to the implied variance index, however, is that it is less noisy over time.

[23]The composite implied volatility indices are often seen as indicators of the (lack of) risk appetite in the economy. A low appetite for risk (high implied volatility) relates to a greater cost of capital for the economy, thus lower investments and lower asset prices, while a high appetite (lower implied volatility) relates to credit and asset price bubbles, increasing the chance for future recessions and stress in the financial system (Cf. Illing and Aaron (2005); Gai and Vause (2006); Aven (2013)). On the other hand, Bekaert et al. (2013) decompose the US counterpart of the index in risk-aversion and expected equity market volatility and show the effect monetary policy has on both components.

5 Robustness Checks

In this section we report on a variety of robustness checks. In Section 5.1 we analyse two alternative approaches to measuring systemic risk: the CoVaR approach of Adrian and Brunnermeier (2016) and the VI (Vulnerablity Index) approach to assessing the probability of multiple concurrent defaults based on Extreme Value theory developed by Zhou (2010). Neither the CoVaR nor the VI measure add up to systemic risk, which makes it impossible to rank banks on their *relative* contribution to overall systemic risk. We see the fact that our approach does allow for such a comparison as a major advantage of the MES based modeling framework we develop in this paper. In Section 5.2 we extend our framework to non-normal distributions. Specifically we first show how to introduce fat tails by transforming the latent variable U into a multi-dimensional student-t distribution. Since Student-t distributions are still symmetric we need to use a different approach to trigger skewness. So in Section 5.2 we apply a different transformation, following Chan and Kroese (2010) who introduce both fat tails and skewness. In the final robustness check we compare our market-data-based approach to a similar attempt to get around the lack of equity prices but using accounting data rather than market data (cf Engle (2018).

5.1 Alternative Measures of Systemic Risk

As we saw in the literature review of Section 2, there is a wide variety of systemic risk measures which make use of market data. As a robustness check, we look at which of those measures comply with the MES and PCES rankings established earlier.

First, we look at the $CoVaR$ measure proposed by Adrian and Brunnermeier (2016) to quantify the tail-dependency between an institution and the system it is part of. As modified by Huang et al. (2012), $CoVaR$ is evaluated as the worst $\alpha\%$ losses of the system, given that institution i is in its worst $\alpha\%$.[24] To align this measure with the

[24] Adrian and Brunnermeier (2016) define $CoVaR$ by conditioning on individual losses being equal to a quantile rather than a region of their distribution as $\mathbb{P}(L_{sys} \geq CoVaR_i | L_i = VaR_i) = \alpha$. This allows the use of quantile regression for the estimation of the measure. On the negative side, such conditioning

concept underlying the MES, we invert its conditioning to get the Exposure $ECoVaR$, which now like MES also quantifies the sensitivity of the institution's losses to a systemic tail event as $\mathbb{P}(L_i \geq ECoVaR_i | L_{sys} \geq VaR_{sys}) = \alpha$

Both the MES and the $ECoVaR$ measure the institution's expected losses if the system ends up in the tail of its potential losses over the coming year. However, in contrast to MES, which measures the *average* loss once the system is its tail, the $ECoVaR$ zooms in deep in the tail of the potential losses of the institution, measuring the α-th quantile not only with respect to the systemic losses but also with respect to the institution's own losses.

Second, we compare the MES results to another measure that is not influenced by the assumptions on how losses are formed and how they correlate between companies, as these assumptions inevitably affect both the MES and the $ECoVaR$ which are driven by the same loss simulations. We use the measure of systemic sensitivity of Zhou (2010) labeled as a Vulnerability Index (VI). It is defined as the probability that institution i will be in distress conditional on having more than one bank in distress in the system, formally: $VI_i = \mathbb{P}(\mathbb{1}_{d_i} = 1 | N_d > 1)$. Note that Zhou (2010) relies on Extreme Value Theory to estimate the proposed measures. Also, we rely on default as an indication of distress, whereas the original measure is constructed to capture large tail movements in the equity value of the institution.

Finally, we consider also w, the relative size of banks' liabilities, as a naive model-free measure of systemic importance.

	w	MES	ECoVaR	VI
w				
MES	0.54			
ECoVaR	0.39	0.80		
VI	0.28	0.74	0.64	

Table 2: Rank Correlations

	w	$PCES$	$w * ECoVaR$	$w * VI$
w				
$PCES$	0.79			
$w * ECoVaR$	0.99	0.85		
$w * VI$	0.89	0.89	0.91	

Table 3: Rank Correlations, Weighted Measures

can give a misleading tail-risk indication when the loss distribution is fat-tailed, by not capturing the probability mass below the VaR quantile. In our case, the losses of the systemic portfolio are strongly non-Gaussian, so we use the modified version of CoVaR, as in Huang et al. (2012), which conditions on $L_i \geq VaR_i$.

Table 2 summarizes the results, first for the indicators unweighted by size. The closest correlation is between the MES and the ECoVaR measures, the correlation between the MES and the VI measure is somewhat lower. All measures have a relatively low correlation to bank size. Table 3 compares the estimated $PCES$, which in fact is a size-weighted version of the MES, to size w again and to the size-weighted variants of $ECOVAR$ and VI. Naturally, the measures now become much more correlated once we all weigh them by the same factor w. Note that the rankings by $PCES$ have a much lower correlation to the rankings by size than the other two measures $ECoVaR$ and VI. The explanation is that the $PCES$, by including the risk components, embeds additional information into the ranking.

We have to emphasize that from the proposed measures, only the individual banks' weighted MES sums up to total systemic risk. This is due to its additivity property (19). The two other risk measures do not have this property and their weighting can therefore only be seen as heuristic; they can produce instructive rankings but they do not indicate how much each bank contributes to overall systemic risk.

5.2 Alternative Dependency Specifications: The Student-t and the Skewed-t Model

Next we verify how the risk attribution is affected by modifying the Gaussian asset returns assumptions. The main drawback of the Gaussian model of Equation 9 is that the thin tails of the normal distribution may underestimate the chance of extreme events happening: there is strong empirical evidence that asset price returns do not tend to favor the normality assumptions, especially with higher frequency returns (Cont, 2001; Haas and Pigorsch, 2009).

Furthermore, since the Gaussian model presented so far is based on the dependency structure of the multivariate normal distribution, it has no tail dependence.[25] As a result,

[24] Formally, this is known as the Euler property of a risk measure. See Hull (2018) (Chapter 12) for details. For a discussion on the additivity property of the VaR in the context of systemic risk, see Puzanova and Düllmann (2013).

[25] Tail dependence shows how extreme events in one random variable are related to extreme events

it may underestimate the clustering of defaults at any moment in time. We address this problem in what follows by employing two fat-tailed alternatives to the Gaussian Copula. Specifically, in order to examine the sensitivity of our baseline model to the realization of extreme risk and dependencies, we modify the factor model in Equation (9) by allowing in Equation 22 for fat-tails and, in addition, for skewness in Equation 23 in the distribution of the latent variable U_i whose behavior governs the credit-worthiness of the underlying banks.

The Student-t Model: To introduce tail risk and tail dependency, we follow a setup by Bassamboo et al. (2008). We add an aggregate multiplicative factor F to the set-up of Equation 9 with F independent of M and Z_i for any i. The new specification of the latent variable then is

$$U_i = \sqrt{h(F)}\left(A_i M + \sqrt{1 - A_i A_i'} Z_i\right) \tag{22}$$

where $h(F) = \frac{\nu}{F}$ with $F \sim \chi^2(\nu)$.[26]

The choice of the distribution of F and the specification of the function $h(F)$ govern the distribution of the latent variable U_i. In general, F can be any positive-valued independent stochastic variable, and $h(.)$ is a continuous function. (McNeil and Embrechts, 2005) provides a discussion on the class of default-threshold models that come with the choice of these specifications, known in general as normal-variance mixtures.[27] For the particular setup that we have selected, however, it can then be shown that U_i will be multivariate student-t distributed with η degrees of freedom.

Even though the factor F is constructed mathematically as a way to impose a certain

in another random variable. In simpler terms, it measures the tendency of variables to move together during extreme events. Bolder (2018) shows that tail dependence goes up with higher correlation but down in degrees of freedom n for a multidimensional t-distribution; for $n \to \infty$ the distribution becomes Gaussian and tail dependency goes to zero.

[26]An equivalent specification, also appearing in the literature and leading to the same multivariate model, is to set $h(F) = \frac{1}{F}$ with $F \sim \Gamma(\frac{\nu}{2}, \frac{\nu}{2})$, where $\Gamma(.)$ is the Gamma distribution. This is the specification that appears, for example in Chan and Kroese (2010).

[27]For example, Bolder (2018) shows that for $h(F) = F$ with $F \sim \Gamma(a, a)$, the latent variable will be variance-gamma distributed, and for $h(F) = F$ with $F \sim GIG(a, a)$ it will be generalized-hyperbolic.

distributional assumption on the latent variables U_i, it also has a convenient interpretation as an additional factor that governs the *intensity* of risk. In our specification, for example, smaller realizations of the variable F, i.e. draws from the χ-squared distribution closer to zero, would imply larger amplification effects on the systematic and idiosyncratic factors M and Z_i respectively.[28] This is how the factor F generates extreme tail dependence, by simultaneously hitting all banks at the same time. Note, that the conditional distribution $U_i|F$ is still Gaussian, however.

The moments of the marginal distribution of U_i that the factor implies can easily be verified to correspond to the student-t distribution (Cf. Annex B.2). As a result, the new specification does not change the structure of the latent variable correlations, their expected values, or their skewness. What changes, however, is the kurtosis of the distribution.

The multivariate Student-t distribution is still symmetric, as can be seen from its specification and the derivations so far. This means that the joint occurrence of extreme positive and extreme negative events is equally likely. This is not always empirically viable, as one might expect dependencies between institutions in a market crash. So in Section 5.2 we extend the model further to allow for different dependencies in positive and negative markets. This is done by referring to a model generalization in the class of the normal-*mean*-variance mixtures (McNeil and Embrechts, 2005).

Skewness and Skewed Dependency: We follow an approach suggested by Chan and Kroese (2010), who modify Equation (23) with an additional factor that induces skewness and asymmetric dependencies. They suggest the following structure of the fat-tailed latent variable

$$U_i = \sqrt{\frac{\nu}{F}} \left(\delta G + A_i M + \sqrt{1 - A_i A_i'} Z_i \right) \tag{23}$$

[28]Chan and Kroese (2010) also suggest a mathematical specification where the amplification can be heterogeneous across systematic vs. idiosyncratic shocks. This however raises significantly the degrees of freedom for fitting the model, so we do not explore that alternative here.

where $G \sim TN\left(-\sqrt{\frac{2}{\pi}}, 1\right)$, with $TN(\mu, \sigma)$ is a normal distribution truncated left at $-\sqrt{\frac{2}{\pi}}$.

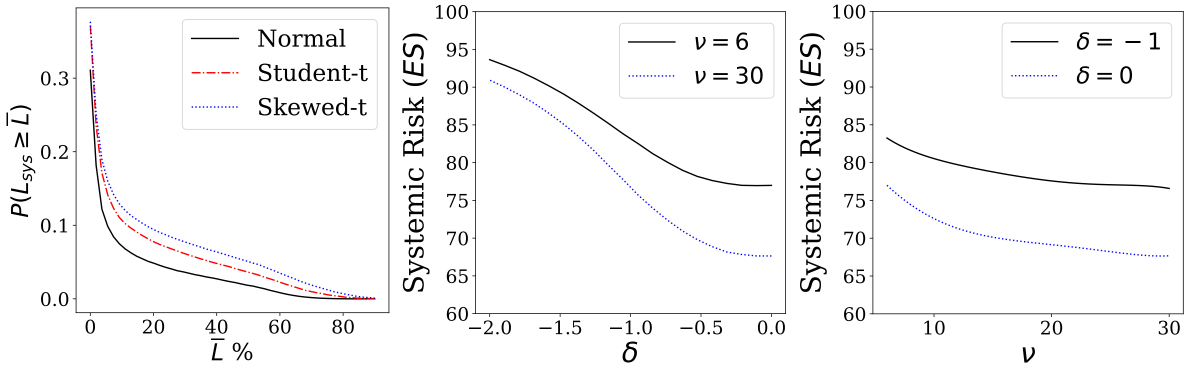
Again, G is an aggregate stochastic factor and it affects all variables in the same way through the shared exposure δ. This non-symmetric distribution of the common factor G then creates the non-linear dependency structure between banks. As a special case, in the symmetric Student-t model then $\delta = 0$, while in the Gaussian case, F is fixed to be one, and δ is zero.

Comparing the consequences of using of alternative distributions: Figure 7 shows how the three models compare against each other in estimating total risk (Cf. also Annex A.6 for an illustration of the simulated scenarios). First, Figure 7a shows the full tail of the distribution of systemic losses, by quantifying the probability that total losses may be larger than a certain threshold $\overline{L}$, with losses measured relative to the size of the aggregate liabilities in the system. We can clearly see that adding a fat-tailed factor in line with specification (22) increases the estimated probability of large losses occurring (the red dashed curve in the figure), compared to the normal model (the black curve). Adding a skew factor on top of that in line with the specification in Equation (23) increases the tail risk even further (the red curve) for the whole range of scenarios. We can see that the tail of the normal model declines much faster than that of the others, indicating that the probability of observing large losses declines to zero much faster than when a fat-tailed or skewed factor is included.

Figures 7b and 7c then show the estimated systemic risk by varying the skewness parameter δ and the tail-fatness parameter ν. It is interesting that the systemic ES appears to be much more sensitive to negative skewness than to symmetric fatter tails, as can be seen in Figure 7c. For ν around 30 and δ of zero, the model becomes normal. The risk estimate then increases sharply with more negative δ, even with otherwise thinner tails (degrees of freedom close to 30). The highest level of risk appears with high skewness and high tail-fatness parameters.

Next, Figure 8 compares the risk attribution numbers across the banks in the uni-

Figure 7: Aggregate Tail Risk, Model Comparison



(a) Tail Comparison (b) Skewed-t (c) Skewed-t

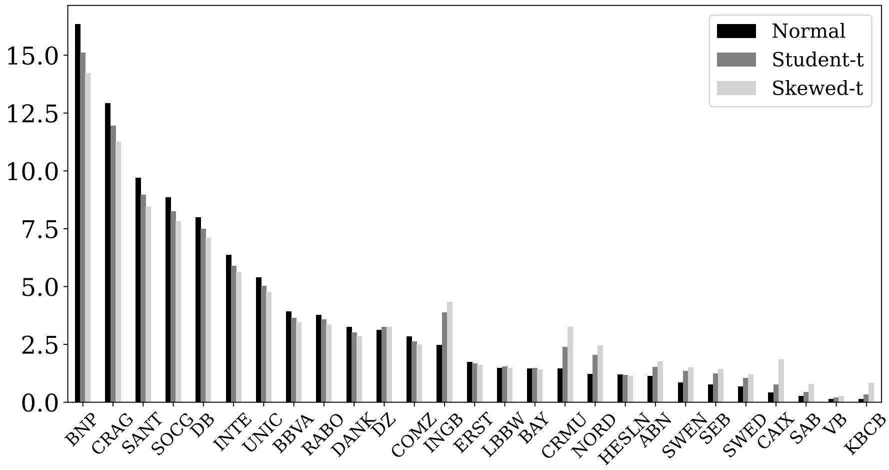
Note. This plot shows the overall risk in the system, measured by the systemic portfolio's ES. Figure (a) compares the probability of realizing a loss larger than a given threshold $\overline{L}$ for the Normal, the Student-t with $\nu = 6$, and the Skewed-t model (with $\nu = 6$ and $\delta = -1$). Figures (b) and (c) show the portfolio's ES as a function of the skewness parameter δ and the tail-fatness parameter ν.

verse. With few exceptions, the $PCES$ estimates follow the overall trend and rankings established earlier with the Gaussian model (Cf. Table 1). This overall tendency can be expected as we model tail risk and skewness as aggregate factors that affect all banks in the same way.

Note, however, that the specification of the nonlinear common factor F has implications both for the factor risk and for the idiosyncratic risk of individual banks, as in equations (22) and (23) it scales both the common linear factor M and the idiosyncratic factor Z_i. Therefore banks which had low exposures to the linear common factors M in the normal-distribution model, and thus low overall correlation with the rest of the universe, now do have an additional source of (nonlinear) common risk. In the very extreme, this means that now even if banks have full exposure to their idiosyncratic risk factors and none to the common linear factors, a low realization of F will drive up their tail risk anyhow. As a result, the common factor F will have the propensity to jointly drive up the tail risk of all banks at the same time after all, even with low or no exposure to the linear factor M.

As a consequence, we observe in the new rankings some relocation of risk attribution from the top contributors in the Normal model (*BNP, CRAG, SANT, SOCG*) to the

Figure 8: Model Comparison PCES



Note. This figure shows the share of systemic risk measured by PCES at 99% confidence level for the three asset returns distributional models. The Student-t model uses degrees of freedom ν of 6 and the Skewed-t uses, in addition, a δ parameter of -1.

lower end of the attribution ranking (e.g. *INGB, CRMU, NORD, CAIX*). The latter is a group of banks that under the normal model have a relatively small proportion of factor risk (Cf. Figure 2). With the additional source of aggregate risk in the form of the nonlinear factor, the interdependency between these banks increases in extreme scenarios and as a result, increases their contribution to the total risk.

Overall, we find that while adding a nonlinear or negatively skewed factor to the model has strong implications on the level of aggregate risk, it does little to change the risk attribution, and the risk rankings within the cross-section. The only exception is that banks with high idiosyncratic risk are now pushed up in the ranking. The implications of these results are twofold. First, given that the estimation of the tail parameters is difficult and subject to misspecification (joint systemic defaults are rarely observed, especially within a reasonable time window of data), working with the Gaussian model of Equation (9) still offers a reliable view into the cross-sectional distribution of risks. At the same time, however, allowing for tail dependence via one of the two extended models could offer a way of performing stress testing. In such a stress test environment, one could evaluate the sensitivity of the projected losses to an increased potential for the materialization of an extreme joint events, while still having the correlations of the model empirically based.

5.3 Alternative approach to missing equity market prices

Engle and coauthors use a different method altogether to uncover the systemic risk contributions of unlisted banks (cf. Engle and Jung (2023); Engle et al. (2023)). Like us they have a sample consisting of both listed and unlisted banks. In what we call a "synthetic approach", they map key balance sheet information into the estimated systemic risk measure for the sub-sample of listed banks. This is then used to extrapolate to a hypothetical value for the systemic risk measure for all non-listed banks. Comparing the results to the banks' projected capital depletion in EU-wide stress tests, they find that this approach offers a realistic alternative to estimation based purely on equity prices.

We take their approach one step further and evaluate how it behaves over time and how sensitive it is to the development of systemic events relative to the credit-based approach that we outlined so far. We use the direct method proposed by Engle et al. (2023) in which the MES is directly estimated from the covariates.[29]

First, we evaluate the MES^l of each listed bank on a rolling-window basis, based on equity data following the approach by Acharya et al. (2017). Then, we run a regression of MES^l on a set of balance sheet variables $X_{i,t}$ over the full time window of the listed sample:

$$MES^l_{i,t} = X_{i,t}\beta + \epsilon_{i,t} \tag{24}$$

Following, Engel et al. the covariates vector $X_{i,t}$ contains Log-value of total assets, Log-value of total assets-squared, Profits over total assets, Equity over total assets, CET1 Ratio, and a Periphery dummy.

Table 5 below shows the results.[30] The signs of the coefficients are as one would

[29]An alternative would be an indirect approach in which the regression is used to estimate a synthetic version of the factor loadings in Equation (9).

[30]Note that we exclude an intercept from the model in order to alleviate potential strong forward-looking bias, which would be induced from regime breaks affecting the mean of the relationship. We also exclude a Liquidity ratio variable from the regression specification due to incomplete of data over time.

expect: lower CET1 ratio, higher size of assets, lower profitability, and lower equity to total assets increase systemic risk (i.e. the MES estimate) given everything else.

Table 4: Pooled OLS Regression Results

	coef	std err	t	P> |t|	[0.025	0.975]
CET1r	-0.0590	0.038	-1.556	0.120	-0.133	0.015
lnTA	1.1393	0.058	19.665	0.000	1.026	1.253
Prof_TA	-2.9278	0.441	-6.636	0.000	-3.794	-2.061
EQ_TA	-1.1870	0.120	-9.879	0.000	-1.423	-0.951
Periphery	3.7121	0.330	11.249	0.000	3.064	4.360

Omnibus:	57.244	**Durbin-Watson:**	1.274
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	79.575
No. Observations:	573	**Prob(JB):**	5.25e-18
R-squared (uncentered):	0.890		

As in Engle and Jung (2023); Engle et al. (2023), the next step is to use the estimated regression to forecast in-sample the synthetic MES for the unlisted banks, $M\hat{E}S^{nl}_{i,t}$:

$$M\hat{E}S^{nl}_{i,t} = X_{i,t}\hat{\beta} \tag{25}$$

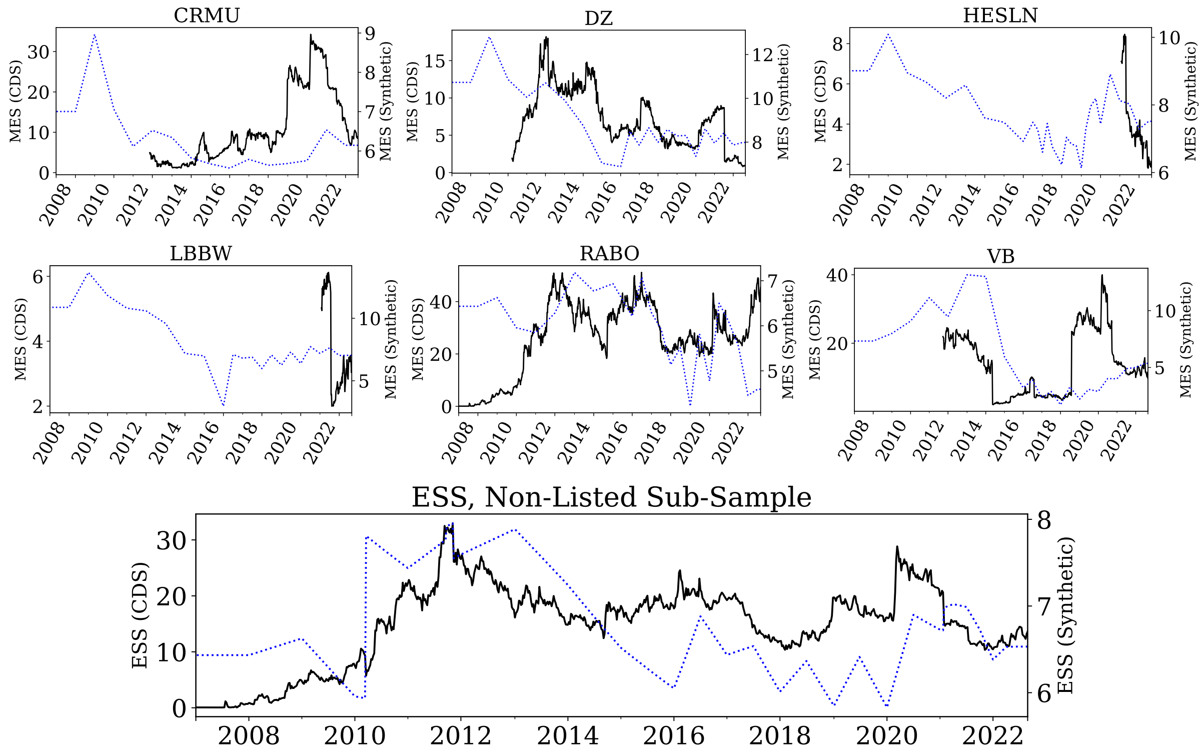
The balance sheet data is available quarterly at best, while we are interested in a comparison that develops at higher frequency; so we interpolate over the balance sheet ratios to allows us to get weekly estimates of $M\hat{E}S^{nl}_{i,t}$.

Figures 9 show, at 95% confidence level (i.e. MES at 5% of the worst potential outcomes) the comparison in the estimated numbers with the CDS approach and with the synthetic approach. Clearly the CDS measure is more sensitive to shorter-run market developments. This is as one should have expected, given that our measure is purely based on market data.

Note that we plot the two measures on different scales. The reason is that we are interested in comparing the development of the trends over time, we are not interested in measuring the size of losses. In fact, there are reasons why the two measures can

In Engle et al., however, the variable is not significant, so we are still close to their approach.

Figure 9: CDS-based vs Synthetic MES of Non-Listed Banks



Note. This set of figures shows the MES model estimates based on the CDS approach (solid line) and based on the Engle et al. synthetic approach (dotted line). The last chart shows the aggregated ESS number for the sub-sample of non-listed banks.

be expected to evolve over a different scale. Because of seniority of debt over equity the LGD embedded in equity prices is different from the LGD embedded in CDS prices. CDS prices are a lot more indicative of tail events.[31]

But overall, we can see that the two approaches may be significantly different in magnitude, but they roughly follow similar patterns over time. This is especially visible in the aggregate ESS estimate in Figure 9. What the CDS approach offers then is the advantage of having estimates that are sensitive to market information that is available on a higher frequency than accounting data.

Of course there may also be circumstances where where the two approaches can be combined by utilizing CDS information whenever it is available, while applying the Engle

[31]Carr and Wu (2011) in fact provide a link between the two by comparing the value of CDS contracts to the value of deep out-of-the-money put options traded on the same company.

accounting based approach for smaller banks for which CDS contracts do not exist.

6 Conclusion

In this paper, we address a common challenge in estimating and monitoring the build-up of systemic risks in many countries and/or regions: a regulator cannot observe the market price of equity for institutions that are privately or state-held. We show that high-frequency data from the CDS market can be used to imply views on co-dependencies and joint losses when equity data are not available. We apply the model to the European banking sector, where many key banks are not publicly traded. The model allows us to rank banks in the Eurozone by their contribution to overall systemic risk in the Euro Area. We find that on a European scale, there is a discrepancy in the capitalization between the largest contributors to systemic risk relative to smaller less systemically important banks. This has important implications for the policy debate on conducting macroprudential policy at a European, rather than a national level.

In contrast to the micro-prudential view, an appropriate macro-prudential policy should monitor not only the risky positions of an institution on its own but also the interdependencies between institutions and the potential for several of them to realize large losses at the same time. In the universe that we consider, we confirm that a risk ranking incorporating tail dependence across banks is different from a ranking based on standalone tail risk. From an aggregate risk point of view, it is clear that a focus on contributions to systemic risk rather than on standalone risk is more important.

In the process, we present a model that addresses systemic risk more from a structured credit angle. The financial institutions in the system are seen as part of a defaultable loan portfolio. Systemic losses occur in the case of the default of one or several institutions. The average tail losses of the portfolio (the ES measure) speak for the magnitude of the systemic risk. The average losses of each institution, given that the system is in its tail, speak for the sensitivity of each institution to systemic risk. Then, the share of the portfolio tail risk that can be attributed to an institution represents its contribution to systemic losses.

We also show that the estimate of aggregate systemic risk in itself, as measured by the expected shortfall ES of the portfolio of systemic banks, is a useful indicator for the development or resolution of systemic risks in the banking sector. We illustrate the sensitivity of this measure to recent events, such as the resolution of the Euro debt crisis, the initiation of the Covid-related market turmoil, and the initiation of the war in Ukraine.

Methodologically, we extend the existing literature, in particular - the approaches used in Huang et al. (2012) and Puzanova and Düllmann (2013), in two ways. First, we allow for risk dependencies to appear not only in the form of default correlations but also in the form of dependencies in the size of default losses. This reflects the empirical evidence collected from the observation of historical defaults, as well as insights from the theory of firesales and contagion. Second, we allow for higher-order common factors to account for asset return fat-tails, skewness, and asymmetric dependencies. This takes into account the potential for extreme events to materialize more often than presumed by the Gaussian distribution, and the tendency of assets to have a higher dependency in crises than in booms.

As an application of the model, we examine the extent to which the capitalization of a large sample of European banks is consistent with their contribution to systemic risk within the Eurozone, rather than, as the current regulatory framework is set up, within national borders. We find that the largest institutions on a European scale in terms of size and risk contribution are less capitalized compared to smaller institutions, both in terms of the regulatory O-SII capital buffers and in terms of the actual CET1 capital. This finding should be relevant in the policy debate on the systemic resilience of the individual banks in the Euro Area and on the adequacy of current capital requirements.

Overall, we conclude that there are strong arguments in favor of embedding market-based implied measures of systemic risk like ours, into the policy process. The approach developed in this paper makes that possible even in the presence of state-owned or coöperative banks that lack the equity listing necessary for the application of the techniques developed in the current academic literature. First of all, such measures provide a way to verify the ranking that policymakers come up with based on EBA's guidelines using regulatory data only. Any discrepancy in the rankings based on the two approaches raises important questions, the answers to which may well improve the regulator's approach to assessing systemic risk. And even if no discrepancy between the two approaches appears, a market-based measure such as the MES and the $PCES$ can be used to assess risks between annual policy assessments. We also touch upon the field of extreme risk, and how risk results are affected by the addition of common factors responsible for skewness and tail-fatness in asset returns. Further research could explore additional features in the systemic risk model.

In fact, our credit portfolio approach can be considered a basic architecture which is extendable to incorporate any specific observed stylized features of asset prices or of the structure of the examined financial network that are relevant for the analysis of systemic risk. Since tail correlations between the institutions are a key driver of systemic contributions, it is worthwhile exploring the non-linear structures of these dependencies. The ability to model large multi-dimensional dependencies is key. Oh and Patton (2018) for example suggests the use of a dynamic factor copula approach. Wang (2021) suggest a deep learning approach. Alternatively, network models could be used to mimic the often observed core-periphery structure of the financial sector (Bräuning and Koopman, 2016; Andrieş et al., 2022). Institutions that constitute the core of the network could well be dominant drivers of systemic risk (Glasserman and Young, 2016; Jackson and Pernoud, 2021).

To sum up, estimating systemic risk contributions properly is essential for the efficient regulation of the financial system. Macroprudential capital requirements need to be applied in line with the extent to which they generate generate systemic externalities, so identifying these institutions and the extent to which they contribute to systemic risks is crucial. The framework we develop in this paper using CDS prices makes it possible to do so in an environment where not all banks are listed on public exchanges.

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A Annex: Charts And Tables

A.1 Data Description

Table 5 provides an overview of the bank CDS rates included in the analysis.

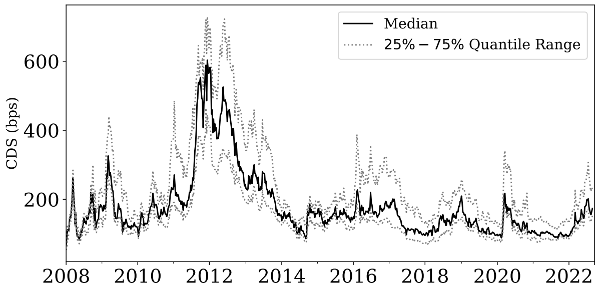
Table 5: Data Sample Descriptive Table

Short Code	Country	Bank Full Name	Public	CDS Type	Start Date
ERST	Austria	Erste Group	Y	SR	1/3/2006
KBCB	Belgium	KBC	Y	SUB	1/4/2006
DANK	Denmark	Danske Bank	Y	SUB	1/10/2006
NORD	Finland	Nordea	Y	SUB	1/3/2006
BNP	France	BNP Paribas	Y	SUB	1/3/2006
CRAG	France	Credit Agricole	Y	SUB	1/3/2006
CRMU	France	Credit Mutuel	N	SUB	2/23/2010
SOCG	France	Societe Generale	Y	SUB	1/3/2006
COMZ	Germany	Commerzbank	Y	SUB	1/3/2006
DB	Germany	Deutsche Bank	Y	SUB	1/3/2006
DZ	Germany	DZ Bank	N	SR	6/30/2008
BAY	Germany	Bayern LB	N	SR	5/13/2019
LBBW	Germany	LBBW	N	SR	5/13/2019
HESLN	Germany	Helaba	N	SR	5/13/2019
INTE	Italy	Intesa Sanpaolo	Y	SUB	1/3/2006
UNIC	Italy	Unicredit	Y	SUB	1/3/2006
RABO	Netherlands	Rabobank	N	SUB	1/3/2006
ABN	Netherlands	ABN Amro	Y	SUB	1/3/2006
INGB	Netherlands	ING Bank	N	SUB	1/3/2006
VB	Netherlands	Volksbank	N	SUB	1/3/2006
CAIX	Spain	Caixabank	Y	SUB	8/12/2016
SAB	Spain	Sabadell	Y	SUB	1/3/2006
SANT	Spain	Santander	Y	SUB	1/3/2006
BBVA	Spain	BBVA	Y	SUB	1/3/2006
SWEN	Sweden	Handelsbanken	Y	SUB	5/14/2008
SEB	Sweden	Skandinaviska Enskilda Banken	Y	SUB	1/3/2006
SWED	Sweden	Swedbank	Y	SUB	1/3/2006

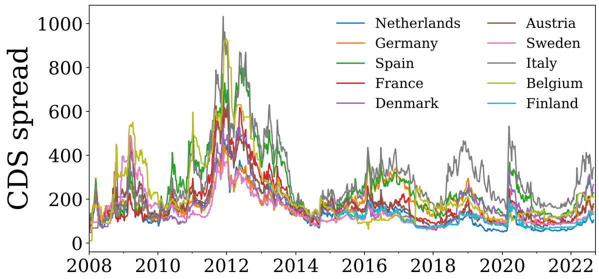
Note. This table shows the basic properties of the dataset: the country to which each bank belongs, the bank short code used throughout this paper, whether the bank's equity is traded on the market ('Y') or it is privately owned ('N'); and the type of CDS spreads used as input for the study (on senior debt (SR) or on subordinate debt (SUB)). Senior Debt CDSs are corrected for the median spread between senior and subordinate debt.

Figure 10 below provides an initial overview of the distribution and evolution of the CDS prices over time. 10b shows the median spreads per country over time, providing a perspective into the possible dependencies across countries, and consequently the potential for (co)occurrence of credit events. At the same time, the level of the CDS spreads indicates which country's banks may be subject to higher credit risk. Figure 10b shows the distribution of cross-bank correlations over time.

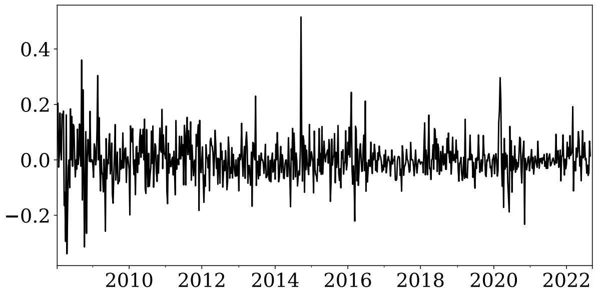
Figure 10: Data Overview: CDS Time Series



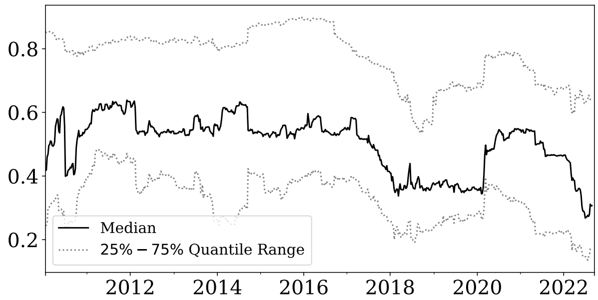
(a) CDS Prices



(b) CDS Prices by Country Median



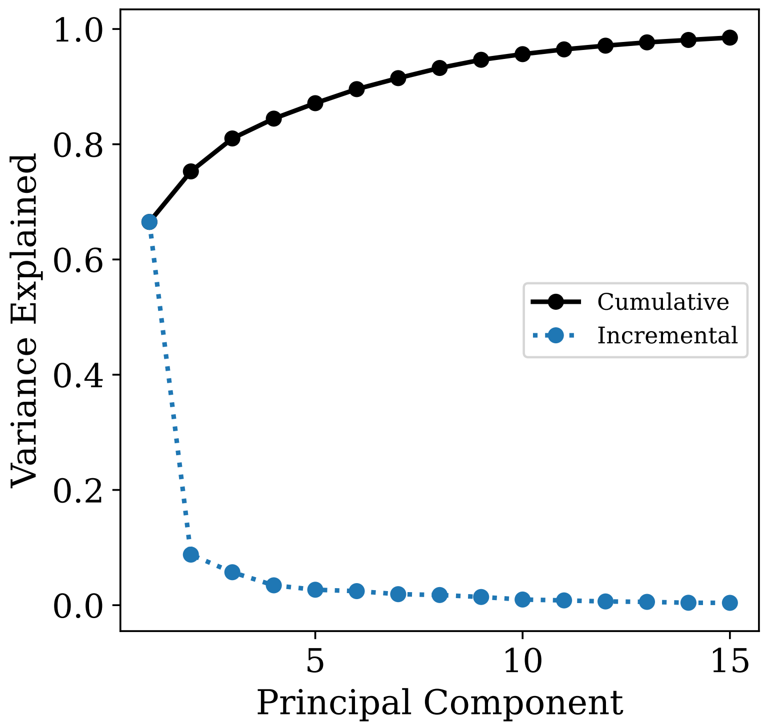
(c) Median Weekly Log-Changes



(d) 2-Year Weekly Rolling Correlation

A.2 PCA-based Factor Selection

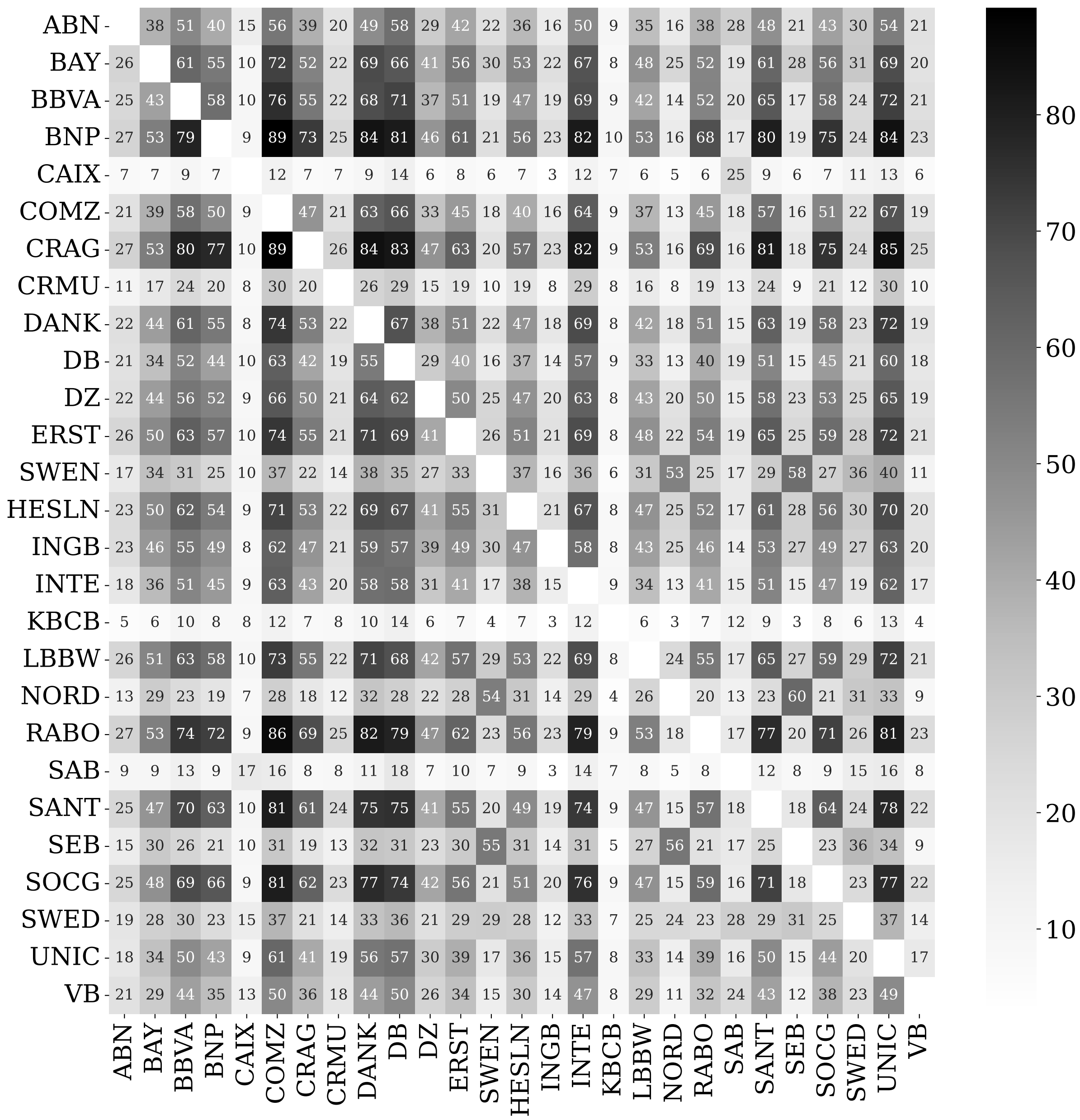
Figure 11: Share of Total Explained Variance



Note. This figure shows the cumulative share of explained variance (the solid curve) and the incremental share (dashed curve) over the CDS price log-changes in our sample.

A.3 Conditional Probabilities of Default

Figure 12: Conditional Probability of Default



Note: This chart shows the risk-neutral probabilities that an institution j may default, conditional on institution i being in default. The estimates are evaluated as of the end of August 2022.

A.4 Rank Comparisons: Asset Correlations vs JPD and CPD

We rank from highest to lowest all bank dependency pairs in terms of asset correlation (AC), JPD, or CPD. This allows us to observe the extent to which each measure gives similar ranking information. Table 6 summarizes these relationships by showing the Spearman rank correlations between these dependency measures, showing that the rankings by asset correlation (AC) are similar to those obtained from the JPD and the CPD.

Table 6: Rank Correlations between Dependency Pairs

	AC	JPD	$CPD_{i\|j}$	$CPD_{j\|i}$
AC				
JPD	0.91			
$CPD_{i\|j}$	0.93	0.88		
$CPD_{j\|i}$	0.92	0.88	0.74	

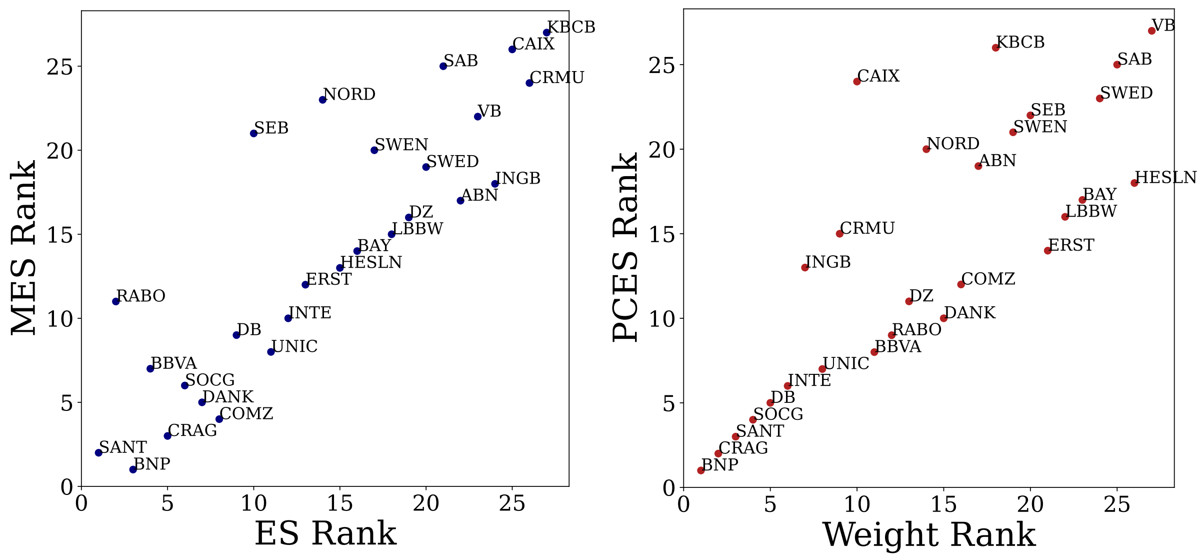
Note. This table shows a matrix of the rank correlations between dependency pairs, where dependency is measured by the implied asset correlations, joint default probability, and conditional default probabilities.

A.5 Rank Comparisons: Size vs. MES vs. PCES

First, we want to point out the relationship between banks' vulnerability rankings with respect to their own risk, and the vulnerability of the system as a whole. Figure 13a plots the rankings between the ES and MES. Overall, the relationship is positive. This is consistent with the fact that higher individual risk will inevitably correspond to a larger threat to the other players in the system when banks are interlinked. However, there are also some notable differences between individual and systemic risk rankings. Most notably, *NORD* and *SEB* are around the middle of the ranking scale in terms of ES, but they are at the bottom of the scale in terms of MES. The banks were identified as having small factor exposure to the main common factor and large negative exposure to the second factor which clusters banks from Sweden and Finland, as shown in Figure 1.

Figure 13b on the other hand shows the positive relationship between size and PCES rankings. It can be expected that for the same level of risk sensitivity (MES), the larger an institution, the larger its PCES will be. This positive relationship is strongly evident at the upper end of the rankings, where the top six banks by size and by weight are the same. After that, however, we start seeing some differences. For example, *INGB* is ranked seventh by size but 13th by contribution to ES. This can be traced back to the fact that it has a lower own risk (ES) and much lower sensitivity (MES) than the rest of the universe (Cf. Table 1). At the middle and bottom end of the rankings, banks are relatively the same size, so now their different MES helps us distinguish between degrees of risk contribution. Such is the case with, for example, *KBCB* which is ranked 18th by size but 27th in terms of contribution to systemic risk due to its low sensitivity to systemic losses, indicated by the MES of 5.37%.

Figure 13: Risk Rankings



(a) (b)

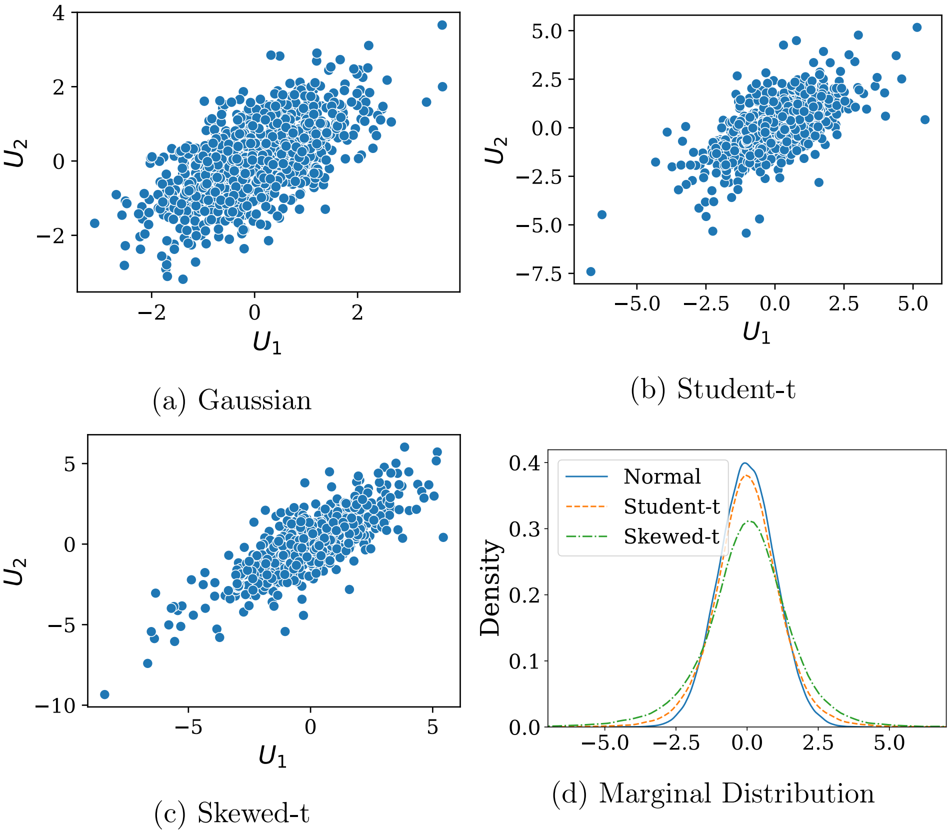
Note: This set of charts shows scatterplots of the rankings in size, risk, risk sensitivity, and risk contribution for the individual banks.

A.6 Simulations of the Factor Copula Models

As an illustration of how the different models behave, Figure 14 below shows a simulation of two latent variables U_i and U_j with each of the three models define so far.

Chart 14a shows clearly how the standard normal multivariate distribution forms between the two variables. The realization of scenarios further away from zero than three standard deviations is not at all likely. Chart 14b on the other hand, shows how symmetric extreme events start to appear, in line with a multivariate Student-t distribution with six degrees of freedom. Figure 14d finally visualizes the multivariate model implied by the specification of Equation (23) with a skewness parameter $\delta = -2$. We can see that in this chart the occurrence of joint large negative events is much larger than that of joint positive events.

Figure 14: Simulated Factor Copula



(a) Gaussian

(b) Student-t

(c) Skewed-t

(d) Marginal Distribution

Note. This plot shows 1,000 simulations using the three specifications of the factor model. Common factor loading of .8 is used in each of the cases. For the Student-t and the Skewed-t versions, we use degrees of freedom of $\nu = 6$, and skewness parameter $\delta = -2$.

B Model Derivations

B.1 Latent Factor Model Estimation

We apply the following algorithm based on Andersen and Basu (2003) to estimate the latent factor model from time-series data of the institutions' CDS prices.

Assume that Σ is an $n \times n$ matrix containing the target asset correlations between the key institutions. Assume the following factor model

$$\boldsymbol{U} = \boldsymbol{A}\boldsymbol{M} + \boldsymbol{Z}$$

where $\boldsymbol{U}$ is an $n \times 1$ vector of standardized asset returns for the n institutions, $\boldsymbol{A}$ is an $n \times f$ common factor loadings matrix, $\boldsymbol{M}$ is an $f \times 1$ vector of common factors and $\boldsymbol{Z}$ is a $n \times 1$ vector of idiosyncratic factors. All factors are independent of each other with zero expectation and unit variance.

The problem is one of solving for $\boldsymbol{A}$ by minimizing the least squared difference of the model correlation matrix to the target one, such that:

$$\min_{\boldsymbol{A}} \left\{ \left(\boldsymbol{\Sigma} - \boldsymbol{A}\boldsymbol{A}' - \boldsymbol{F} \right) \left(\Sigma - \boldsymbol{A}\boldsymbol{A}' - \boldsymbol{F} \right)' \right\}$$

where $\boldsymbol{F}$ is a diagonal matrix such that $diag(\boldsymbol{F}) = 1 - diag(\boldsymbol{A}\boldsymbol{A}')$.

The numerical solution algorithm then is

1. Guess $\boldsymbol{F}^0$

2. Perform PCA on $\boldsymbol{\Sigma} - \boldsymbol{F}^i$ and compute $\boldsymbol{A}^i = \boldsymbol{E}^i \sqrt{\boldsymbol{\Lambda}}^i$, where i is an iterations counter, $\boldsymbol{E}$ is a matrix of the normalized column eigenvectors of $\boldsymbol{\Sigma} - \boldsymbol{F}$, $\sqrt{\boldsymbol{\Lambda}}$ is Cholesky decomposition of the diagonal matrix containing the f largest eigenvalues of $\boldsymbol{\Sigma} - \boldsymbol{F}$.

3. Calculate $\boldsymbol{F}^{i+1}$

4. Continue with Step 2, until $\boldsymbol{F}^{i+1}$ is sufficiently close to $\boldsymbol{F}^i$.

B.2 Moments of the Student-t Model

We can imply the moments for the latent variable for the Student-t model.

Define as U_i^n the random variable that results from specification (9); and define as U_i^{st} the student-t specification of (22). Due to independence between the multiplicative factor F and all other factors in the specification, the factor model still implies expectation of zero for the latent variable

$$\mathbb{E}(U_i^{st}) = 0$$

The variance is

$$\begin{aligned}\mathbb{V}\mathrm{ar} U_i^{st} &= \mathbb{E}\left(h(F)\,(U_i^n)^2\right) = \mathbb{E}\left(h(F)\right)\mathbb{E}\left(U_i^n\right)^2 \\ &= \mathbb{E}\left(h(F)\right) \\ &= \mathbb{E}\left(\frac{\nu}{F}\right) = \frac{\nu}{\nu-2}\end{aligned}$$

where the last step follows from the expectation of an inverse chi-squared distribution, and we use the shorthand notation introduced earlier where $U_i^n = \left(A_i M + \sqrt{1 - A_i A_i'} Z_i\right)$.

Similarly, it can be shown that $\mathbb{C}\mathrm{ov}(U_i^{st}, U_j^{st}) = \mathbb{E}(U_i^{st} U_j^{st}) - \mathbb{E}(U_i^{st})\mathbb{E}(U_j^{st}) = A_i A_j' \mathbb{E} h(F)$ which implies that he correlation is invariant to the factor F:

$$\mathbb{C}\mathrm{orr}(U_i^{st}, U_j^{st}) = A_i A_j' \tag{26}$$

Following Bolder (2018) we can derive the skew $\mathcal{S}(U_i)$:

$$
\begin{aligned}
\mathcal{S}(U_i^{st}) &= \mathbb{E}\left(\left(\frac{U_i^{st} - \mathbb{E}(U_i^{st})}{\sqrt{\mathbb{V}\mathrm{ar} U_i^{st}}}\right)^3\right) \\
&= \mathbb{E}\left(\left(\frac{U_i^{st}}{\sqrt{\mathbb{V}\mathrm{ar} U_i^{st}}}\right)^3\right) \\
&= \left(\frac{\mathbb{E}h(F)\mathbb{E}U_i}{\sqrt{\mathbb{V}\mathrm{ar} U_i^{st}}}\right)^3 = 0
\end{aligned}
\tag{27}
$$

where the last equality follows from $\mathbb{E}U_i = 0$.

Finally, the kurtosis of $\mathcal{K}(U_i)$ can be determined as

$$
\begin{aligned}
\mathcal{K}(U_i) &= \mathbb{E}\left(\left(\frac{U_i - \mathbb{E}(U_i)}{\sqrt{\mathbb{V}\mathrm{ar} U_i}}\right)^4\right) \\
&= \frac{\mathbb{E}\left((U_i^{st})^4\right)}{(\mathbb{V}\mathrm{ar}(U_i^{st}))^2} \\
&= \frac{\mathbb{E}(h(F)^2)\mathbb{E}\left(U_i^4\right)}{(\mathbb{E}(h(F))^2} \\
&= 3\frac{\mathbb{E}\left(h(F)^2\right)}{\left(\mathbb{E}h(F)\right)^2}
\end{aligned}
$$

Again, considering the fact that from the inverse chi-squared distribution we have

$$
\begin{aligned}
\mathbb{E}h(F) &= \mathbb{E}\left(\frac{\nu}{F}\right) = \frac{\nu}{(\nu-2)} \\
\mathbb{E}(h(F)^2) &= \mathbb{E}\left(\frac{\nu^2}{F^2}\right) = \frac{\nu^2}{(\nu-2)(\nu-4)}
\end{aligned}
\tag{28}
$$

Then we have

$$
\mathcal{K}(U_i^{st}) = 3\frac{\nu-2}{\nu-4}, \quad \text{for } \nu > 4
$$

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