Survey Data and Subjective Beliefs in Business Cycle Models

Anmol Bhandari[1] Jaroslav Borovička[2] Paul Ho[3]

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[1] University of Minnesota

[2] New York University

[3] Federal Reserve Bank of Richmond

INTRODUCTION

- Household forecasts contain large and systematic biases
  - biases for various macro variables exhibit co-movement at business cycle frequency
  - common variation interpretable as time-varying pessimism/optimism
- What drives these biases?
- Do they matter for macroeconomic outcomes / asset valuations?

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This paper

- A general framework where households' subjective beliefs are
  - jointly endogenously determined with equilibrium outcomes
  - disciplined using survey data on household expectations
- A quantitative application with frictional labor markets

OUTLINE

- Survey evidence
- Framework for subjective beliefs
  - time-varying concerns for model misspecification
  - tractable solution
- Structural business cycle model
  - frictional labor market
  - monetary policy
- Role of fluctuations in subjective beliefs

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SURVEY DATA

Data source

- University of Michigan Survey: households
- New York Fed Survey of Consumer of Expectations

Time series

- Unemployment rate
- Inflation rate

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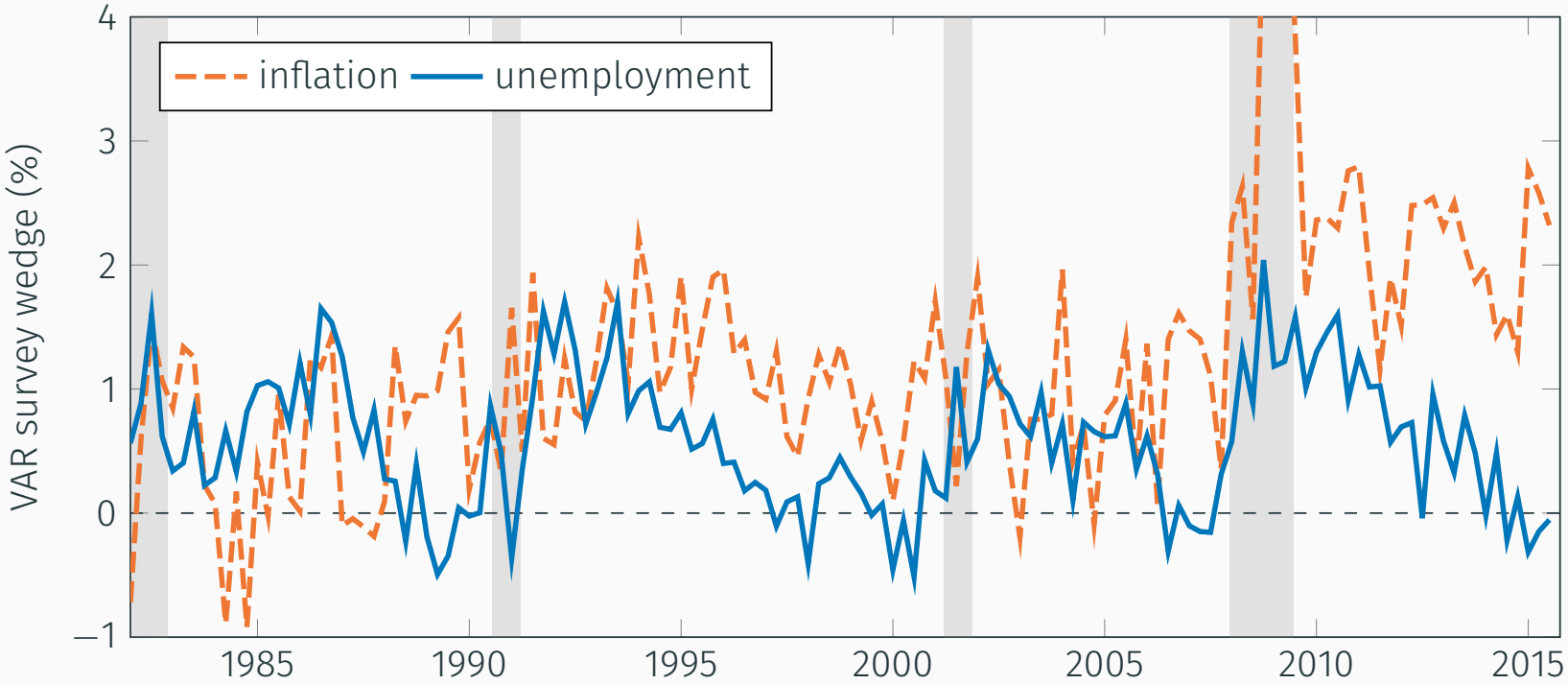
Construct **belief wedges**

- **Michigan Survey** minus **rational forecast**
- rational forecast: model-implied forecast (VAR) or SPF

BELIEF WEDGES: MICHIGAN SURVEY MINUS VAR



1981Q2–2015Q4	mean	std	correlation matrix $\Delta(u)$	$\Delta(\pi)$	output gap	GDP growth
unemp. wedge $\Delta(u)$	0.58	0.54	1.00	0.23	−0.54	−0.32
inflation wedge $\Delta(\pi)$	1.25	1.03		1.00	−0.37	−0.53

ROBUSTNESS

Findings consistent with

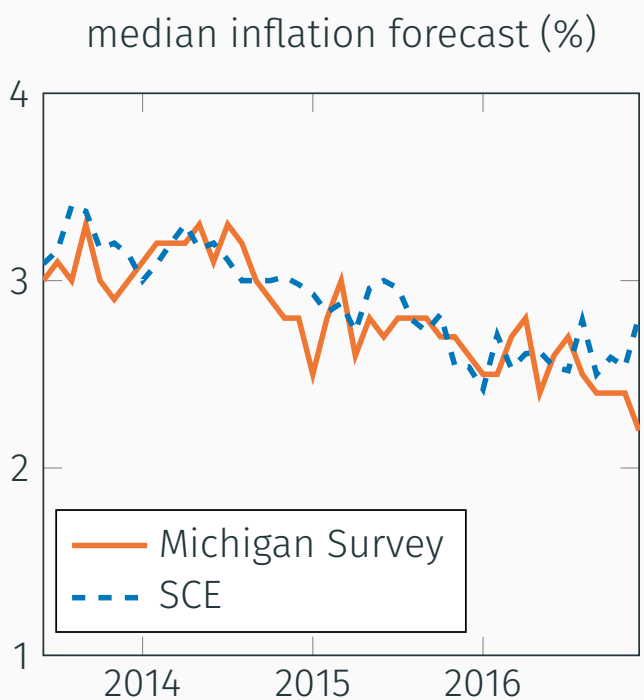
- New York Fed Survey of Consumer Expectations
- Bank of England Inflation Attitudes Survey

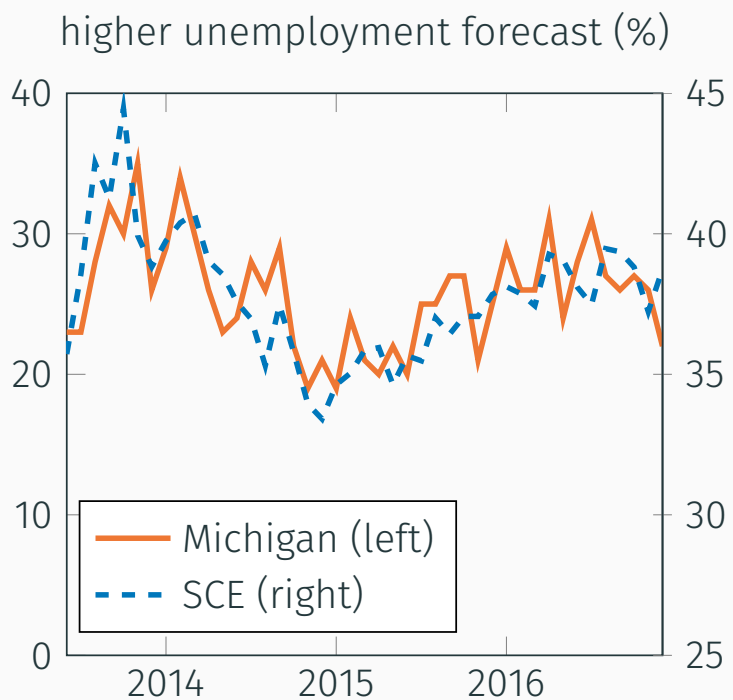
Patterns robust to

- SPF instead of VAR forecasts
- median household forecasts instead of average
- alternative sample periods

Same patterns appearing in cross-sectional household data

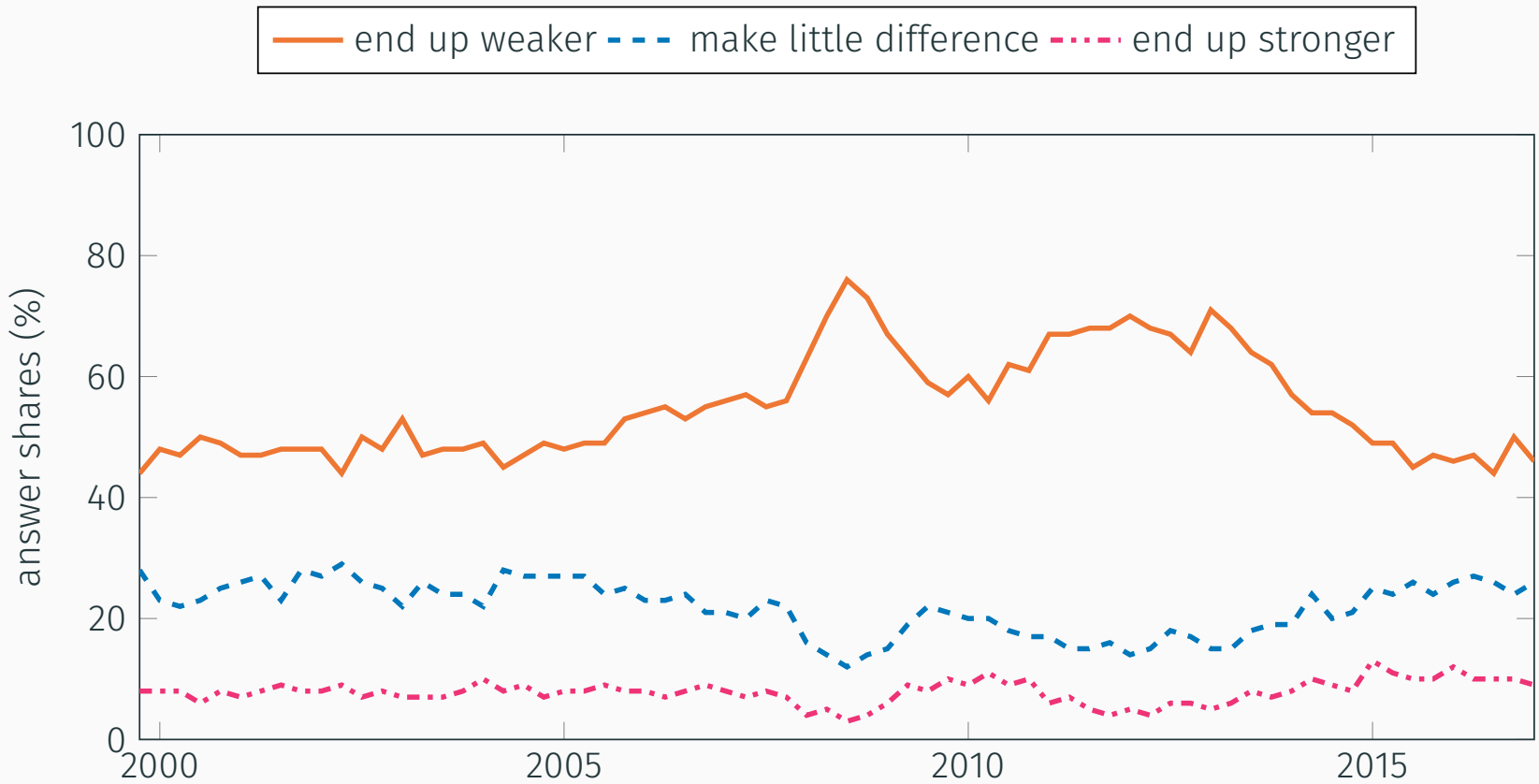
MICHIGAN SURVEY AND FRBNY SURVEY OF CONSUMER EXPECTATIONS





Left panel: Median inflation forecast in the Michigan Survey and in the New York Fed Survey of Consumer Expectations. ***Right panel:*** Share of respondents in the Michigan Survey stating that unemployment will be higher during the next 12 months, and the mean probability that unemployment will be higher one year from now in the New York Fed Survey.

BANK OF ENGLAND INFLATION ATTITUDES SURVEY



Bank of England Inflation Attitudes Survey, shares of answers to the question: "*If prices started to rise faster than they do now, do you think Britain's economy would ...*" Data sample 1999Q4–2017Q1.

CROSS-SECTIONAL EVIDENCE

Substantial cross-sectional dispersion in forecasts.

Systematic relationship between and unemployment and inflation forecasts.

- Households who overpredict inflation more also overpredict unemployment more.
  - True for demographic groups, individual households, on average over time (pooled sample) as well as in month-by-month regressions.
- FRBNY data also confirm this cross-sectional relationship for other variables.
  - Forecasts of earnings growth, job separation rates, job finding rates and stock prices, with expected signs.

Interpretation: Household-level heterogeneity in magnitude of belief biases but these biases share a common origin.

Details on cross-sectional evidence

SUBJECTIVE BELIEFS AND DECISION-MAKING

Assumption: Agents make decisions according to their subjective beliefs, which are also reported in surveys.

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Asset pricing

- **Greenwood and Shleifer (2014)**: return expectations and subsequent realized returns
- **Nagel and Xu (2018)**: stock market dynamics

Corporate finance

- **Gennaioli, Ma, Shleifer (2015)**: managers' surveys and firm investment

Macroeconomics

- **Malmendier and Nagel (2016)**: Michigan survey responses and borrowing and lending decisions
- **Bachmann, Berg and Sims (2015)**: Michigan survey responses and consumer spending
- **Crump, Eusepi and Tambalotti (2015)**: New York Fed survey responses and planned consumption

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MODEL OF TIME-VARYING PESSIMISM/OPTIMISM

- preferences of an agent with concern for model misspecification

$$V_t = u_t + \beta E_t [V_{t+1}]$$

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  - penalty parameter θ_t (rational expectations $\theta_t = 0$)

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- implied belief distortion m_{t+1} defines a probability measure $\widetilde{P}$

$$m_{t+1} = \frac{\exp\left(-\theta_t V_{t+1}\right)}{E_t \left[\exp\left(-\theta_t V_{t+1}\right)\right]}$$

- θ_t controls the magnitude of the belief distortion
  - sign of θ_t determines whether beliefs are pessimistic or optimistic

Belief wedges

$$\Delta_t^{(1)} = \widetilde{E}_t\left[x_{t+1}\right] - E_t\left[x_{t+1}\right] = Cov_t\left(m_{t+1}, x_{t+1}\right) \approx -\theta_t Cov_t\left(V_{t+1}, x_{t+1}\right)$$

- $Cov_t\left(V_{t+1}, x_{t+1}\right)$ endogeneously determines which states are 'bad'
  - $\widetilde{P}$ overweighs states with low continuation utility V_{t+1} (when θ_t positive)
- $-\theta_t$ is the time-varying scaling of the distortions

Theory of subjective beliefs

- can be embedded in GE framework with endogenous x_{t+1} and V_{t+1}
- discipline from cross-equation restrictions
- discipline from survey data

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MODEL STRUCTURE AND FORWARD-LOOKING DECISIONS

Household endowed with subjective beliefs exhibiting time-varying pessimism/optimism

- consumption Euler equation

DMP labor market

- vacancy-posting decision on the side of firms

Monopolistic producers under sticky prices

- firms' price-setting decision

Taylor-type monetary policy rule

REPRESENTATIVE HOUSEHOLD

Preferences

$$V_t = \min_{\substack{m_{t+1} \\ E[m_{t+1}]=1}} \log C_t + \beta E_t \left[m_{t+1} V_{t+1} \right] + \beta \frac{1}{\theta_t} E_t \left[m_{t+1} \log m_{t+1} \right]$$

Family of a unit mass of workers

- unemployed workers search
- employed workers supply a unit of labor

Labor service sector

- hires workers l_t in a market s.t. search frictions with Nash bargaining
- supplies labor services h_t competitively

Intermediate goods sector

- monopolistically competitive producers hire labor input

$$Y_{i,t} = A_t h_{i,t} - \phi$$

- subject to Calvo pricing frictions

Final goods producers

- aggregate intermediate goods using CES technology

$$Y_t = \left[\int_0^1 (Y_{i,t})^{\frac{1}{\lambda}} \, di \right]^{\lambda}$$

Monetary policy rule

$$\log \frac{R_t}{\overline{R}} = \rho_r \log \frac{R_{t-1}}{\overline{R}} + (1-\rho_r)\left[r_\pi \log \frac{\pi_t}{\overline{\pi}} + r_y \log \frac{Y_t}{Y^*}\right] + \sigma_r w_t^r$$

Neutral technology process

$$\log A_{t+1} = \rho_a \log A_t + \sigma_a w_{t+1}^a$$

Belief dynamics

$$\theta_{t+1} = (1-\rho_\theta)\,\overline{\theta} + \rho_\theta \theta_t + \sigma_\theta w_{t+1}^\theta$$

EQUILIBRIUM

Vector of **equilibrium conditions**

$$0 = \widetilde{E}_t \left[\mathrm{g} \left(x_{t+1}, x_t, x_{t-1}, w_{t+1}, w_t \right) \right]$$

- forward-looking equations under the subjective belief $\widetilde{P}$

$\widetilde{P}$ determined endogenously using the **continuation value**

$$m_{t+1} = \frac{\exp\left(-\theta_t V_{t+1}\right)}{E_t \left[\exp\left(-\theta_t V_{t+1}\right)\right]}$$

- heterogeneous subjective beliefs possible

Fixed point yields **equilibrium law-of-motion**

$$x_{t+1} = \psi \left(x_t, w_{t+1} \right)$$

Details on solution method

EXAMPLE: VACANCY-POSTING DECISION

Value of a match to the firm

$$J_t = \vartheta_t - \xi_t + \rho \widetilde{E}_t \left[s_{t+1} J_{t+1} \right].$$

- marginal product ϑ_t, wage ξ_t

Role of subjective beliefs in the frictional labor market

- value of the match is forward-looking
- incentives to search and hire depend on subjective forecasts

Equivalently for other forward-looking decisions

CALIBRATION: GOODS AND LABOR MARKET AND POLICY PARAMETERS

Parameters		Value
β	Discount factor	0.994
ε	Elasticity of substitution intermediate goods	6.00
χ	Calvo price stickiness	0.75
λ	Wage rigidity	0.925
$\bar{\pi}$	Monetary policy rule: intercept	0.01
ρ_r	Monetary policy rule: smoothing	0.84
r_π	Monetary policy rule: loading on inflation	1.60
r_y	Monetary policy rule: loading on output	0.028
$100\sigma_r$	Volatility of monetary policy shock	0.078
ρ	Job survival probability	0.89
μ	Matching efficiency	0.67
ν	Curvature of matching function	0.72
η	Worker's bargaining weight	0.72
κ	Vacancy posting costs	0.09
D	Flow benefits of unemployment	0.57

CALIBRATION: TFP PROCESS AND BELIEF FLUCTUATIONS

TFP process disciplined by Fernald (2014) data

	TFP process	
ρ_a	Persistence of TFP shock	0.84
$100\sigma_a$	Volatility of TFP shock	0.568

Dynamics of θ disciplined by belief wedge data

- ρ_θ using persistence of the principal component of belief wedges
- $\overline{\theta}$ and σ_θ using mean and volatility of belief wedges.

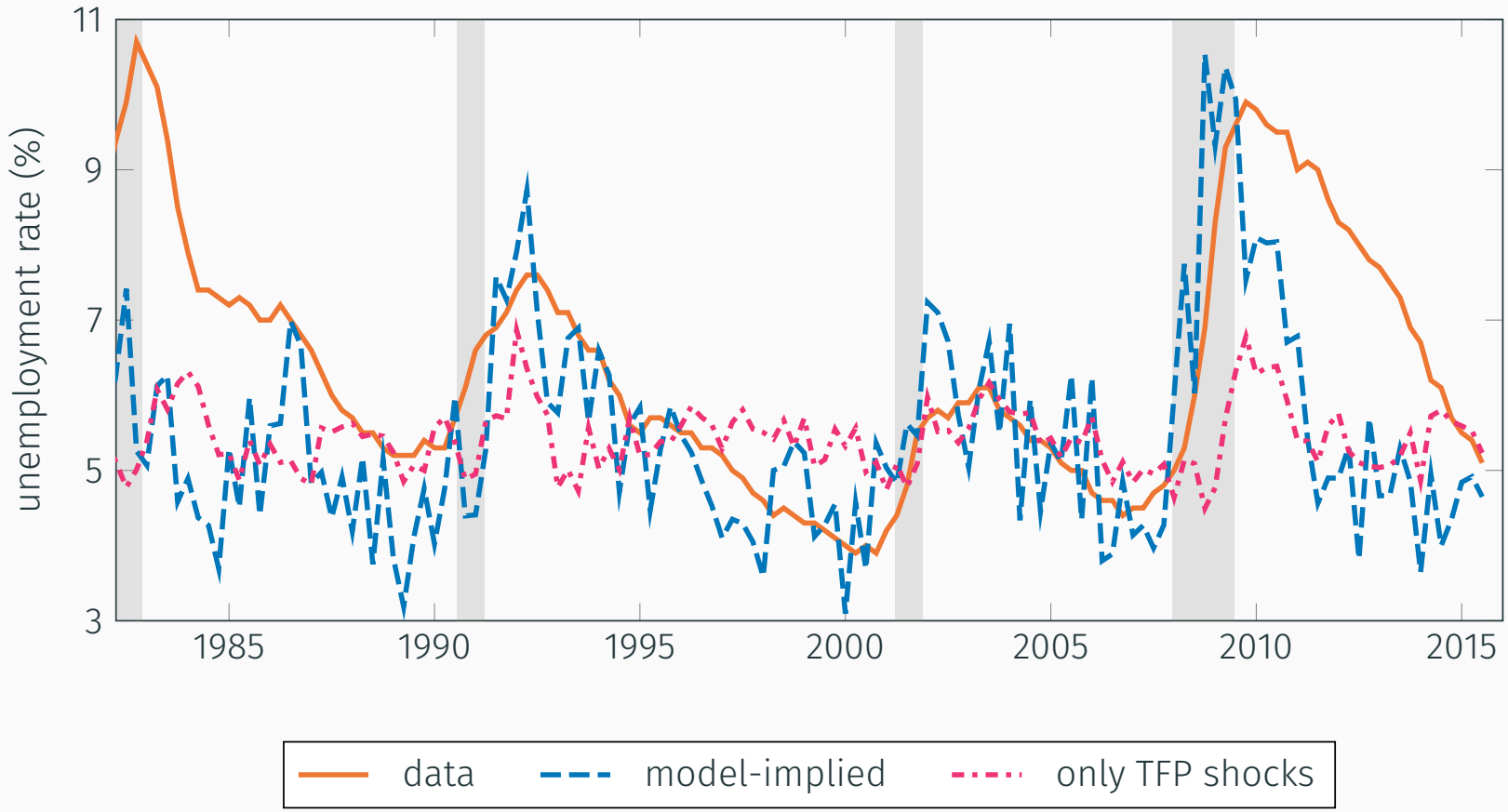
	Shocks	
$\overline{\theta}$	Mean belief wedges	5.64
ρ_θ	Persistence of PC (wedges)	0.714
σ_θ	Volatility of belief wedges	4.3

MODEL FIT

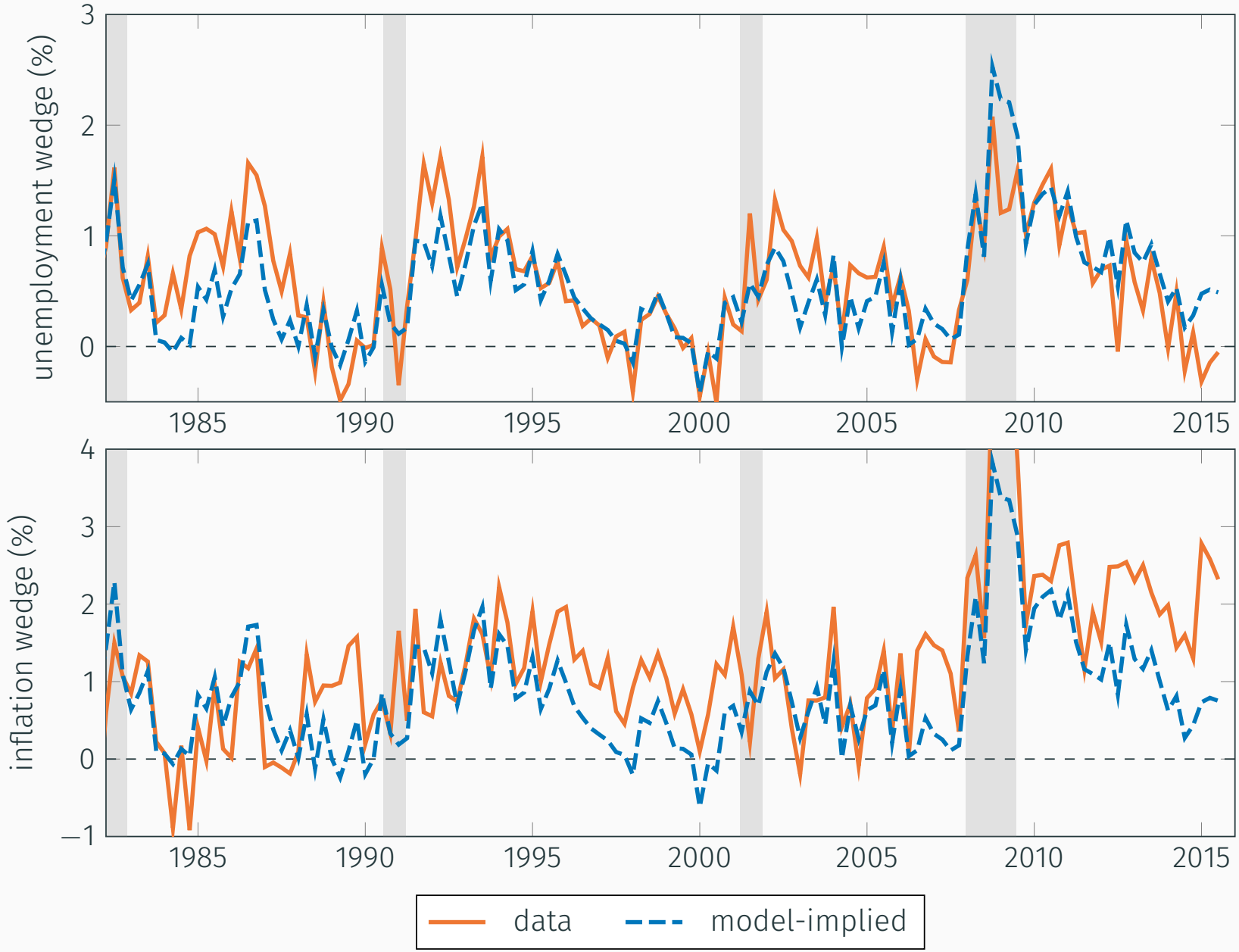
Moment	Data	Model		
		full	no θ_t	no TFP
Mean of inflation wedge	1.25	0.90	0.00	−0.32
Mean of unemployment wedge	0.54	0.55	0.00	0.54
Volatility of inflation wedge	1.03	0.73	0.00	0.26
Volatility of unemployment wedge	0.45	0.45	0.00	0.43
Volatility of inflation	1.40	1.41	1.15	0.82
Volatility of output	2.32	2.22	1.55	1.09
Volatility of unemployment	1.65	1.39	0.55	0.87

Data and model-implied theoretical moments for macroeconomic quantities and belief wedges. The sample period for the Data column is 1982Q1–2015Q4. Values in all columns are in percentages and annualized.

MODEL FIT: UNEMPLOYMENT DYNAMICS



MODEL FIT: BELIEF WEDGES



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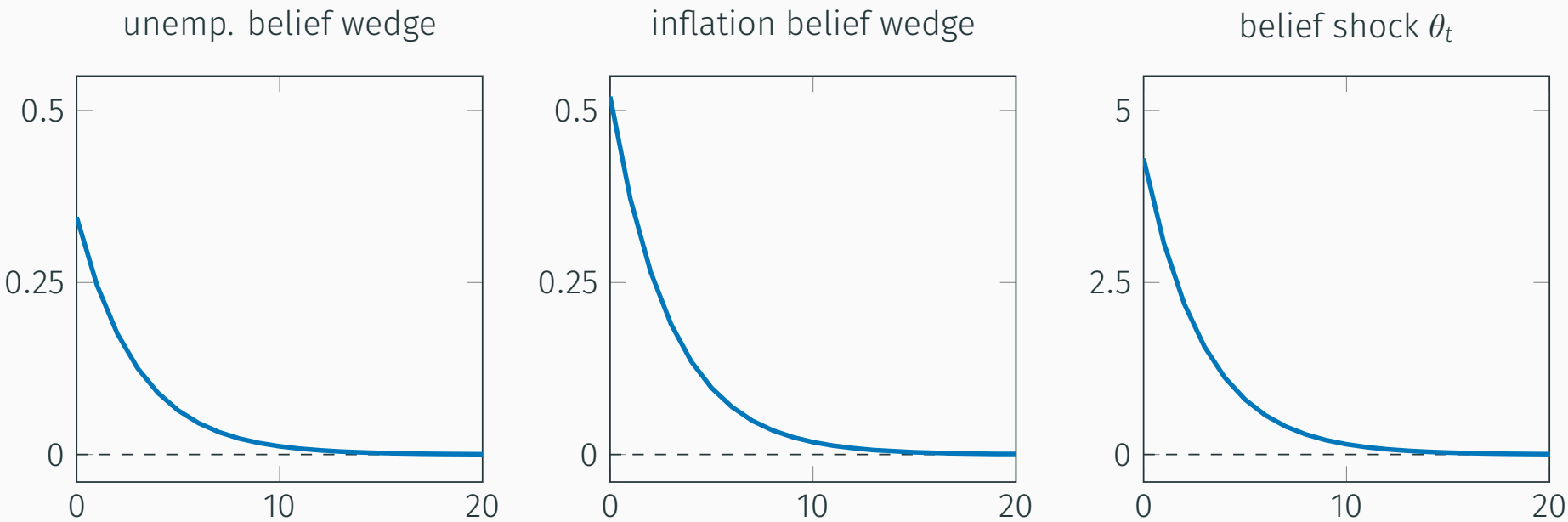
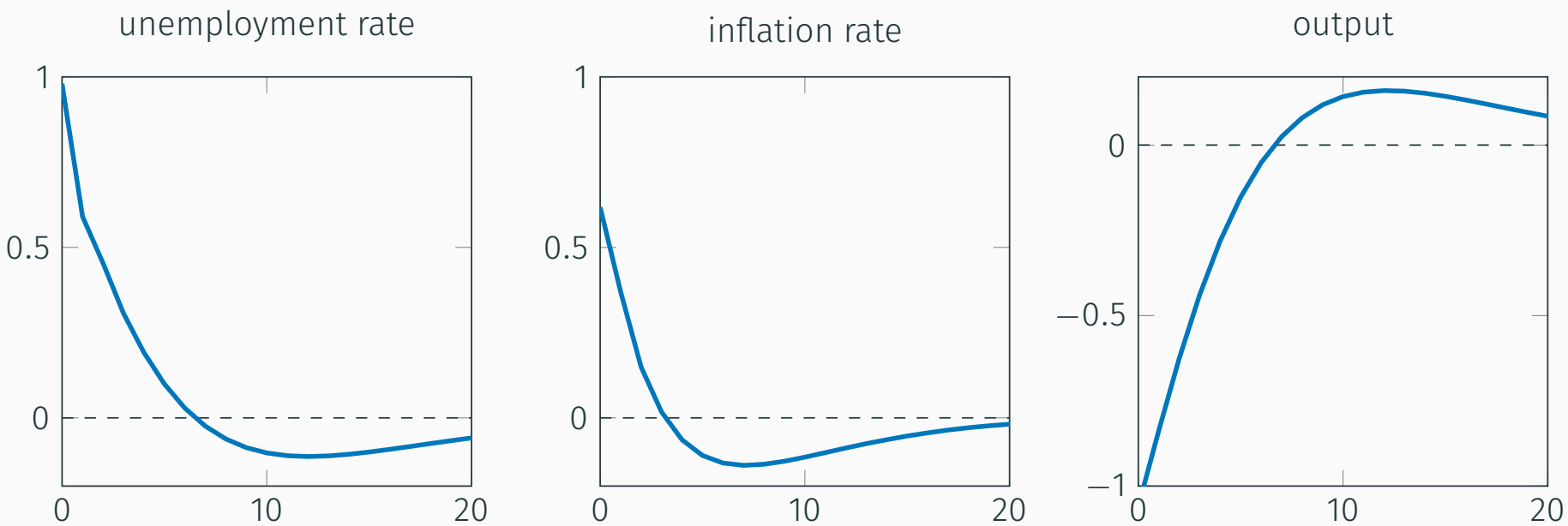


An increase in model uncertainty makes the subjective model more pessimistic relative to the data-generating process.

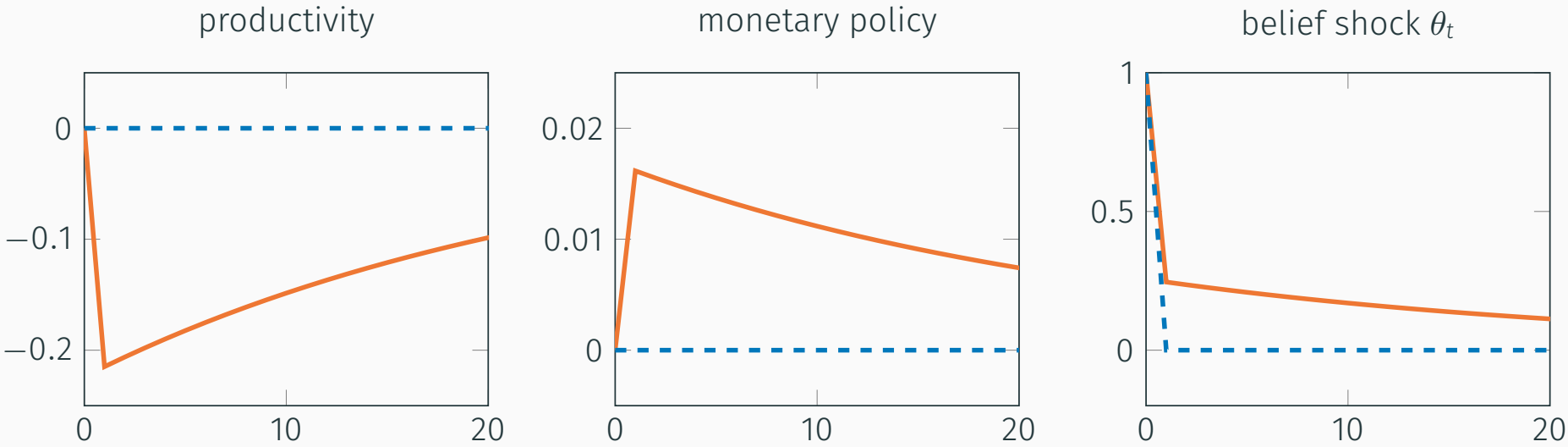
- stronger overprediction of inflation and unemployment rate
- stronger underprediction of GDP growth

MACROECONOMIC VARIABLES



- increase in model uncertainty is contractionary
- contrary to a typical demand shock, inflation does not fall

SHOCKS UNDER DATA-GENERATING AND SUBJECTIVE MODEL



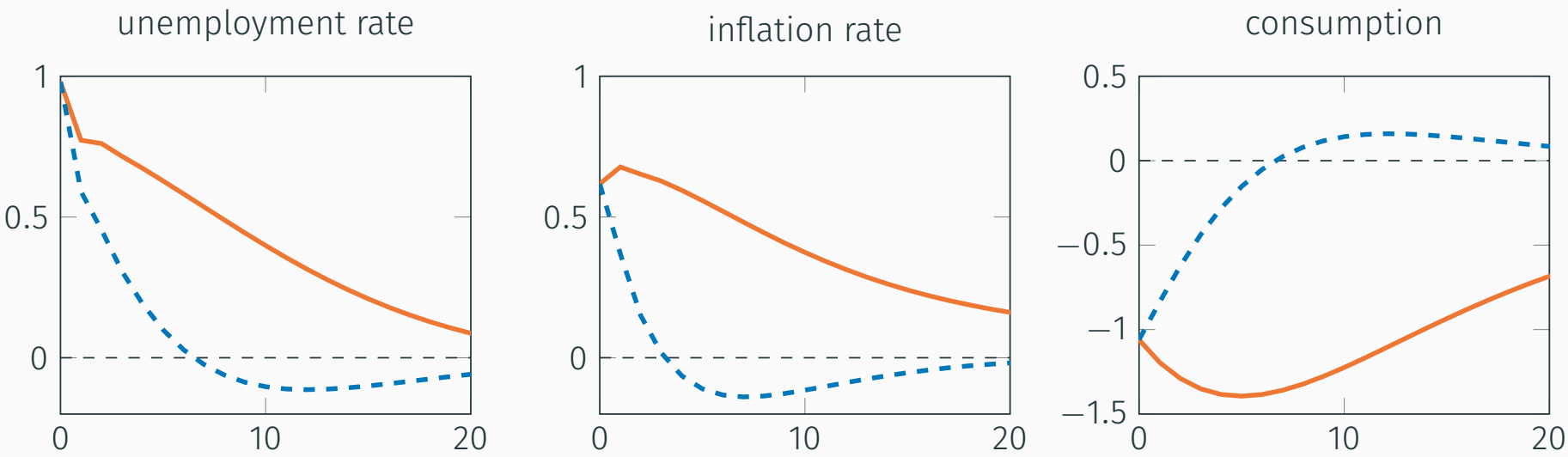
Increase in pessimism alters subjective beliefs about fundamental shocks.

- under the data-generating measure, shocks are independent
- subjective model induces a correlated structure on the innovations

Increase in pessimism is associated, under the subjective model, with

- slower technological growth
- monetary tightening

MACROECONOMIC VARIABLES UNDER DGP AND SUBJECTIVE MODEL



Increased pessimism means higher fear of future adverse shocks.

- fear of future adverse TFP shocks dominates
- agents fear more the 'high unemployment-high inflation' scenario
- consistent with positive comovement of unemployment and inflation wedges

SUBJECTIVE BELIEFS OF CONSUMERS, WORKERS, AND FIRMS

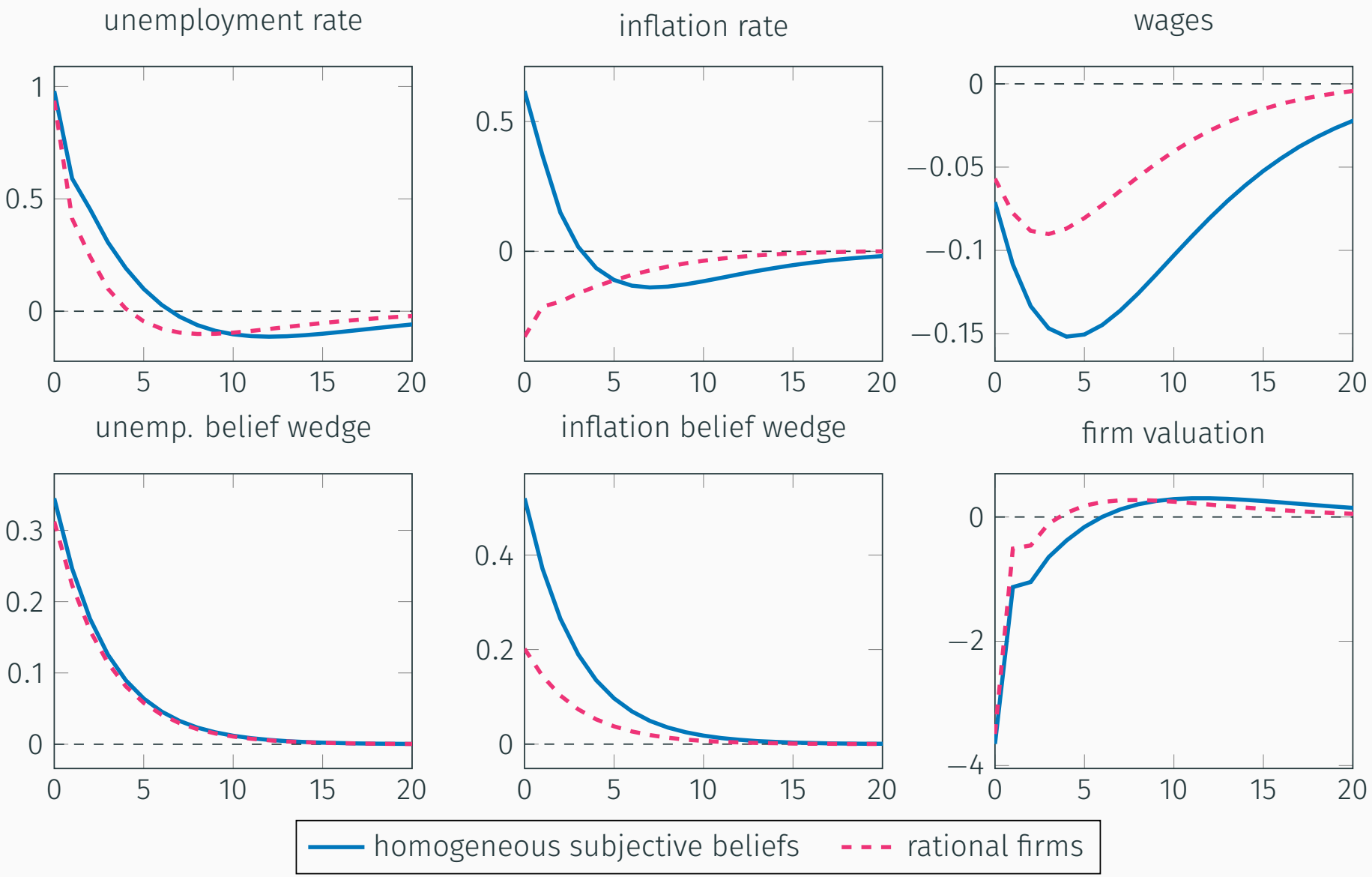
Subjective beliefs affect outcomes through forward-looking behavior of consumers, workers and firms

- Consumers' distorted Euler equation lowers aggregate demand
- Goods market firms expect higher marginal costs and future inflation
- Workers' and firms' subjective valuations alter outside options in bargaining
- Labor market firms' lower valuation of match surplus lowers vacancies

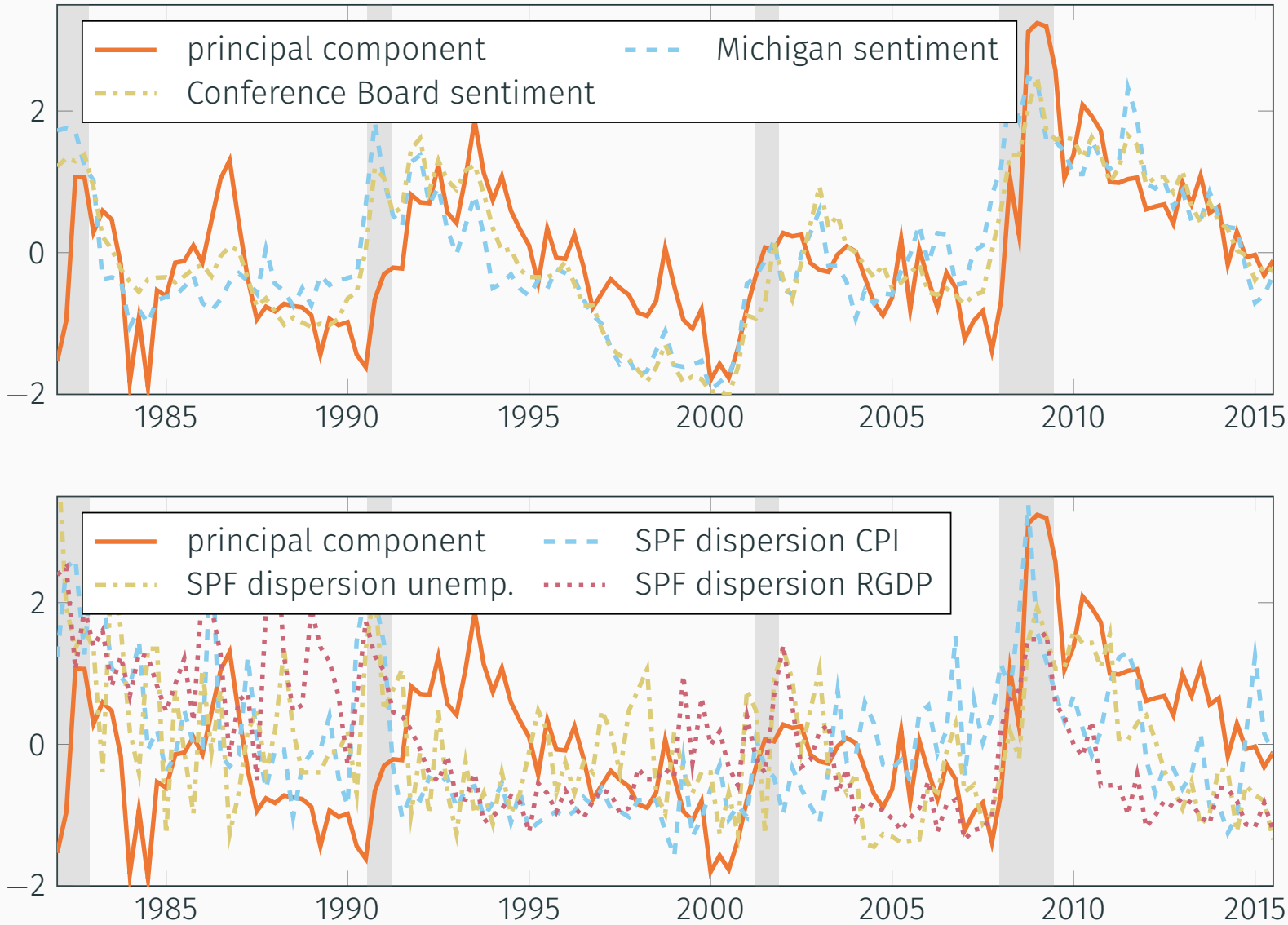
What is the role of subjective beliefs imposed on individual agents?

- Impose correct beliefs on the side of the firms

ROLE OF FIRMS' BELIEFS



BELIEF WEDGES AND MEASURES OF UNCERTAINTY



CONCLUSION AND FURTHER RESEARCH

Developed a framework linking

- survey data that document fluctuations in subjective beliefs
- theoretical framework that determines and endogenous model of time-varying pessimism/optimism
- equilibrium model that propagates fluctuations in subjective beliefs into the macroeconomy

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Next steps

- heterogeneity in subjective beliefs
  - uninsurable idiosyncratic risk: exposure to risk and subjective beliefs will differ in the cross-section
- optimal policy
  - role of expectations management
- asset pricing
  - rich data from investors' surveys

ENDOGENOUS BELIEF DISTORTIONS IN THE FACTOR MODEL

$$y_{t+1} = \psi_y y_t + \psi_{yf} f_{t+1} + \psi_{yw} w^y_{t+1}$$
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Households' expectational errors for $w_{t+1} = \left(\left(w^y_{t+1} \right)', w^f_{t+1} \right)'$

$$\theta_t = (F_y, F_f) \begin{pmatrix} y_t \\ f_t \end{pmatrix} \qquad \widetilde{E}_t \left[w_{t+1} \right] = H \theta_t$$

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$$\Delta^{(\tau)}_t = G^{(\tau)}_x x_t + G^{(\tau)}_0 \qquad x_t = \left(y'_t, f_t\right)'$$

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$$F_y = 0 \qquad \Longrightarrow \qquad \theta_t = f_t$$

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back to factor model

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$$\Delta_t^{(1)} = \psi_w \widetilde{E}_t\left[w_{t+1}\right] = -F\left(\bar{x} + x_{1t}\right)\left(\psi_w \psi_w'\right) V_x'.$$

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back to robust preferences

LINEAR APPROXIMATION: EXPANSION DETAILS

Small noise approximation of x_t

$$x_t(\mathrm{q}) = \psi\left(x_{t-1}(\mathrm{q}), \mathrm{q}w_t, \mathrm{q}\right)$$

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$$x_t(\mathrm{q}) \approx \bar{x} + \mathrm{q}x_{1t} + \frac{\mathrm{q}^2}{2}x_{2t} + \ldots$$

Approximation of V_t

$$V_t(\mathrm{q}) = u\left(x_t(\mathrm{q}), \mathrm{q}\right) - \beta \frac{\mathrm{q}}{F\left(\bar{x} + x_{1t}\right)} \log E_t\left[\exp\left(-\frac{F\left(\bar{x} + x_{1t}\right)}{\mathrm{q}}\right) V_{t+1}(q)\right]$$

Approximation of distortion

$$m_{t+1}(q) = \frac{\exp\left(-\frac{\theta_t}{q} V_{t+1}(q)\right)}{E_t\left[\exp\left(-\frac{\theta_t}{q} V_{t+1}(q)\right)\right]} \approx m_{0,t+1} + q m_{1,t+1}$$

LINEAR APPROXIMATION: DISTORTIONS AND EQUILIBRIUM

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Expansion of equilibrium conditions

$$0 = E_t\left[m_{0,t+1} g_{0,t+1}\right]$$
$$0 = E_t\left[m_{0,t+1} g_{1,t+1}\right] + E_t\left[m_{1,t+1} g_{0,t+1}\right] = E_t\left[m_{0,t+1} g_{1,t+1}\right]$$

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- First-order expansion is a system of linear second-order stochastic difference equations
- Solve for ψ_q, ψ_x, ψ_w by comparing coefficients

Linear approximation and solution

$$x_t \approx \bar{x} + x_{1t}$$

- $\bar{x}$: zero-th order dynamics — deterministic steady state
- x_{1t}: first order dynamics — linear dynamics

$$x_{1t+1} = \psi_q + \psi_x x_{1t} + \psi_w w_{t+1}$$

- ψ_q, ψ_x, ψ_w functions of dynamics of θ_t

LINEAR APPROXIMATION

Key idea

- perturbation approximation scales down the volatility of shocks

$$x_{t+1}(\mathrm{q}) = \psi\left(x_t(\mathrm{q}), \mathrm{q}w_{t+1}\right) \qquad \mathrm{q} \to 0$$

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- subjective belief distortion vanishes in the limit

$$m_{t+1} = \frac{\exp\left(-\theta_t V_{t+1}(\mathsf{q})\right)}{E_t\left[\exp\left(-\theta_t V_{t+1}(\mathsf{q})\right)\right]} \to 1$$

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- perturbation approximation scales down the volatility of shocks

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- compensate by **scaling up** the model misspecification concern θ_t

$$m_{t+1} = \frac{\exp\left(-\frac{\theta_t}{\mathsf{q}} V_{t+1}(\mathsf{q})\right)}{E_t\left[\exp\left(-\frac{\theta_t}{\mathsf{q}} V_{t+1}(\mathsf{q})\right)\right]}$$

Non-degenerate limit $\Longrightarrow$ first-order effects of subjective beliefs.

SUBJECTIVE BELIEFS AND BELIEF WEDGES

Equilibrium dynamics

$$x_{1t+1} = \psi_q + \psi_x x_{1t} + \psi_w w_{t+1}$$

- Distribution of shocks under subjective belief

$$w_{t+1} \sim N\left(-\theta_t \left(V_x \psi_w\right)', I_{k\times k}\right)$$

Belief wedges

$$\Delta_t^{(1)} = \widetilde{E}_t\left[x_{1t+1}\right] - E_t\left[x_{1t+1}\right] = \psi_w \widetilde{E}_t\left[w_{t+1}\right] = -\theta_t\left(\psi_w \psi_w'\right) V_x'.$$

- V_x — exposure of value function to state variables
- ψ_w — exposure of state variables to shocks
- $-\theta_t$ — time-varying scaling of the distortions

Back to equilibrium

DETECTION ERROR PROBABILITIES

Alternative calibration for $\bar{\theta}$

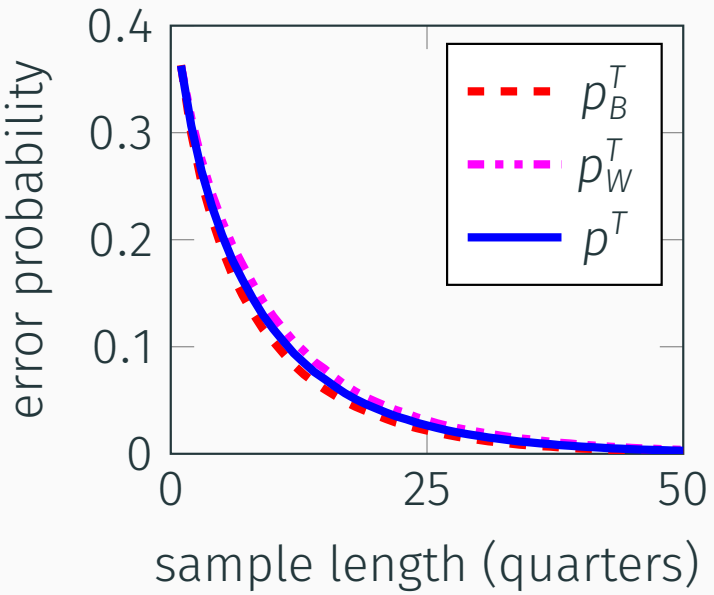
- consider probability of classification error

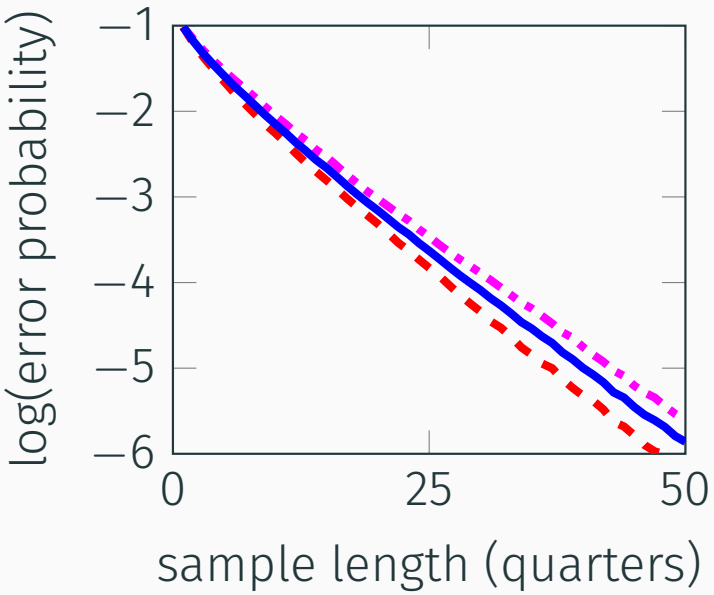
$$p_B^T = P\left(\log L_W^T > \log L_B^T \mid B\right)$$
$$p_W^T = P\left(\log L_W^T < \log L_B^T \mid W\right)$$

- detection error probability

$$p^T = \frac{1}{2}\left(p_B^T + p_W^T\right)$$

DETECTION ERROR PROBABILITIES





Detection error probability of 5% after 5 years of data

- lower than existing calibrations in finance

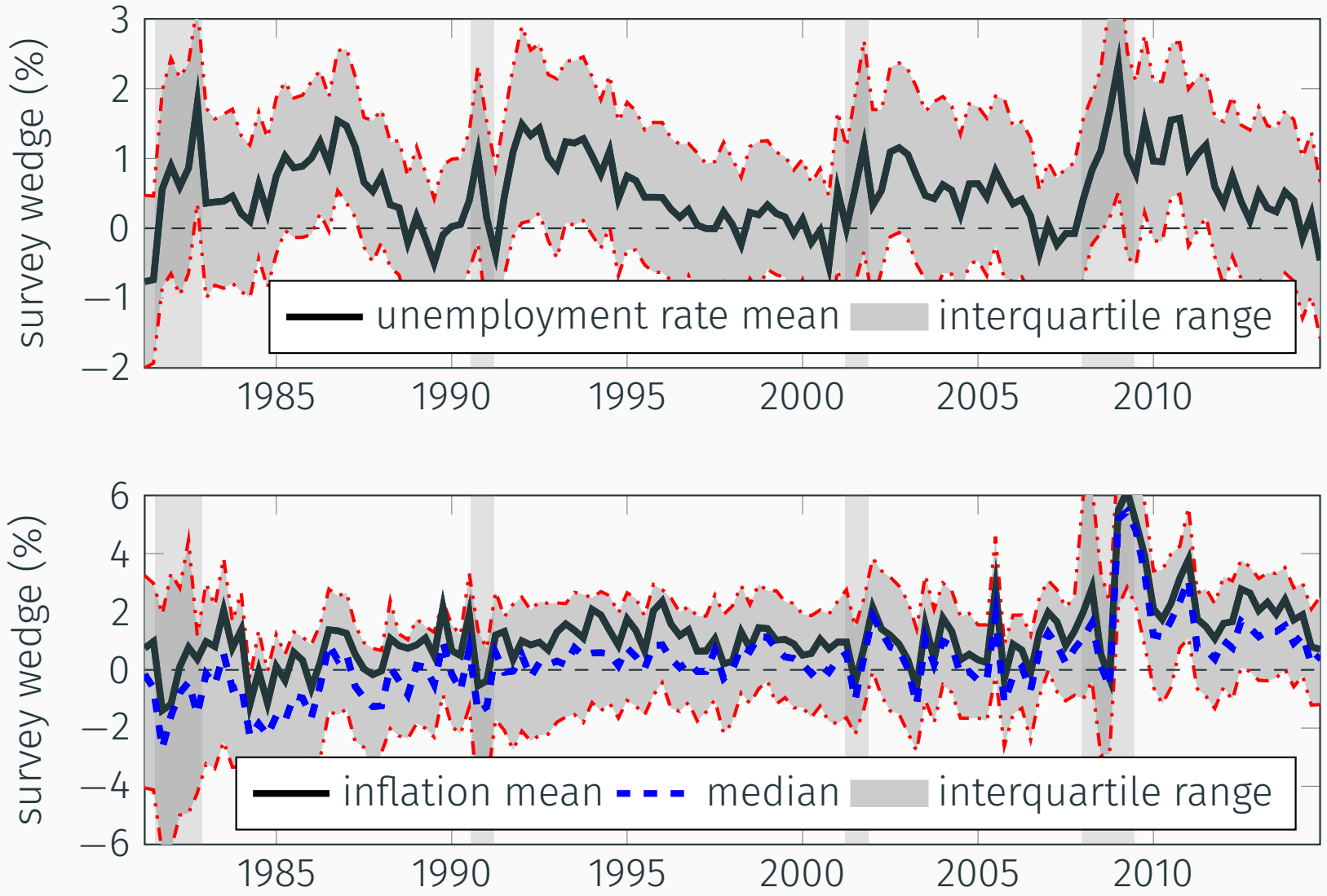
back

MICHIGAN SURVEY BY DEMOGRAPHIC GROUP

	actual	SPF	all	18-34	35-44	45-54	55-64	65+	W	NC	NE	S
π	2.82	3.01	4.05	4.14	4.07	4.02	3.86	3.96	3.98	4.00	4.00	4.18
u	6.34	6.34	6.88	6.74	6.90	6.97	6.98	6.88	6.86	6.87	6.93	6.86
g_y	2.70	2.74	1.07									
			male	female	bottom	2nd Q	3rd Q	top	HS	SC	COL	GS
π			3.47	4.56	4.99	4.28	3.77	3.27	4.58	3.98	3.57	3.46
u			6.79	6.95	7.03	6.90	6.85	6.73	6.97	6.88	6.77	6.79

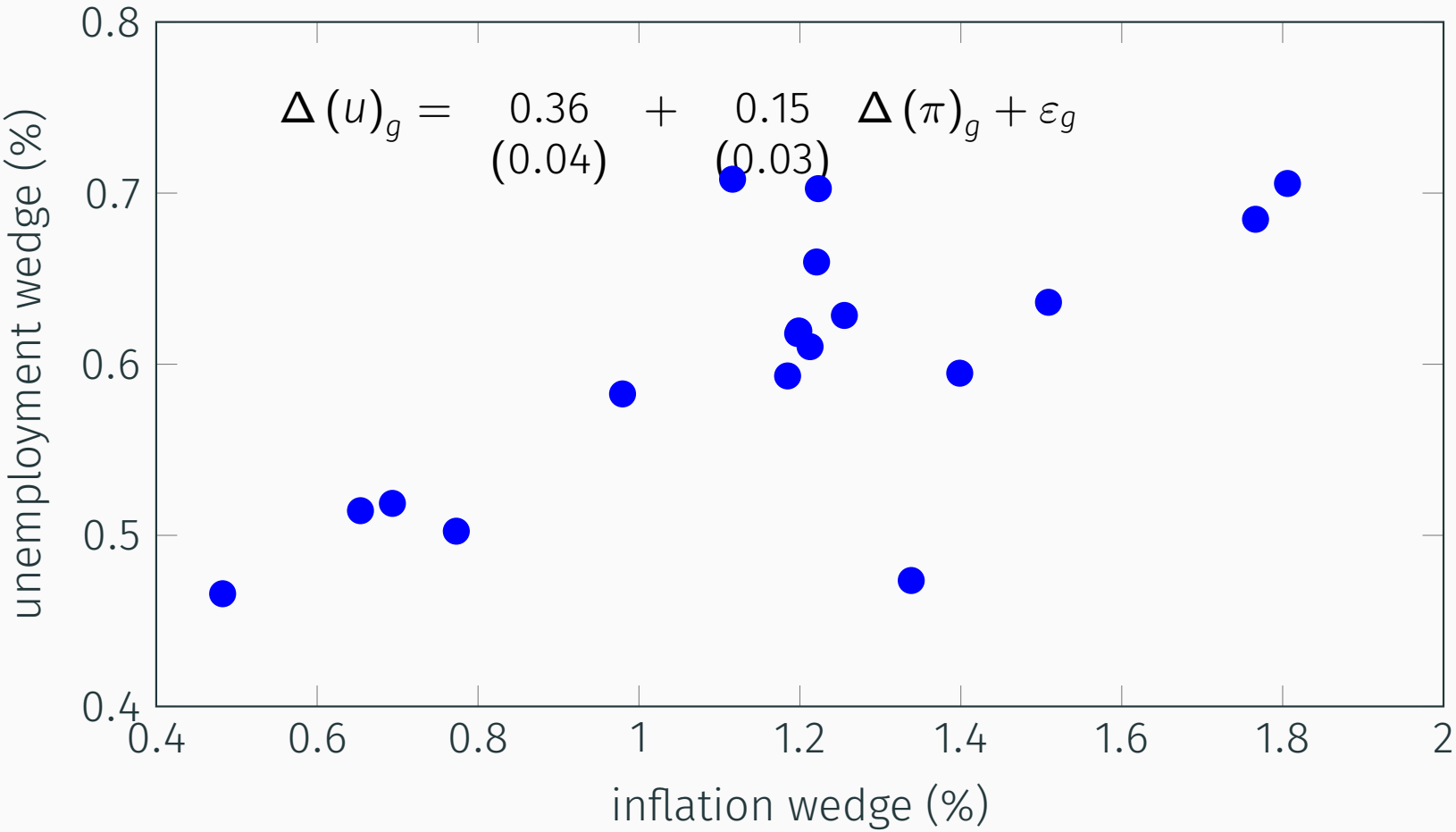
Table 1: Demographic characteristics of households' expectations on inflation and unemployment rate. Time-series averages, all values are in percent, time period 1981Q2–2015Q4. *Actual*: actual average inflation and unemployment rate; *SPF*: average SPF forecast; *all*: average household forecast; *18-34 etc*: age groups; *W*: West region; *NC*: North-Central; *NE*: North-East; *S*: South; *bottom, 2nd Q, 3rd Q, top*: income quartiles; *HS*: high-school education; *SC*: some college; *COL*: college degree; *GS*: graduate studies.

MICHIGAN SURVEY FORECAST DISPERSION



DIFFERENCES ACROSS DEMOGRAPHIC GROUPS

*Do **demographic groups** which **on average** overpredict inflation relatively more also overpredict unemployment relatively more?*



DIFFERENCES ACROSS DEMOGRAPHIC GROUPS

*In times when **group** g overpredicts inflation more **relative to population**, does it also overpredict unemployment more relative to population?*

$$\Delta(u)_{g,t} - \Delta(u)_t = \alpha_g + \beta_g \left[\Delta(\pi)_{g,t} - \Delta(\pi)_t\right] + \varepsilon_{g,t}$$

	18-34	35-44	45-54	55-64	65+	W	NC	NE	S
$\widehat{\beta}_g$	3.14	1.91	1.16	1.83	1.84	2.21	0.65	1.74	2.02
std. err.	0.59	0.51	0.57	0.43	0.30	0.58	0.59	0.54	0.63

	male	female	bottom	2nd Q	3rd Q	top	HS	SC	COL	GS
$\widehat{\beta}_g$	2.53	2.32	1.52	0.55	0.65	0.69	3.16	4.22	2.08	3.59
std. err.	0.69	0.67	0.48	0.58	0.60	0.91	0.70	0.58	0.65	0.83

DIFFERENCES ACROSS DEMOGRAPHIC GROUPS

*In times when **individual households** overpredict inflation more **relative to population/group,** do they also also overpredict unemployment relatively more?*

- Pooled sample relative to population, demographic controls

$$\Delta(u)_{i,g,t} - \Delta(u)_t = \underset{(0.03)}{1.26} \left[\Delta(\pi)_{i,g,t} - \Delta(\pi)_t\right] + \varepsilon_{i,g,t}$$

DIFFERENCES ACROSS DEMOGRAPHIC GROUPS

In times *when* ***individual households*** *overpredict inflation more* ***relative to population/group,*** *do they also also overpredict unemployment relatively more?*

- Sample relative to demographic group

$$\Delta(u)_{i,g,t} - \Delta(u)_{g,t} = \beta_g \left[\Delta(\pi)_{i,g,t} - \Delta(\pi)_{g,t}\right] + \varepsilon_{i,g,t}$$

	population	education	income	region	age	sex
$\hat{\beta}_g$	1.22	1.19	1.29	1.22	1.22	1.18
std. err.	0.02	0.02	0.03	0.02	0.02	0.02

DIFFERENCES ACROSS DEMOGRAPHIC GROUPS

In times *when* ***individual households*** *overpredict inflation more* ***relative to population/group,*** *do they also also overpredict unemployment relatively more?*

- Month-by-month regressions relative to demographic group

$$\Delta(u)_{i,g,t} - \Delta(u)_{g,t} = \beta_{g,t}\left[\Delta(\pi)_{i,g,t} - \Delta(\pi)_{g,t}\right] + \varepsilon_{i,g,t}$$

	population	education	income	region	age	sex
average $\hat{\beta}_{g,t}$	2.13	2.08	2.11	2.13	2.13	2.08
std. dev. $\hat{\beta}_{g,t}$	1.28	1.22	1.26	1.28	1.27	1.27

FRBNY DATA: DIFFERENCES ACROSS DEMOGRAPHIC GROUPS

Correlations between relative overpredictions for alternative forecasts across demographic groups

	infl.	earn.	inc.	sep.	find.	stock	unemp
inflation	1						
earning gr. (–)	0.87	1					
income gr (–)	0.85	0.82	1				
job separation	0.56	0.50	0.45	1			
job finding (–)	0.70	0.80	0.85	0.58	1		
stock prices (–)	0.78	0.90	0.64	0.44	0.66	1	
unemployment	0.27	0.29	0.11	0.49	0.22	0.21	1

Back to survey data

REPRESENTATIVE HOUSEHOLD

Preferences

$$V_t = \min_{\substack{m_{t+1} \\ E[m_{t+1}]=1}} \max_{C_t, B_{t+1}} u(x_t) + \beta E_t [m_{t+1} V_{t+1}] + \beta \frac{1}{\theta_t} E_t [m_{t+1} \log m_{t+1}]$$

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Period utility function

$$u(x_t) = \log C_t$$

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Process for robust concerns θ_t

$$\theta_{t+1} = (1 - \rho_\theta)\,\overline{\theta} + \rho_\theta \theta_t + \sigma_\theta w^\theta_{t+1}$$

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$$\theta_{t+1} = (1 - \rho_\theta)\,\overline{\theta} + \rho_\theta \theta_t + \sigma_\theta w^\theta_{t+1}$$

Subjective belief

$$m_{t+1} = \frac{\exp(-\theta_t V_{t+1})}{E_t [\exp(-\theta_t V_{t+1})]}$$

LABOR MARKET

Unit mass of workers, l_t employed, $u_t = 1 - l_t$ unemployed

- employed workers keep job with probability ρ
- unemployed search for jobs, job finding probability j_t
- firms post vacancies v_t, hiring rate η_{t+1}

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Law of motion for employed

$$l_{t+1} = (\rho + \eta_{t+1})\, l_t$$

Labor market clearing

$$\eta_{t+1} l_t = j_{t+1}\,(1 - \rho l_t)$$

Matching function

$$f\left(e_t, \frac{v_t}{u_t}\right) = \chi_m \left(\alpha_m e_t^{\psi_m} + (1 - \alpha_m)\left(\frac{v_t}{u_t}\right)^{\psi_m}\right)^{1/\psi_m}$$

VALUE FUNCTIONS

Value of employed worker

$$J_t^w = \xi_t + \widetilde{E}_t \left[s_{t+1} \left(\left(\rho + (1-\rho) j_{t+1} \right) J_{t+1}^w + (1-\rho) \left(1 - j_{t+1} \right) U_{t+1} \right) \right]$$

- stochastic discount factor s_{t+1}, subjective belief $\widetilde{P}$

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- stochastic discount factor s_{t+1}, subjective belief $\widetilde{P}$

Value of being unemployed

$$U_t = D_t + \widetilde{E}_t \left[s_{t+1} U_{t+1}^S \right]$$

$$U_t^S = \max_{e_t} -\frac{c_0}{2} e_t^2 + f\left(e_t, \frac{v_t}{u_t}\right) J_t^w + \left(1 - f\left(e_t, \frac{v_t}{u_t}\right)\right) U_t$$

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Value of a match to the firm

$$J_t = \vartheta_t - \xi_t + \rho \widetilde{E}_t \left[s_{t+1} J_{t+1} \right].$$

- marginal product ϑ_t, wage ξ_t

BARGAINING AND CONTINUATION VALUES

Nash bargaining

- surplus sharing rule

$$J_t = \frac{1-\eta}{\eta}\left(J_t^W - U_t\right)$$

BARGAINING AND CONTINUATION VALUES

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Role of subjective beliefs in the frictional labor market

- value of the match is forward-looking
- incentives to search and hire depend on subjective forecasts

PRODUCTION

- Monopolistic producers of intermediate goods hire workers

$$Y_{i,t} = A_t h_{i,t} - \phi_t$$

- Competitive final good producers

$$Y_t = \left[\int_0^1 (Y_{i,t})^{\frac{1}{\lambda}} \, di \right]^{\lambda}$$

- Inflation determined by the Calvo price setting mechanism
- Aggregate resource constraint

$$C_t + \left(\frac{\kappa_v}{q_t} + \kappa_h \right) \eta_t l_{t-1} = Y_t$$

Monetary policy rule

$$\log \frac{R_t}{\overline{R}} = r_\pi \log \frac{\pi_t}{\overline{\pi}} + r_y \log \frac{Y_t}{Y^*} + \sigma_R w_t^R$$

Neutral technology process

$$\log A_{t+1} = \rho_a \log A_t + \sigma_a w_{t+1}^a$$

Belief dynamics

$$\theta_{t+1} = (1 - \rho_\theta)\,\overline{\theta} + \rho_\theta \theta_t + \sigma_\theta w_{t+1}^\theta$$

back