DNB Working Paper

No 800/ February 2024

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DeNederlandscheBank

EUROSYSTEEM

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* Views expressed are those of the authors and do not necessarily reflect official positions of De Nederlandsche Bank.

Working Paper No. 800

February 2024

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Optimal normalization policy under behavioral expectations*

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Abstract

We examine optimal normalization strategies for a central bank confronted with persistent inflationary shocks and a potential de-anchoring of expectations. Our analysis characterizes optimal monetary policy, when the central bank uses both the short-term interest rate and the balance sheet, in a framework in which agents' expectations can deviate from the rational expectations benchmark. Optimal policy is developed using a sufficient statistics approach, highlighting the dynamic causal effects of changes in each policy instrument on the central bank's targets. Three key insights emerge: first, the interest rate is identified as the key instrument for managing inflationary pressures, outperforming balance sheet adjustments. Second, having anchored expectations about the path of quantitative tightening (QT) is crucial to mitigate economic downturns and controlling inflation, within the framework of an optimal balance sheet strategy set under a predefined interest rate rule. Lastly, when both the interest rate and QT are set optimally, expectations are found to significantly influence the optimal interest rate trajectory, whereas their impact on the optimal QT path is comparatively minimal.

Keywords: Optimal monetary policy, de-anchored expectations, normalization strategy

JEL Codes: E52, E71, D84

*We are grateful to Guido Ascari, Maurice Bun, Gabriele Galati, Cars Hommes, Christian Matthes, Falk Mazelis, Leonardo Melosi, Roberto Motto, Annukka Ristiniemi for valuable comments. We are also grateful to seminar participants at various places. The views expressed do not reflect those of De Nederlandsche Bank or the Eurosystem.

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1 Introduction

In response to historically low interest rates following the global financial crisis, central banks in advanced economies adopted unconventional monetary policies, notably large-scale asset purchase programs, which have been extensively used over the last decade. Such measures significantly expanded central bank balance sheets and brought them to historically high levels. Recently, the resurgence of high inflation has prompted central banks to normalize their monetary policy stance. As a result, they raised their key interest rates and initiated plans for reducing their enlarged balance sheets (quantitative tightening, or QT henceforth).

In addition to these challenges, central banks also confronted the risk that inflation expectations among market participants might become de-anchored. After a decade of persistently below-target inflation, inflation trends during the rate hike phase risked shifting expectations to the upside. This development led major central banks to closely monitor expectations during the recent rate hike phase, aware of the risks associated with an inadequate policy response that could lead to sustained inflationary pressures. Simultaneously, the process of reducing their enlarged balance sheets poses additional issues, especially as rate hikes exert downward pressure on economic activity. In such an environment, monetary policy faces the complex task of finding a suitable combination of short-term rate-setting and balance sheet strategy. This combination must effectively control inflation and ensure the softest possible economic landing, all while being robust to various forms of inflation expectations. This paper grapples with this challenge.

There is a vast literature addressing the importance of expectations formation for the traditional inflation-output trade-off or for the interest rate setting (see Dennis and Ravenna, 2008; Gaspar et al., 2006, 2010; Gáti, 2023; Molnár and Santoro, 2014; Orphanides and Williams, 2005, among others). However, the interplay between expectations formation and normalization policy — encompassing interest rate hikes and the unwinding of central bank balance sheets — remains less explored. This paper aims to bridge this gap. We examine optimal normalization policies, involving both interest rates and central bank asset holdings, under the condition of possibly de-anchored expectations, in a context marked by rising inflation due to price cost-push shocks.

Our contributions to this question are twofold. First, we analytically characterize policy trade-offs for the two monetary policy instruments under both rational expectations (RE) and de-anchored expectations. We derive optimal policy rules in the form of *forecast targeting criteria* applicable to any economic shock. Second, through numerical simulations, we investigate how a central bank should optimally respond to price cost-push shocks while in the process of normalizing its policy stance.

For these purposes, we use the four equation New-Keynesian model of Sims et al. (2023), extended to incorporate a resource cost for central bank bond holdings as in Kabaca et al. (2023) and Karadi and Nakov (2021). In our model, agents may hold de-anchored expectations, defined as the misperception of the long-run mean of macroeconomic variables, which are influenced by short-term forecast errors, following the framework of Eusepi et al. (2020).

We emphasize the following three main takeaways. First, we underscore the primary role of the short-term policy rate in addressing inflationary pressures, which we regard as being more effective and less costly than the balance-sheet to curb inflation, independent of the private sector's expectations. Second, we emphasize the importance of *having anchored QT expectations*, to mitigate economic downturns and control inflation. This aspect is crucial when determining the optimal balance sheet trajectory for a given interest rate rule. Lastly, in scenarios in which both policy instruments are set under optimal policy, expectations exert a considerable influence on the optimal interest rate path, particularly necessitating greater monetary policy tightening when more agents hold de-anchored expectations. Conversely, expectations play a limited role in shaping the optimal QT trajectory.

Our analysis begins with an examination of optimal policy for each monetary policy instrument, comparing our findings to existing literature on rational and de-anchored expectations. For tractability, we adopt initially a standard ad-hoc dual-mandate loss function in our analytical derivations. Similar to Gaspar et al. (2010) and Molnár and Santoro (2014), we show analytically that the central bank faces a more pronounced inflation-output gap trade-off under de-anchored expectations, when employing only the short-term policy rate to minimize the dual-mandate optimal policy problem.

Continuing with our analytical study, we establish optimal policy rules using a sufficient statistics perspective, a method parallel to the approach of McKay and Wolf (2022) and Barnichon and Mesters (2023), extending it to incorporate behavioral expectations. We introduce a *trade-off matrix* as a simple yet effective tool to summarize the dynamic trade-off faced by the policymaker over the policy horizon. This matrix, derived from the impulse responses of inflation (the primary objective in the dual-mandate loss function) and the output gap (the secondary objective) to policy changes, weighted by their importance in the central bank's loss function, offers a clear visual representation of the intertemporal trade-offs faced by policymakers.

Using this approach, we establish the forecast targeting criterion in the case of an optimal QT, for a given interest rate rule. Our findings indicate that the short-term interest rate is more effective in stabilizing inflation following cost-push shocks, as the trade-off becomes more acute under QT. Additionally, our research underscores that in the context of an optimal dual-mandate balance sheet policy, for a given interest rate rule, central banks would benefit from adopting a credible commitment strategy. This strategy would involve an initial reduction in the size of the balance sheet, followed by a subsequent expansion above its long-run level. Such a course of action is essential to counter the negative effects on economic activity observed at the initial stages of such policy. In this context, effective communication regarding the future trajectory of the balance sheet can therefore play a pivotal role from a macroeconomic stabilization perspective.[1] Furthermore, we derive the optimal trade-offs under both de-anchored and rational expectations for each policy instrument, to highlight the impact of the expectations formation mechanism.

In the second part of our results, we turn to numerical simulations. In the line with the methodology of Rotemberg and Woodford (1997), we proceed with the derived model-consistent welfare criterion from the second-order approximation of the utility functions of the parent and the child in our extension of Sims et al. (2023). These simulations are focused on a case in which a central bank is facing persistent price cost-push shocks. Similar to McKay and Wolf (2022), we leverage on our sufficient statistics representation of optimal policy rules to solve for the interest rate and balance-sheet sequences that implement our model consistent forecast targeting criteria.

Consistent with our analytical results, our findings suggest that when QT is the sole instrument used in an optimal setting, it proves to be less effective and more costly economically compared to policy rate adjustments. However, our analysis also reveals that the simultaneous optimization of both the policy rate and the balance sheet leads to a more

[1] The importance of anchoring QT for setting the stance of monetary policy is discussed in Altavilla et al. (2023).

tempered application of each policy tool. This coordinated strategy enhances inflation stabilization, and minimizes adverse effects on the output gap compared to scenarios in which only QT is set optimally.

We conduct a detailed examination of the role of expectations in this context. Our primary finding is that the degree of de-anchoring is a critical determinant for setting the optimal policy rate trajectory, while its impact on QT is comparatively minimal, particularly when the central bank uses both instruments to minimize the loss function. As described in the literature (e.g. Gaspar et al., 2010), the central bank needs to be more aggressive in its optimal interest rate setting decisions when inflation expectations are de-anchored. With both instruments set optimally, interest rate paths show substantial variation depending on the degree of de-anchoring of expectations, whereas balance sheet trajectories remain consistent across the different expectation formation mechanisms. This is a novel insight in the literature.

Finally, we show that large credit shocks can push central banks to shorten the QT process, to limit the negative effects on financial stability. Furthermore, we find that when both monetary instruments are set up to respond optimally to a combination of inflationary and credit shocks, the central bank uses the policy rate to control inflation, while it uses the balance sheet to maintain financial stability, aligning each instrument with the shock for which it is most effective.

As of the writing of this paper, major central banks are still hiking or have concluded, at least at the moment, their rate hikes. Concurrently, they are continuing their QT policy by further reducing the sizes of their balance sheets. During their last strategy review, major central banks reviewed their workhorse models acknowledging the importance of deviations from the RE benchmark in the analysis of monetary policy transmission.[2] This paper touches thus upon issues lying at the heart of current policy debates.[3] It highlights that the significance of these deviations from RE varies depending on the monetary policy

[2]In her speech on September 30, 2020, at the ECB and its watchers XXI conference, Christine Lagarde alluded to the relevance of models that depart from the RE assumption by stating, *"while make-up strategies may be less successful when people are not perfectly rational in their decisions—which is probably a good approximation of the reality we face—the usefulness of such an approach could be examined."*

[3]ECB staff has analyzed the impact of policy normalization using a suite of models, among them structural and semi-structural ones (see Motto et al., 2023). A key finding is the uncertainty about the impact of monetary policy normalization on the economy and this is partly due to the sensitivity of the models considered on the assumptions about the expectations formation mechanism. The significance of anchoring inflation expectations and demonstrating commitment to achieving the target during recent episodes of inflation has been emphasized in the recent BIS report from Amatyakul et al. (2023).

instrument employed.

Related literature:

The ongoing policy normalization of major central banks has given impetus to a growing literature on optimal monetary policy normalization. Benigno and Benigno (2022) analyze optimal monetary policy normalization in a linear quadratic framework and show that liquidity provided by the central bank should be withdrawn at an early stage, implying that QT starts before the liftoff of the policy rate. Additionally, they show that the withdrawal of liquidity takes place at a slower pace relative to the normalization of the policy rate. In a similar vein, Karadi and Nakov (2021) analyze optimal asset purchase policies in a model featuring a banking sector and argue that the optimal exit from large central bank balance sheets should be gradual due to the induced flattening of the yield curve and the slowing down of the banking sector recapitalization. Focusing on the portfolio balance channel and abstracting from a normative analysis, Cantore and Meichtry (2023) provide a quantitative assessment of the normalization policy in model with borrowers and savers and, contrary to Benigno and Benigno (2022), show that interest rate hikes should start materializing prior to QT.

This paper differs from the above contributions in the following aspects. First, we consider deviations from the RE benchmark whereas the papers above feature RE. Subsequently, we design optimal monetary policy both under RE and under de-anchored expectations (bounded rationality). Contrary to Benigno and Benigno (2022), we are not interested in the optimal timing of QT relative to the optimal timing of a liftoff. Instead, we are interested in the implications of the expectations formation mechanism for the optimal interest rate setting and the optimal path of the central bank balance sheet. We also analyze the properties of each monetary policy instrument both in isolation and jointly. That is, by restricting our focus on cost-push shocks, we derive optimal monetary policy when only the short-term rate is set optimally, when only the central bank balance sheet is set optimally and when both instruments are set optimally.

As mentioned in the introduction, there is a vast literature on the design of optimal monetary policy under bounded rationality (see Dennis and Ravenna, 2008; Eusepi and Preston, 2018; Gaspar et al., 2006, 2010; Molnár and Santoro, 2014; Mele et al., 2020; Hommes et al., 2023, among others). Using our dual-mandate loss function, we corroborate some established results in the literature, namely that the traditional inflation-output trade-off includes an intertemporal nature under de-anchored expectations, originally documented in Molnár and Santoro (2014) and Gaspar et al. (2010). By deriving a second order approximation of the welfare of the households in the model, we show that this coincides with a more aggressive optimal interest rate trajectory, as in Gáti (2023) and Molnár and Santoro (2014), compared to the RE benchmark. Notably, our model does not incorporate long-run interest rate expectations, diverging from the findings of Eusepi et al. (2020) who argue that in their presence, aggressive policy responses can be sub-optimal due to induced volatility in long-term interest rates. Additionally, different from the existing literature, our analysis also focuses on the optimal balance sheet policy under de-anchored expectations, an insight little is known about.

Our study extends the existing body of research by examining the individual and combined effectiveness of two monetary policy instruments – the interest rate and the balance sheet – in minimizing the central bank's loss function. This approach allows us to contribute valuable insights into the literature regarding the interplay and substitutability between conventional and unconventional monetary policy measures. Specifically, we show that following cost-push shocks, setting only the balance sheet policy optimally - while the policy rate follows a predefined feedback rule - stabilizes inflation equally well as when only the policy rate is set optimally, at the expense though of a markedly deeper contraction. Notably, we show that this result holds both under RE and under de-anchored expectations. Embarking on the same model as we do, Sims and Wu (2020) explore the substitutability between conventional monetary policy and QE at the effective lower bound (ELB) and find that, when the policy rate is fixed, QE can indeed serve as an effective substitute and be utilized to achieve price stability, albeit with different implications for the output gap compared with conventional policy. Apart from our focus on QT and cost-push shocks, our paper differs from Sims and Wu (2020) in that it considers de-anchored expectations and it rests on optimal monetary policy to draw conclusions. Moreover, QT (or QE) in our paper is costly which gives a more realistic picture about the degrees of freedom that the central bank has in its ability to adjust its balance sheet. Given that our welfare criterion is model consistent variations in balance sheet of the central bank become welfare relevant.

Our paper is also linked to the literature on optimal monetary with two policy instruments, namely central bank asset holdings and the policy rate (see Sims et al., 2023; Bonciani and Oh, 2021; Harrison, 2017; Kabaca et al., 2023, among others). The focus of those papers though is rather on the design under RE of optimal QE at the ELB and the subsequent optimal policy rate liftoff rather than on QT away from the ELB.

Our characterization of optimal policy rules leverages on the recent work by McKay and Wolf (2022), which we extend to a behavioral model. Besides, we introduce the concept of trade-off matrix, to visually summarize the intertemporal trade-off between the central bank targets using sufficient statistics, in the spirit of Barnichon and Mesters (2023). The sufficient statistics approach can be seen as a key input in the context of forecast-targeting rules (e.g., Svensson and Woodford, 2004,; Woodford, 2003). Different from a Taylor (1993)-type instrument rule, a forecast targeting rule specifies that the policy path must be adjusted such that the forecasts for the policy objectives satisfy the first-order conditions of optimality. This condition is fully summarized by the trade-off matrix. Our computational solution relies on the use of sequence-space linear-quadratic policy problems similar to McKay and Wolf (2023), de Groot et al. (2021) and Hebden and Winkler (2021).

The paper is organized as follows. Section 2 describes the extension of the model by Sims et al. (2023) to account for costs associated with central bank asset holdings. Section 3 presents analytical results from the design of optimal monetary policy as well as the extension of the sequence space approach by McKay and Wolf (2022) to a behavioral model. Section 4 presents our simulations from optimal policy using a model consistent welfare criterion. Section 5 concludes.

2 Model

The model is based on Sims et al. (2023), which adds a role for central bank's asset purchases in a standard three-equation New-Keynesian model. The non-reduced framework consists of two types of households, patient (referred to as the "parent") and impatient (the "child"), financial intermediaries modeled as in Gertler and Karadi (2011), and a standard production side of the economy split into three sectors (final output, retail output, and wholesale output). To this baseline model, we add a cost for QE, as a proxy for unmodeled distortions and political costs of maintaining a positive central bank balance sheet as in Karadi and Nakov (2021) and Kabaca et al. (2023). In its log-linearized form, the model boils down to four endogenous equations. Further details on the model and the introduction of the QE cost in the baseline model can be found in Appendix A. Equations (1) and

(2) are the IS and the New Keynesian Philips curves:

$$x_t = \tilde{\mathbb{E}}_t x_{t+1} - \frac{1-z}{\sigma}\left(r_t^s - \mathbb{E}_t \pi_{t+1} - r_t^f\right) - \left(z\bar{b}^{cb} + \frac{\tau QE}{Y}\right)\left(\tilde{\mathbb{E}}_t qe_{t+1} - qe_t\right)$$
$$- z\bar{b}^{FI}\left(\tilde{\mathbb{E}}_t \theta_{t+1} - \theta_t\right) \quad (1)$$

$$\pi_t = \beta \tilde{\mathbb{E}}_t \pi_{t+1} + \gamma\zeta x_t - \frac{\gamma\sigma}{1-z}\left(z\bar{b}^{cb} + \frac{\tau QE}{Y}\right) qe_t - \frac{\gamma\sigma z}{1-z}\bar{b}^{FI}\theta_t + cp_t \quad (2)$$

The variable π_t denotes inflation, while $x_t = y_t - y_t^*$ is the output gap, with y_t^* potential output.[4] The short-term nominal interest rate is denoted r_t^s. The real value of the central bank bond portfolio is captured by qe_t.[5] The term $\tilde{\mathbb{E}}_t$ denotes the fact private agents might have non-rational expectations, which we detail below. The inverse intertemporal elasticity of substitution is α, β is the subjective discount factor, γ is the elasticity of inflation with respect to real marginal cost and ζ is the elasticity of real marginal cost with respect to the output gap. The parameter $z \in [0, 1)$ represents the share of impatient households in the total population, while $\bar{b}^{cb}$ and $\bar{b}^{FI}$ are the steady state long-term bond holdings of the central bank and the financial intermediaries, respectively, as a share to total outstanding long-term bonds. The cost for QE/QT is denoted τ, and $\frac{QE}{Y}$ is the steady state ratio of bond holdings over quarterly real GDP.

The economy is subject to shocks to the natural rate r_t^f, to price cost-push shocks cp_t and to credit shocks θ_t. These shock process are exogenous AR(1) processes, with persistence rho_j and standard deviation s_j for $j = \{f, cp, \theta\}$:

$$r_t^f = \rho_f r_{t-1}^f + s_f \varepsilon_{f,t} \quad (3)$$

[4]Note that, unlike in the standard New Keynesian model, in Sims et al. (2023), the equilibrium level of potential output is consistent with price flexibility and no credit shocks, as both frictions distort the competitive equilibrium.

[5]Although we are interested in normalization policy and hence in QT, we use the notation of Sims et al. (2023) whose focus instead is on QE. Given our focus on QT, we assume that a central bank has positive asset holdings at the steady state or above the steady state at the initialization of our simulation exercises as we explain in detail in section 4.1. In her speech on March 27, 2023, at an event organized by Columbia University and SGH Macro Advisors, Isabel Schnabel stated that "*the size of our balance sheet will not return to the levels seen before the global financial crisis*" and concluded by stating "*Ultimately, our obligation to act in line with the principle of an open market economy implies that, in the steady state, the size of our balance sheet should only be as large as necessary to ensure sufficient liquidity provision and effectively steer short-term interest rates towards levels that are consistent with price stability over the medium term*". Given the modeling of the central bank balance sheet in Sims et al. (2023), we treat as plausible the assumption of positive central bank asset holdings at the steady state.

$$cp_t = \rho_{cp} cp_{t-1} + s_{cp}\varepsilon_{cp,t} \tag{4}$$

$$\theta_t = \rho_\theta \theta_{t-1} + s_\theta \varepsilon_{\theta,t} \tag{5}$$

Our analysis primarily focuses on optimal monetary policy rules. However, for certain scenarios, we examine the optimal path of QT under a predetermined interest rate policy. In these scenarios, we assume that the central bank employs a Taylor rule (6) for setting the short-term nominal interest rate, taking into account the ELB. Additionally, when looking at optimal interest rate policy only, we assume that the balance sheet follows a stationary AR(1) process (7):[6]

$$r_t^s = \max\left[0\,;\phi_\pi(\pi_t - \pi^T) + \phi_x(x_t - x^T) + s_r\varepsilon_{r,t}\right] \tag{6}$$

$$qe_t = \rho_{qe} qe_{t-1} + s_q \varepsilon_{q,t} \tag{7}$$

Expectations

In this model, private agents can have non-rational expectations. The economy is populated by two types of agents: a first group acts as anchored, and uses the RE (i.e. minimum state variable, or MSV) solution. Anchored agents have expectations that are well-aligned with the central bank's stated targets. Importantly, they understand the communication (e.g. forward guidance) of the central bank. This understanding allows the policymaker to commit to a specific policy strategy, which these agents will, at least partially, understand and factor into their expectations. The higher the fraction of anchored agents, the more the central bank can rely on a commitment strategy. They forecast the evolution of the economy following,

$$\tilde{\mathbb{E}}_t^a\left(z_{t+1}\right) = \left(\mathbf{a} + \mathbf{b}y_t + \mathbf{c}w_t\right) \tag{8}$$

with $z_t = [\pi_t, x_t, qe_t, \theta_t]'$ the expectational variables, the vector $\mathbf{a}$ represents the constant terms including the central bank's inflation target, the matrix $\mathbf{b}$ maps the impact of contemporaneous variables $y_t = [\pi_t, x_t, qe_t, r_t^f, cp_t, \theta_t]'$, and the matrix $\mathbf{c}$ accounts for the effect of each shock $w_t = [\varepsilon_{f,t}, \varepsilon_{cp,t}, \varepsilon_{\theta,t}, \varepsilon_{q,t}]'$ on expectations.

[6] Using an exogenous AR(1) process for QE/QT allows us to isolate the impact of setting the interest rate in an optimal way. This approach helps to clearly identify the specific effects of optimal interest rate adjustments, excluding the influence of simultaneous balance sheet changes.

The remaining share of agents has de-anchored expectations, i.e. they learn to forecast the long-run mean of endogenous variables through simple constant gain learning, as in Eusepi et al. (2020) and Molnár and Santoro (2014). The beliefs of de-anchored agents follow :

$$\mathbb{E}_t^d z_{t+1} \equiv \omega_t^z = \omega_{t-1}^z + \bar{g}\left(z_{t-1} - \omega_{t-1}^z\right) \tag{9}$$

in which $\bar{g}$ is the learning gain parameter, which governs the extent to which expectations are affected by short-term forecast errors. These agents misperceive the unobserved long-run mean (or drift) ω_t^z of variables $z_t = [\pi_t, x_t, qe_t, \theta_t]'$, and revise their expectations based on previous short-term forecast errors $\left(z_{t-1} - \omega_{t-1}^z\right)$. They are therefore backward looking, and do not necessarily believe that the central bank can credibly commit to reach its announced target.

Aggregate expectations are therefore the sum of anchored and de-anchored agents' expectations, weighted by their relative fractions $n^{a,z}$ and $n^{d,z}$:[7]

$$\begin{aligned}\tilde{\mathbb{E}} z_{t+1} &= n^{a,z}\, \tilde{\mathbb{E}}^a\left(z_{t+1}\right) + n^{d,z}\, \tilde{\mathbb{E}}^d\left(z_{t+1}\right) \\ &= n^{a,z}\Big[\mathbf{a} + \mathbf{b} y_t + \mathbf{c} w_t\Big] \;+ n^{d,z}\, \Big[\omega_{t-1}^z + \bar{g}\left(z_{t-1} - \omega_{t-1}^z\right)\Big]\end{aligned} \tag{10}$$

with $n^{a,z} + n^{d,z} = 1$.[8]

Baseline calibration

For our optimal policy exercises, we mainly parameterize the model following Sims et al. (2023). The discount factor and elasticity of substitution follow standard values from existing literature. The consumption share of the child in total consumption z is set to one-third.[9] The steady state bond holdings for the central bank and financial intermediaries, $\bar{b}^{cb} = 0.3$ and $\bar{b}^{FI} = 0.7$, are derived from the calibrations of other steady state parameters. The parameters γ and ζ, representing the elasticity of inflation and the output gap to real marginal cost, are based on structural parameters of the non-linear model, including the Calvo pric-

[7]In cases in which there are both anchored and de-anchored agents, the matrix $\mathbf{b}$ summarizing the effect of contemporaneous variables on the expectations of anchored households must also include the expectations $\boldsymbol{\omega}_t^z$ for each variable z forecasted by de-anchored households $y_t = [\pi_t, x_t, qe_t, r_t^f, cp_t, \boldsymbol{\omega}_t^z, \theta_t]'$.

[8]In this paper, we calibrate these fractions at a fixed level, uniformly applied across all variables.

[9]Sims et al. (2023) described this parameter as calibrated in such a way that it approximately reflects the proportion of durable goods consumption and private investment in the overall private non-government domestic spending.

ing parameter, as described in Sims et al. (2023).[10] The resource cost of QE follows Kabaca et al. (2023). We set the gain equal to the value estimated in Eusepi et al. (2020), in the period after 1999. In the baseline case, we set the interest rate rule such that it only targets inflation with $\phi_\pi = 1.5$. We set all standard deviation of shocks equal to 0.01, while first-order autocorrelation of all shocks are set to 0.8.

Table 1: Calibrated Parameters

Parameter	Value or Target	Description
β	0.995	Discount Factor
z	0.33	Consumption share of child
σ	1	Inverse elasticity of substitution
$\bar{b}^{cb}$	0.3	Steady state share of central bank's LT bond holdings
$\bar{b}^{FI}$	0.7	Steady state share of fin. intermediary's LT bond holdings
γ	0.086	Elasticity of inflation with regard to marginal cost
ζ	2.49	Elasticity of marginal cost with regard to output gap
τ	0.01	Resource cost of QE
$\frac{QE}{4Y}$	0.1	Steady state ratio of QE to annualized output
ϕ_π	1.5	Inflation reaction coefficient in the Taylor rule
ϕ_x	0	Output gap reaction coefficient in the Taylor rule
ρ_{qe}	0.8	Peristence of the QE/QT process
$\bar{g}$	0.02	Learning gain of de-anchored agents
ρ_f	0.8	Persistence of natural rate shocks
ρ_{cp}	0.8	Persistence of cost push shocks
ρ_θ	0.8	Persistence of credit shocks

3 Analytical results, in the case of a dual mandate

In this section, we examine how bounded rationality, of the form described in Section 2, impacts optimal monetary policy, with a particular focus on both interest rate and balance-sheet policies. Specifically, we explore the extent to which traditional trade-offs, such as the inflation-output trade-off faced by the central bank under rational expectations, are altered when agents have de-anchored expectations.

For tractability, we begin our analysis assuming an ad-hoc loss function, in which the policymaker follows a dual mandate approach. This framework facilitates comparisons

[10] In particular, $\gamma = \frac{(1-\phi)(1-\phi\beta)}{\phi}$ in which $\phi \in (0,1]$ is the probability of non-price adjustment. Besides, $\zeta = \frac{\chi(1-z)+\sigma}{1-z}$ in which χ is the inverse Frisch labor supply elasticity for the "parent".

with existing literature on rational expectations (Giannoni and Woodford, 2002), and on learning or de-anchored expectations (Molnár and Santoro, 2014 and Eusepi et al., 2020). Our analysis proceeds in two steps. First, we derive the standard *forecast targeting criterion* using the structural equations of the model, following the approach of Svensson (2003) and Woodford (2007). Unlike a Taylor-type rule, formulating the optimal policy problem as a forecast targeting criterion provides a realistic, forward-looking perspective. It helps handle shock uncertainty, analyze trade-offs, and address time consistency issues. This approach therefore offers valuable policy implications and allows for comparative analysis of different policy rules. We then derive these optimal policy rules using a sufficient statistics approach similar to McKay and Wolf (2022) and Barnichon and Mesters (2023), in the context of a behavioral model. This is achieved through the use of an analytical tool we term the *trade-off matrix*.

In our subsequent numerical exercises, we derive a model consistent welfare criterion that the central bank seeks to maximize, obtained from a second order approximation of the utility of the two types of agents, the "parent" and the "child".

3.1 Deriving the trade-off using the model structural equations

In this section, we derive analytical results showing how fully de-anchored expectations affect optimal monetary policy relative to the case of fully anchored expectations (i.e., RE). As mentioned above, we initially assume a simple ad-hoc loss function for the central bank to keep tractability and to illustrate the impact of de-anchored expectations for optimal monetary policy in a simpler way. In this section, for the sake of exposition, we assume that the central bank follows a simple loss function with two objectives, namely inflation and output gap stabilization:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left(\frac{\lambda_\pi}{2} \pi_t^2 + \frac{\lambda_x}{2} x_t^2 \right) \tag{11}$$

We assume that the central bank always has full knowledge of the actual law of motion of the economy and the way agents form their expectations, whether they are rational or not, as in Gaspar et al. (2010) and Gáti (2023). We analyze two extreme scenarios regarding expectations formation: one in which all agents have anchored expectations, allowing the central bank to fully commit to a pre-announced plan that agents understand, and another where all agents have de-anchored expectations, with their forecasts being influenced by short-term forecast errors. If the central bank can set the interest rate optimally, it thus minimizes (11) subject to the Phillips curve equation (2). Under commitment and RE, minimization of the above loss function subject to the Phillips curve results in the following trade-off.

$$-\lambda_\pi \pi_t = \frac{\lambda_x}{\gamma\zeta}\left(x_t - x_{t-1}\right) \tag{12}$$

for $t = 1, 2, 3, ...$, which is the standard trade-off identified in the literature. By committing to the above trade-off, the central bank is able under rational expectations to anchor inflation expectations in the case of supply shocks, for instance. Rational agents are fully aware of the current and future policy path of the central bank. An important element of the optimal targeting criterion under rational expectations is the history dependence, a well-known result in the literature. This allows it to anchor expectations and to achieve its inflation target effectively. Agents in this case take into account the entire path of monetary policy as well as the forecasted path based on a targeting or instrument rule, and given the commitment of the central bank, adjust their expectations accordingly. As argued in Giannoni (2014), in a model with rational expectations, optimal monetary policy under commitment targets the price level since it incorporates more history dependence than inflation itself. In fact, it is easy to show that under rational expectations, the targeting criterion (12) iterated backwards reads as follows (as also shown in Galí, 2008):

$$-\lambda_\pi \hat{p}_t = \frac{\lambda_x}{\gamma\zeta} x_t \tag{13}$$

for $t = 0, 1, 2, ...$, where $\hat{p}_t \equiv p_t - p_{-1}$ is the log-deviation between the price level and an implicit target given by the price level that prevails a period before the central bank decides upon its optimal plan, as in Galí (2008). However, is this policy sufficient when agents have de-anchored expectations? Mistakes in their expectations as well as the backward-looking element, as described in (9) can potentially complicate an otherwise traditional inflation-output trade-off. This is because agents' expectational errors and their associated persistence may make the objective of inflation stabilization more costly.

As described in Sims et al. (2023), when instead the central bank designs its optimal policy under discretion with rational expectations the usual optimal targeting criterion receives the form:

$$-\lambda_\pi \pi_t = \frac{\lambda_x}{\gamma\zeta} x_t \tag{14}$$

which clearly lacks any history dependence given that the central bank takes in this case agents' expectations as given. When agents' have de-anchored expectations instead, their expectations cannot be taken as given. This is because of the endogenous persistence that de-anchored expectations introduce to the model. In this case and given its full knowledge, the central bank knows that its decisions today will have an impact on the economy tomorrow regardless of whether it forms its policy under commitment or under discretion. As mentioned above, de-anchored expectations may involve additional costs that a central bank with full knowledge must take into account. We summarize this possibility in the following proposition.

Proposition 1. *Under de-anchored expectations, the central bank faces an intertemporal trade-off summarized by the optimal targeting rule:*

$$-\lambda_\pi \pi_t + \frac{\lambda_x}{\gamma\zeta} \bar{g} \beta^2 \sum_{s=0}^{\infty} \beta^s \left(1 - \bar{g}\right)^s x_{t+s+1} = \frac{\lambda_x}{\gamma\zeta} x_t \tag{15}$$

Proof. In appendix B.1 ■

What Proposition 1 states is the well-known result under learning or de-anchored expectations, as also shown in Gaspar et al. (2010), Gáti (2023) and Molnár and Santoro (2014). That is, the central bank faces an intertemporal trade-off, as opposed to the intratemporal trade-offs that it faces under rational expectations. In this case, the central bank needs to consider how its decisions at time t influence inflation expectations and thus the future intratemporal trade-offs. When expectations are de-anchored the effects of a shock build up gradually, while at the same time, it takes longer until the monetary policy effects are fed into inflation and output expectations. Agents in this case do not consider the actual law of motion of the economy ignoring thereby the current or future optimal monetary policy reaction. Instead, they use past information about their variable of interest, which weakens the contemporaneous monetary transmission through expectations.[11] The central bank has to consider this feature when stabilizing current inflation and has thus to account for the future output gaps. As such it does not have to bear the full trade-off in the current period only. The central bank thus must take into account the induced expectational errors and the fact that the effects of its decisions are incorporated into agents' expectations with a lag.

As shown in Eusepi et al. (2020), when the Kalman gain parameter $\bar{g}$ approaches zero, beliefs nest rational expectations. In this case, the targeting criterion (15) clearly collapses to (14), namely the discretionary rational expectations solution, a result also discussed in Molnár and Santoro (2014). De-anchored expectations thus introduce additional costs on top of those associated with discretionary monetary policy (e.g., the inflation bias). Not only is optimal policy now subject to an inflation bias, but also inflation stabilization is achieved at a larger cost (i.e., a larger drop in the output gap).

3.2 The trade-off throughout the policy horizon: a sufficient statistics approach

In this subsection, we show that optimal policy rules can be characterized using a sufficient statistics approach, which could be compared to empirically measurable elasticities, similar to Barnichon and Mesters (2023). We extend the results of McKay and Wolf (2022) and derive the forecast targeting criterion in a behavioral model with potentially de-anchored expectations. Using the first-order conditions of the optimal policy problem, we show that, optimal policy rules in responses to exogenous shocks can be expressed as one measure, denoted the *trade-off matrix*. This matrix takes the dynamic causal effects of a set of monetary policy shocks on the central bank's primary objective, inflation, and maps them onto the dynamic causal effects of the same set of shocks on the secondary objective, the output gap. These effects are weighted by their relative importance within the loss function. This matrix summarizes all the complexity of the intertemporal optimal policy problem, and facilitates the analysis of the role of expectations for policy. Additionally, it offers two key benefits: i) it delivers a clear visual interpretation of the trade-off faced by the policymaker within

[11] Similar to our result of an intertemporal trade-off, Gaspar et al. (2006) show that when private expectations are subject to adaptive learning optimal monetary policy responds more strongly and more persistently following cost push shocks, the higher is the initial level of perceived inflation persistence by the private sector.

the policy horizon, ii) it simplifies the comparison with empirical estimates (e.g. from VARs and LPs).

To obtain this object, we recast the model as a linear-quadratic control problem and use perfect-foresight transition sequences, similar to McKay and Wolf (2023), de Groot et al. (2021) and Hebden and Winkler (2021).[12] Under perfect foresight, using the Structural Vector Moving Average (SVMA) representation of the model, we can write the variables of the model in the sequence space as:

$$Z = Z^B + \mathcal{A}^{Z,\varepsilon_y}\varepsilon_y = \mathcal{A}^{Z,\varepsilon_s}\varepsilon_s + \mathcal{A}^{Z,\varepsilon_y}\varepsilon_y \tag{16}$$

with $Z^B = \left\{Z_t^B\right\}_{t=0}^H = \left\{Z_0^B, ..., Z_H^B\right\}$ a baseline path for each variable over all periods within some projection horizon H, which consists of impulse responses $\mathcal{A}^{Z,\varepsilon_s}$ to a set of structural shocks ε_s (e.g., the impulse response to a cost-push shock). The object $\mathcal{A}^{Z,\varepsilon_y}$ collects the impulse responses of each variable Z under the prevailing baseline policy rules, to a set of contemporaneous and expected (i.e. news) monetary policy shocks ε_y.[13]

The problem of dual mandate policymaker under commitment is to choose the path of the desired policy instrument to minimize the loss function (11), while being subject to the constraints imposed by the equilibrium relationships between variables in the model, such as the NKPC equation (2) and the IS curve (1). We re-cast these constraints using their impulse response form, as expressed in (16).

We can subsequently write the optimal policy problem as follows:

$$\begin{aligned}\min_{\{\Pi,X,\varepsilon_y\}} \; & \mathrm{E}_0\Big[\frac{1}{2}\Pi'\Lambda_\Pi\Pi + X'\Lambda_X X \\ & + \Xi^{\Pi'}\left(-\Pi + \Pi^B + \mathcal{A}^{\Pi,\varepsilon_y}\varepsilon_y\right) + \Xi^{X'}\left(-X + X^B + \mathcal{A}^{X,\varepsilon_y}\varepsilon_y\right)\Big]\end{aligned} \tag{17}$$

with $\Pi = \{\pi_t\}_{t=0}^H = \{\pi_0, ..., \pi_H\}$ the perfect-foresight sequence of inflation, and $X = \{x_t\}_{t=0}^H = \{x_0, ..., x_H\}$ the perfect-foresight sequence of output gap. Additionally, $\Xi \equiv \mathrm{diag}\left(1, \beta, \ldots, \beta^T\right) \otimes \xi$ are the Lagrange multipliers to both constraints, and $\Lambda \equiv \mathrm{diag}\left(1, \beta, \ldots, \beta^T\right) \otimes$

[12] As presented in e.g. Fernández-Villaverde et al. (2016) and Auclert et al. (2021), by certainty equivalence, the first-order perturbation solution of models with aggregate risk is identical to the solution of the model in linearized perfect-foresight.

[13] de Groot et al. (2021) show how to obtain these matrices from the state-space representation of the model.

λ are the weights associated with each term in the loss function, with discount factor $\beta \in (0, 1)$.

The first-order conditions of the linear-quadratic problem are:

$$\Lambda_\Pi \, \Pi = \Xi^\Pi \tag{18}$$

$$\Lambda_X \, X = \Xi^X \tag{19}$$

$$\mathcal{A}^{\Pi,\varepsilon_y\prime} \, \Xi^\Pi + \mathcal{A}^{X,\varepsilon_y\prime} \, \Xi^X = 0 \tag{20}$$

Combining them gives:

$$\mathcal{A}^{\Pi,\varepsilon_y\prime} \, (\Lambda_\Pi \, \Pi) + \mathcal{A}^{X,\varepsilon_y\prime} \, (\Lambda_X \, X) = 0 \tag{21}$$

In line with McKay and Wolf (2022), equation (21) summarizes the trade-off faced by a policymaker, independently of the non-policy shocks hitting the economy. It expresses how a policymaker can set available policy instruments to align the projected path of key macroeconomic variables—such as inflation and the output gap—with the central bank's objectives. This equation can be reformulated as:

$$\Pi + \left(\mathcal{A}^{\Pi,\varepsilon_y\prime} \Lambda_\Pi\right)^{-1} \left(\mathcal{A}^{X,\varepsilon_y\prime} \Lambda_X\right) \, X = 0 \tag{22}$$

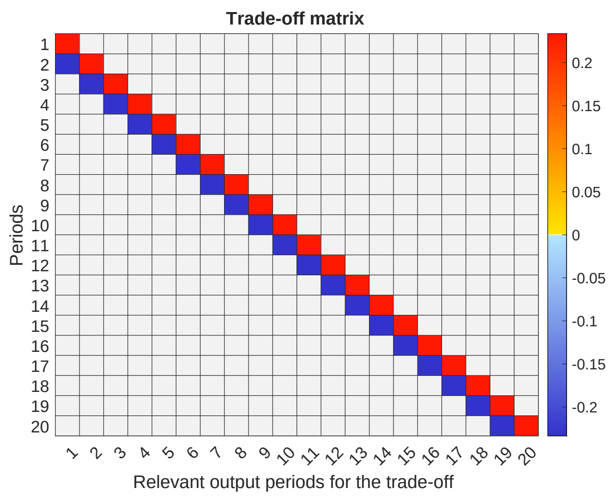
We denote $\mathcal{T}^{\varepsilon_y} = \left(\mathcal{A}^{\Pi,\varepsilon_y\prime} \Lambda_\Pi\right)^{-1} \left(\mathcal{A}^{X,\varepsilon_y\prime} \Lambda_X\right)$, as the trade-off matrix, which takes the impulse responses of the central bank's primary target, inflation $\mathcal{A}^{\Pi,\varepsilon_y}$, to a set of current and contemporaneously announced future policy deviations ε_y, and maps them to the impulse responses of the secondary target, the output gap $\mathcal{A}^{X,\varepsilon_y}$, to the same set of shocks. These impulse responses are weighted by their relative importance in the central bank loss function stacked in Λ_Π and Λ_X. All the required information about the dynamic relationship between the central bank's targets under optimal policy is contained within this matrix. Throughout this subsection, we use this matrix as a sufficient statistic to provide a general characterization of optimal policy independently of the shocks faced by the economy, under different expectation formation mechanism (i.e. RE and de-anchored expectations), and for each policy instrument (i.e. the short-term policy rate and the balance-sheet).

Under rational expectations:

We start with the case of the interest rate policy, under rational (i.e. fully anchored) expectations. The first-order condition (21) can be expressed as a linear combination of impulse responses of inflation $\mathcal{A}^{\Pi,\varepsilon_r}$ and the output gap $\mathcal{A}^{X,\varepsilon_r}$, to a vector of contemporaneous as well as news (i.e. forward guidance) interest rate shocks ε_r:

$$\mathcal{A}^{\Pi,\varepsilon_r\prime}\ (\Lambda_\Pi\ \Pi) + \mathcal{A}^{X,\varepsilon_r\prime}\ (\Lambda_X\ X) = \Pi + \mathcal{T}^{\varepsilon_r}\ X = 0 \tag{23}$$

Figure 1: Trade-off matrix for interest rate policy, under RE



Notes: Trade-off matrix $\mathcal{T}^{\varepsilon_r}$, in the baseline model. Each row is one period in the sequence, each line shows a relevant period of the output gap entering in the optimal policy rule. The intensity of the color corresponds to the magnitude of the trade-off.

The matrix $\mathcal{T}^{\varepsilon_r}$, displayed in Figure 1 summarizes the intertemporal trade-off between inflation and economic activity in the context of an optimal interest rate policy based on a dual mandate loss function, over a selected policy horizon of 20 periods. Each row in the matrix corresponds to a single period within the optimal policy problem, while each column reflects the relevance of the corresponding period of the variable X, the output gap, in the optimal policy rule. In the context of this simple model, as in the textbook New Keynesian model, the shape of this matrix can be used to directly analyze trade-offs and understand the role of central bank commitment. Notably, the sign and magnitude of the entries in the main diagonal provide information on the contemporaneous behavior of the "slack" variable in relation to inflation. Besides, any nonzero values in the lower triangular portion of this matrix indicate that past information matters for the optimal policy path. Conversely, nonzero entries in the upper triangular section emphasize the importance of expectations in shaping the optimal policy trajectory.

In more details, we can write this equation for a sequence of $T = 20$ periods. The relationship in the optimal interest rate rule under RE between the time-paths for inflation and the output gap is therefore summarized by:

$$\{\pi_t\}_{t=0}^{20} + \mathcal{T}^{\varepsilon_r} \ \{x_t\}_{t=0}^{20} = 0$$

or:

$$\begin{bmatrix} \pi_0 \\ \pi_1 \\ \vdots \\ \pi_{20} \end{bmatrix} + \mathcal{T}^{\varepsilon_r} \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_{20} \end{bmatrix} = 0$$

In this specific case, the trade-off matrix $\mathcal{T}^{\varepsilon_r}$ is such that:

$$\mathcal{T}^{\varepsilon_r} = \begin{pmatrix} \frac{\lambda_x}{\gamma\zeta\lambda_\pi} & 0 & \dots & 0 \\ -\frac{\lambda_x}{\gamma\zeta\lambda_\pi} & \frac{\lambda_x}{\gamma\zeta\lambda_\pi} & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & -\frac{\lambda_x}{\gamma\zeta\lambda_\pi} & \frac{\lambda_x}{\gamma\zeta\lambda_\pi} \end{pmatrix} \tag{24}$$

Starting from steady-state, this means that, in period 1, optimal policy must satisfy the following condition, described in Proposition 1, that depends on inflation in period 1 and output gap in periods 1 and 0:[14]

$$\lambda_\pi \pi_1 + \frac{\lambda_x}{\gamma\zeta}(x_1 - x_0) = 0$$

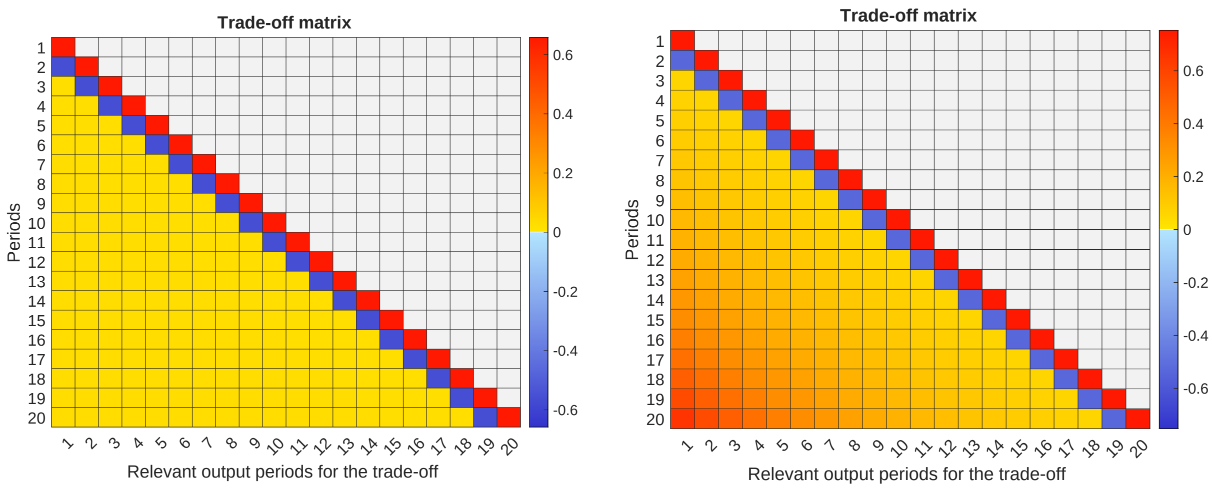
This single equation, whose implications are described in Section 3.1, succinctly represents the trade-off faced by the policymaker in a given period. Examining the entirety of the rows in the matrix extends this understanding across the entire policy horizon.

Under the framework of rational expectations, analogous conditions can be established for an optimal balance sheet policy.[15] Figure 2 presents the dynamic relationship between

[14]Since the first two entries of the matrix are nonzero, while the remaining entries are equal to 0.

[15]In this case, as interest rates are not set freely but follow a Taylor rule, the IS curve would become a constraint in the optimal policy problem.

Figure 2: Trade-off matrices for QT, under RE, for a given interest rate rule



(a) Interest rate rule reacting to inflation only

(b) Interest rate rule reacting to inflation and output gap

Notes: Trade-off matrices $\mathcal{T}^{\varepsilon_{qt}}$ of an optimal QT policy under a given interest rate rule. Each row is one period in the sequence, each line shows a relevant period of the output gap entering in the optimal policy rule. The left panel is with a Taylor rule focusing on inflation only ($i_t = 1.5\pi_t$), the right panel is with a dual mandate Taylor rule ($i_t = 1.5\pi_t + 0.125x_t$). The intensity of the color corresponds to the magnitude of the trade-off.

inflation and the output gap for an optimal QT policy, contingent on a specific interest rate policy rule. The left panel is a case in which the underlying, interest rate policy rule focuses solely of inflation. The right panel, in contrast, presents a case in which the short-term policy rate depends on both inflation and the output gap.

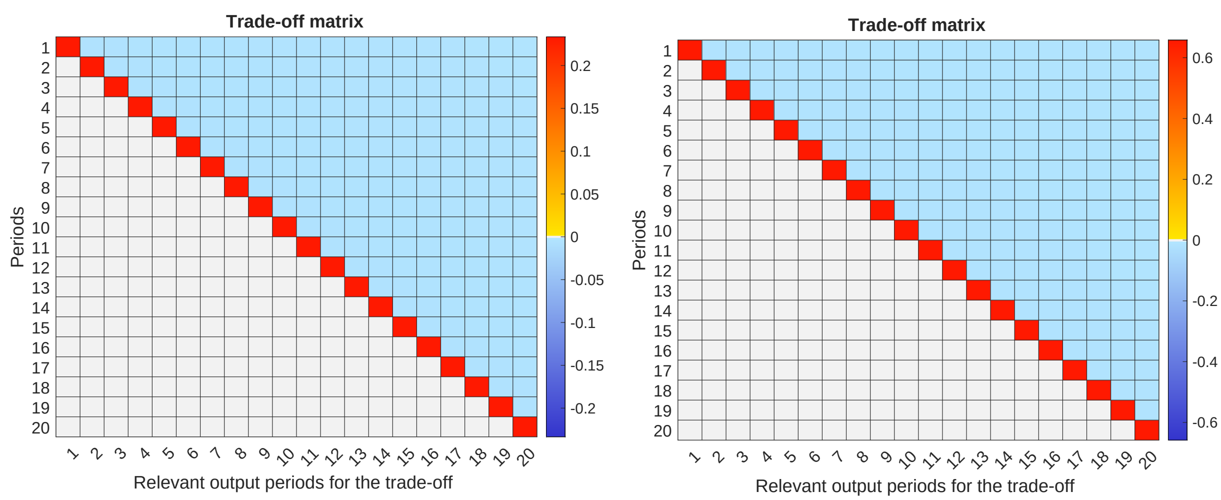
The results in Figure 2 show that central bank's trade-off under optimal QT for a given interest rate rule, are distinct from those where only the interest rate follows an optimal rule. First, the contemporaneous trade-off is more pronounced (as indicated by the larger scale in the diagonal entry, in comparison to Figure 1), suggesting that the optimal QT policy incurs a larger output loss for a given reduction in inflation.[16] Second, the role of history dependence for the trade-off also changes. Unlike in the case of the short-term rate r_t^s, in the model qe_t enters both via its value each period, and as an expectation, through their effects on the long-term rate. Consequently, for optimal QT, the central bank must consider not only the current output gap but also all past values of the output gap, resulting in a lower triangular matrix. This history dependence implies that, just as the traditional result associated with optimal interest rate policy, commitment about future policy decisions plays an essential role in the context of optimal QT. Lastly, the underlying interest rate policy

[16]See Appendix B.2 for further detail on the marginal effect of a QT versus and interest rate shock.

is pivotal in shaping the optimal balance sheet trajectory. A central bank which sets the interest rate following a Taylor rule with dual mandate (right panel) puts a higher weight on past periods output gaps, compared to a central bank which solely focuses on inflation with its interest rate policy rule (left panel).

Under de-anchored expectations:

Figure 3: Trade-off matrices under de-anchored expectations



(a) Optimal interest rate policy (b) Optimal balance sheet policy

Notes: Trade-off matrices for each policy instrument $\mathcal{T}_d^{\varepsilon_y}$ under de-anchored expectations. Each row is one period in the sequence, each line shows a relevant period of the output gap entering in the optimal policy rule. The central bank is assumed to follow a Taylor rule focusing on inflation only. The left panel shows the case under interest rate policy, the right panel is the case under balance sheet policy. The intensity of the color corresponds to the magnitude of the trade-off.

We now examine scenarios in which expectations are de-anchored. In such environments, the trade-off matrix can be expressed as $\mathcal{T}_d^{\varepsilon_y} = \left(\mathcal{A}_d^{\Pi,\varepsilon_y \prime} \Lambda_\Pi\right)^{-1} \left(\mathcal{A}_d^{X,\varepsilon_y \prime} \Lambda_X\right)$, with $\mathcal{A}_d^{Z,\varepsilon_y}$ the impulse responses of the variable Z to a set of policy shocks ε_y in the model with fully de-anchored expectations.[17] Figure 3 shows the trade-off matrices for each optimal policy instrument under these conditions.

The left panel of Figure 3 illustrates the optimal interest rate policy. As detailed in Section 3.1, the optimal interest rate rule involves managing two trade-offs. An intratemporal

[17]Note that in this scenario, agents are entirely backward-looking and are therefore influenced solely by shocks from the point at which they occur, without any anticipation. This contrasts with the case of RE, in which agents react to anticipated future events. Consequently, in the case of de-anchored expectations, impulse response matrices are lower triangular.

trade-off between stabilizing current output alongside current inflation, and a intertemporal trade-off, balancing current inflation against future output gap values. The intratemporal trade-off is reflected in the non-zero values along the main diagonal of the matrix, while the intertemporal trade-off is represented by the upper triangular.

Similarly, in the case of setting an optimal QT rule with a predetermined interest rate policy, as depicted in the right-hand panel of Figure 3, the central bank's trade-off also includes current inflation and output gap, as well as future output gap values. The structure of the forecast targeting criterion remains consistent across both policy instruments when expectations are de-anchored. This uniform structure suggests that, in this scenario, the central bank's methodology for addressing these expectations does not change with the policy instrument. However, a critical observation is the difference in the scale of the main diagonal between the two panels, suggesting that QT is relatively less effective than the interest rate, also in the case of de-anchored expectations.

4 Simulations under a model consistent loss function

In this section, we turn to quantitative simulations. In particular, we focus on the effects of cost-push shocks on the policy decisions of a central bank that has embarked on a normalization of its policy stance. This implies the assumption that the interest rate is above the ELB, and that the balance sheet is expected to steadily decline. Section 4.1 begins with the derivation of a welfare loss function, based on a second-order approximation of the utility functions of the parent and the child, as in Rotemberg and Woodford (1997), and provide details on our solution method for solving for the sequence of each monetary policy instrument that implements optimal policy. In Section 4.2, we subsequently study the optimal policy path for the two policy instruments - the short-term policy rate and the balance sheet - under scenarios of both RE and fully de-anchored expectations In Section 4.3, we analyze the role of expectations in further details, while in Section 4.4 we focus on optimal policy when the economy is subject to both price cost-push and negative credit shocks.

4.1 Ramsey problem and introductory details

We derive the welfare loss function based on a second-order approximation of the utility functions of the two types of households, the "parent" and the "child". This analysis takes into account the costs associated with changes in the central bank's asset holdings, which are present even at the steady state. Consequently, our optimal policy analysis will be based on deriving a welfare measure and then designing optimal monetary policy under a distorted steady state, in the spirit of Benigno and Woodford (2005). The welfare criterion consists of the sum of utilities of the "parent" and the "child":

$$W_t = V_t + V_{b,t} \tag{25}$$

where now V_t and $V_{b,t}$ represent second-order approximations of the utility of the "parent" and the "child", respectively. Our derivation of the welfare loss function is summarized in the following proposition.

Proposition 2. *The discounted sum of the household utilities is given by:*

$$\begin{aligned} W_t = W - U_C Y \Bigg\{ & (1-z)\left(\frac{C}{2}\right) c_t^2 + z\left(\frac{C_b}{2}\right) c_{b,t}^2 + \frac{Y\epsilon\phi}{2(1-\phi\beta)(1-\phi)}\pi_t^2 \\ & + \frac{1}{2}(\tilde{v}_t^p)^2 + \tau\frac{Qb^{cb}}{Y} qe_t + \tau Qb^{cb}\frac{Qb^{cb}}{2Y} qe_t^2 \Bigg\} + t.i.p. + O\left(||\xi^3||\right) \end{aligned}$$

Proof. In appendix C.1 ■

A couple of important observations stand out from the model-consistent welfare criterion in Proposition 2. First, fluctuations in the consumption of each type are costly on top of those in aggregate output. Contrary thus to the ad-hoc loss function previously used, the central bank accounts for those when setting its optimal policy. Second, the central bank's bond portfolio, as measured by qe_t, enters the loss function. Fluctuations in it matter thus for the central bank and as such they have to be set optimally rather than being set by an ad-hoc rule, when the central bank minimizes the loss function. But there is an indirect channel as well which is related to the way changes in central bank's bond holdings affect the consumption of each group. If, for instance, the central bank decides to unwind its balance sheet, the consumption of the child, $c_{b,t}$, will shrink because the central bank will buy

less of its bonds and hence the child will will borrow less to finance its consumption ceteris paribus.[18] On the other hand, given that the parent provides full bailout to the child, lower borrowing implies lower bailout which in turn boosts parent's consumption. The decision thereby of the central about its portfolio, qe_t, determines this trade-off. Observing the loss function, it is clear that the consumption shares captured by z and $1-z$ play a key role on whose consumption fluctuations weigh more in that decision. Clearly, the steady state asset holdings (i.e. the size of the balance sheet) of the central bank play a key role in the design of optimal monetary policy, as we will see below as well.
The loss function $\mathcal{L}_t$ consistent with the Ramsey problem is:

$$\mathcal{L}_t = \left\{ (1-z)\left(\frac{C}{2}\right)c_t^2 + z\left(\frac{C_b}{2}\right)c_{b,t}^2 + \frac{Y\epsilon\phi}{2(1-\phi\beta)(1-\phi)}\pi_t^2 + \frac{1}{2}(\tilde{v}_t^p)^2 + \tau\frac{Qb^{cb}}{Y}qe_t + \tau Qb^{cb}\frac{Qb^{cb}}{2Y}qe_t^2 \right\} \tag{26}$$

Following our baseline calibration, we can write the weights in the loss function as:

$$\omega_C = (1-z)\left(\frac{C}{2}\right) = \frac{0.67}{1}\left(\frac{0.67}{2}\right) = 0.2244$$
$$\omega_{C_b} = z\left(\frac{C_b}{2}\right) = \frac{0.33}{1}\left(\frac{0.33}{2}\right) = 0.0545$$
$$\omega_\pi = \frac{Y\epsilon\phi}{2(1-\phi\beta)(1-\phi)} = 65.0246$$
$$\omega_{qe} = \tau Qb^{cb}\frac{Qb^{cb}}{2Y} = 5e-05$$

In the optimal rule of the Ramsey planner, inflation stabilization has sizeable role, while stabilizing the consumption of the parent (i.e. patient household) weights more than stabilizing the consumption of the child (i.e. impatient household). The QE cost introduces a small weight on the path of the balance sheet.

Introduction for our simulations:

To replicate an environment with a high balance sheet that gradually returns to a steady state (10% of annualized output), we begin by introducing a persistent positive QE shock.[19]

[18] Recall that in the original model, the total stock of long-term bonds issued by the child are held by the financial intermediaries or the central bank.

[19] The argument of a positive balance sheet in the steady-state has been recently emphasized in policy

In period 5, which is four quarters after the initial QE shock, we assume a one standard deviation price cost-push shock. In the same period, we compute optimal policy in the different cases. When looking at the effect of a credit shock, we assume that it occurs in period 9. Each case of optimal policy is therefore analyzed in a context of balance sheet normalization, representing the transition to a lower central bank's balance sheet.

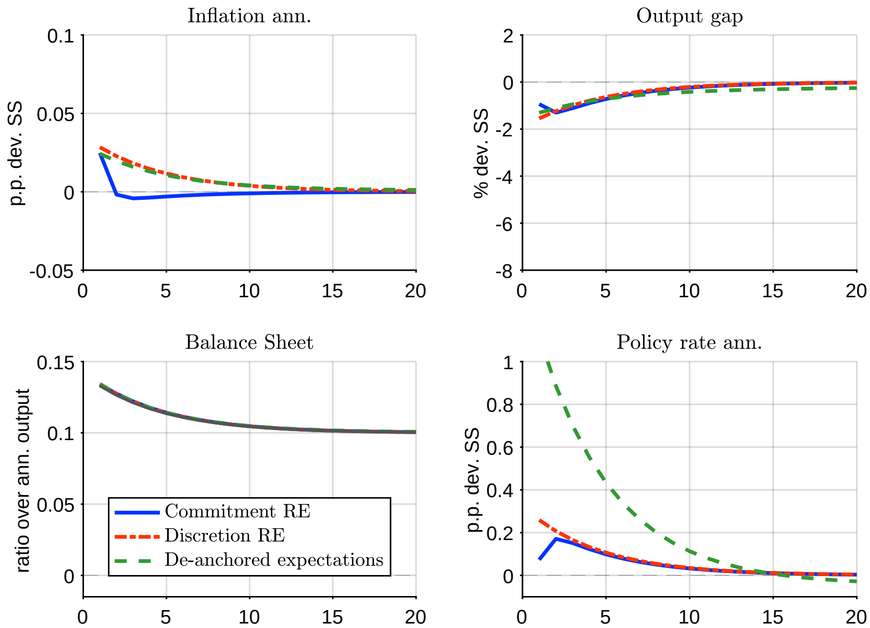
Computational details:

As described in 3.2, our linear-quadratic approach to optimal policy, drawing on de Groot et al. (2021), McKay and Wolf (2023) and Hebden and Winkler (2021), allows us to derive model-consistent forecast targeting criteria, both under de-anchored and rational expectations. We leverage on the first-order conditions of our optimal policy formulation, in which the constraints for each variable are defined using the impulse response representation of the model. We subsequently solve for the set of monetary policy shocks that satisfies the optimal policy problem.[20] This means that we find the interest rate and balance sheet sequences that implement with our forecast targeting criteria. Additional details are presented in Appendix C.2.

4.2 A persistent cost-push shock

First, we analyze how the central bank can implement its monetary policy instruments so as to minimize our model consistent loss function, in response to a persistent price cost-push shock that induces upward pressure on inflation. Our analysis begins by examining the case in which the central bank can optimize its primary instrument, the short-term nominal interest rate. Then, we turn to the a scenario in which the central bank can only implement the path of the balance sheet in a optimal way, for a given interest rate rule. Finally, we look at the way both instruments should be set jointly.

Figure 4: Optimal interest rate policy only, in response to a cost-push shock



Notes: Optimizing the interest rate only, in response to a one standard deviation persistent ($\rho_{cp} = 0.8$) cost-push shock. QT is inactive in the baseline. The solid black line shows the baseline QT trajectory.

4.2.1 Optimal interest rate policy

Figure 4 shows the responses of the short-term policy rate, the balance sheet, inflation and the output gap, in the case in which only the interest rate is used to minimize our model-consistent loss function (26).[21] The trajectories prescribed in each set up are closely aligned with the benchmark established in the literature. The case of commitment under RE, depicted by the solid blue line, presents the standard pattern in the literature e.g. Galí (2008), Woodford (2003). Optimal interest rate policy under commitment results in a lower and shorter-lived inflation compared to the policy under discretion, depicted in red dashed. The forward-looking nature of inflation allows the policymaker to credibly commit

discussions. See, e.g. 9 November 2023, Philip R. Lane, Member of the Executive Board of the ECB, at the ECB Conference on Money Markets "In particular, the appropriate level of central bank reserves can be expected to remain much higher and be more volatile in this new steady state compared to the relatively-low levels that prevailed before the global financial crisis (GFC)."

[20]Our computations benefit from the extensive work outlined in the COPPs toolkit developed by de Groot et al. (2021), which provided valuable insights for solving our optimal policy problems. The toolkit can be accessed here: https://github.com/COPPsToolkit/COPPs.

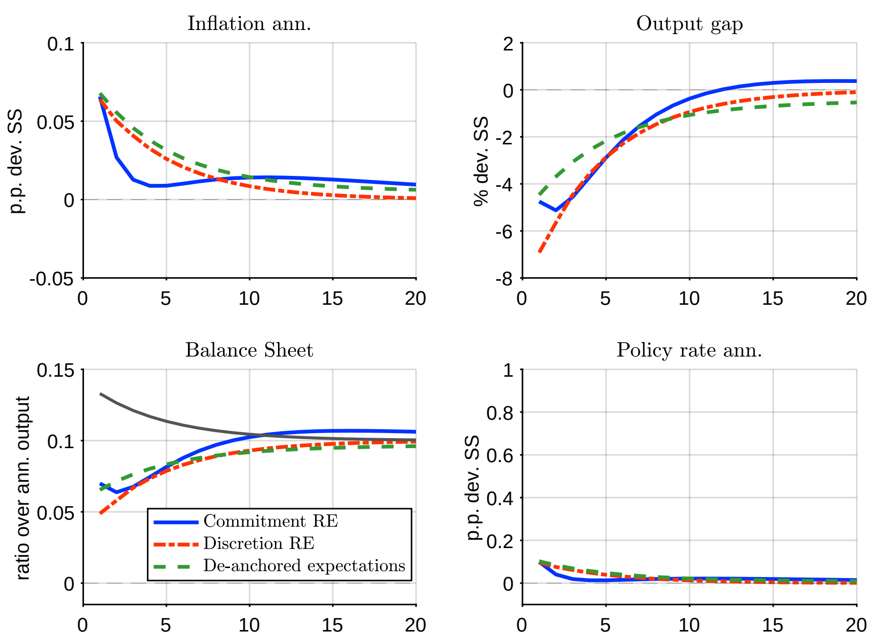
[21]In cases in which the interest rate is the sole instrument used optimally, the balance sheet remains inactive, allowing for a focused examination of the interest rate policy's impact.

to reducing both current and future output gaps, mitigating the stabilization bias inherent in a discretionary policy framework.

Consistent with the findings of Molnár and Santoro (2014) and Gáti (2023) our analytical results described in Section 3 underscore an intertemporal trade-off introduced by de-anchored expectations. In line with these studies and as highlighted in Figure 4, this situation necessitates a more aggressive increase in the interest rate by policymakers under de-anchored expectations.[22] While the trajectory of inflation remains similar to scenarios with discretionary policy, a dampening of inflation under de-anchored expectations results in heightened output losses due to the more pronounced and persistent interest rate adjustments.

4.2.2 Optimal QT policy for a given interest rate rule

Figure 5: Optimal balance sheet policy, in response to a cost-push shock



Notes: Optimizing QT only, in response to a one standard deviation persistent ($\rho_{cp} = 0.8$) cost-push shock. Interest rates follow the baseline Taylor rule reacting to inflation only. The solid black line shows the baseline QT trajectory.

[22]In our model, expectations about interest rates do not play a role. However, when agents misperceive the long-term mean of interest rates as well, as Eusepi et al. (2020) show, they tend to inherit the volatility of inflation expectations, particularly when the central bank's response is aggressive. Consequently, under these conditions, optimal interest rate policy tends to be less aggressive than in scenarios with rational expectations.

Turning to QT, how should the central bank optimally conduct its balance sheet policy in response to a persistent cost-push shock, for a given interest rate rule? We analyze this question in an environment in which the central bank adheres to a standard Taylor rule exclusively targeting inflation, while exploring the optimal path for its balance sheet.

Figure 5 presents the optimal balance policy projections for commitment and discretion under RE, and de-anchored expectations. The analysis reveals that, in response to a cost-push shock, the central bank is required to significantly contract the size of its balance sheet when QT is the sole tool used to minimize our model-consistent loss function. As a consequence, this reduction in the balance sheet size dampens inflation; however, it comes at the expense of considerable output reductions. These findings align with the discussions in Section 3.2, in which we established that QT, although set optimally, is less effective and a more costly instrument than the interest rate to fight inflationary pressures.

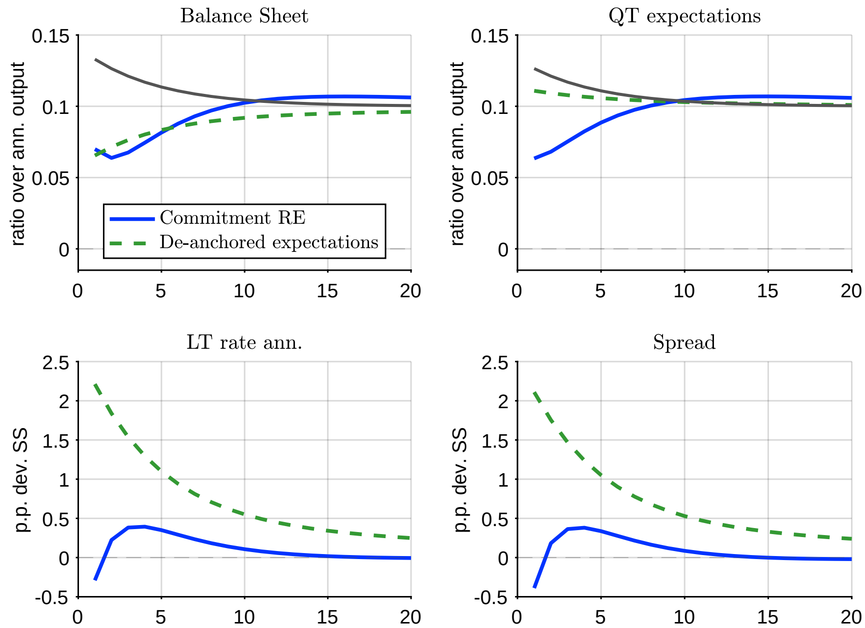
Furthermore, the trajectories under commitment with RE or de-anchored expectations, exhibit important differences. In the RE framework, the central bank can offset the initial fall in the output gap by sustaining a balance sheet above the steady-state once inflation approaches the target. In contrast, with de-anchored expectations, the central bank balance sheet remains below the steady-state throughout the policy horizon, perpetuating a lower output gap. Therefore, the central bank can commit to future positive output gap recovery as inflation decreases, provided that past reductions have been sufficient. In the case of de-anchored expectations, such commitment is not feasible, thus necessitating a consistently lower balance sheet, with no subsequent elevation above the steady-state.[23]

Figure 6 provides further details on the mechanism through which an optimal QT strategy affects inflation and the output gap, emphasizing the role of expectations in shaping the effectiveness of the policy. When expectations are well-anchored, agents' perception track precisely the announced path by the central bank, enabling the central bank to effectively manage long-term rates.[24] This precise control allows the central bank to calibrate QT to the necessary degree, tightening long-term rates just enough to counteract the elevated inflation. However, this is associated with a high output cost for a relatively modest reduction in inflation.

[23]This results from the fact that the trade-off matrix is lower triangular and has positive entries for past output gaps.

[24]As detailed in Appendix A, real long term rates are determined by the expected changes in the balance sheet $\mathbb{E}_t r^b_{t+1} - \mathbb{E}_t \pi_{t+1} = \sigma \left[\mathbb{E}_t c_{b,t+1} - c_{b,t}\right] = \sigma \left[\bar{b}^{FI} \left(\mathbb{E}_t \theta_{t+1} - \theta_t\right) + \bar{b}^{cb} \left(\mathbb{E}_t qe_{t+1} - qe_t\right)\right]$.

Figure 6: Optimal balance sheet policy, QT expectations

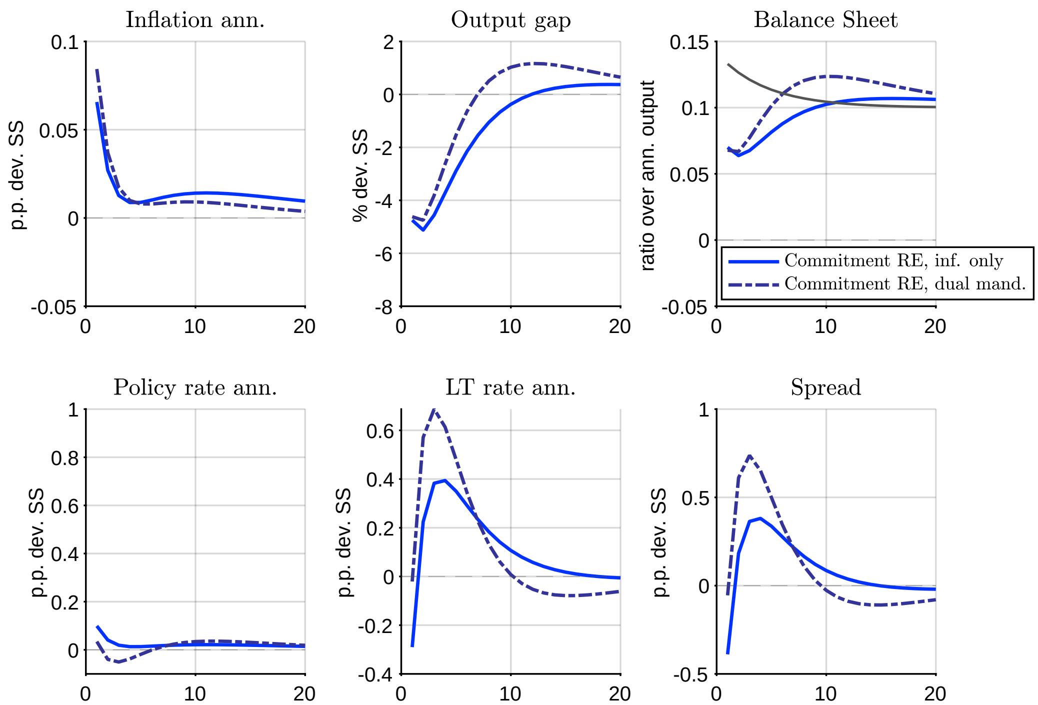


Notes: The dynamics of QT expectations when only the path of the balance-sheet is set optimally for a given (baseline) interest rate rule, in response to a one standard deviation persistent ($\rho_{cp} = 0.8$) cost-push shock. The solid black line shows the baseline QT trajectory.

In the case of de-anchored expectations, expectations are sluggish and agents do not understand the central bank's announcement of the balance sheet reduction path. This results in a significant rise in long-term rates due to the misalignment between the actual QT level and its expectation. Consequently, while inflation falls, output suffers a substantial decline.

Importantly, in the case of an optimal QT strategy, the path of the long-term interest rate resembles the pattern of the short-term policy rate under optimal interest rate policy. Under RE, the long-term rate path is hump-shaped and quickly reverts to the steady state, while it rises quickly and stays elevated under de-anchored expectations. Therefore, although the central bank does not set the level of long-term rates explicitly, it can exert influence over them when expectations are well-anchored. By communicating the anticipated trajectory of its balance sheet, the central bank can shape market expectations, effectively guiding the long-term rates through its balance sheet policy. This highlights the importance of *having anchored QT expectations*, allowing the central bank to significantly influence real economic transmission through its forward guidance on balance sheet policies.

Figure 7: Optimal balance sheet policy in the case of a dual-mandate interest rate policy rule

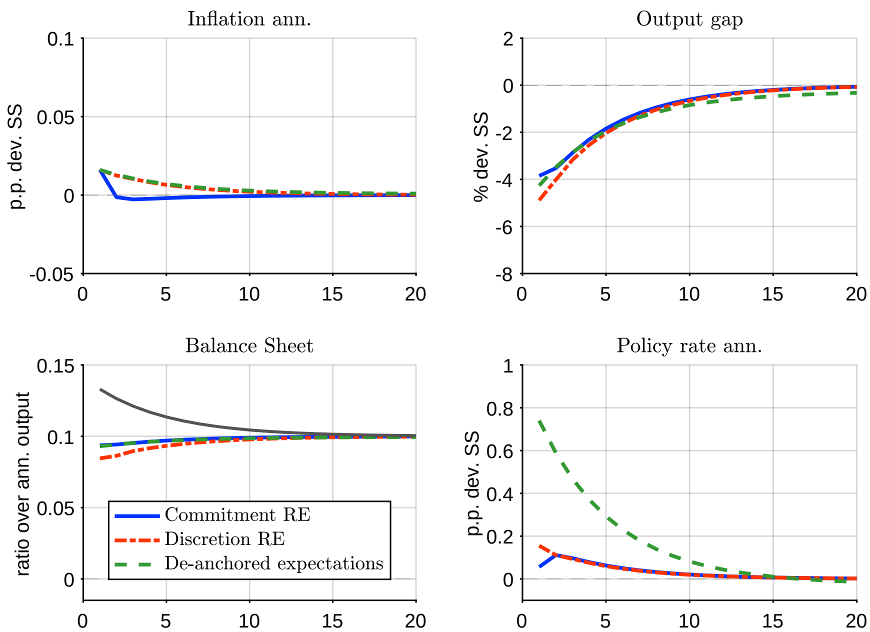


Notes: Optimizing QT only, in response to a one standard deviation persistent ($\rho_{cp} = 0.8$) cost-push shock. The solid line shows the scenario with interest rates which follow the baseline Taylor rule reacting to inflation ($i_t = 1.5\pi_t$). The dashed line shows an interest rate policy which follows a standard Taylor rule reacting to inflation and the output gap ($i_t = 1.5\pi_t + 0.125x_t$). The solid black line shows the baseline QT trajectory.

As detailed in Section 3.2, the form of the systematic short-term interest rate policy rule is crucial for the conduct of optimal balance sheet policy, as it increases the importance of output gap recovery through the commitment device. In the baseline case, depicted in Figure 5, the central bank solely focuses on inflation in the interest rate rule ($i_t = 1.5\pi_t$). However, when the central bank's interest rate rule incorporates a response to the output gap ($i_t = 1.5\pi_t + 0.125x_t$), the commitment strategy with anchored expectations necessitates a less aggressive QT trajectory, as shown in Figure 7. In practice, this translates into a more pronounced initial reduction in the size of the balance sheet, followed by a steeper rise above the steady state value. This approach induces a more substantial but transient spike in the long-term interest rate, fostering a quicker rebound in economic activity.

4.2.3 Optimal monetary policy mix

Figure 8: Optimal monetary policy mix, in response to a cost-push shock



Notes: Optimizing QT and the interest rate, in response to a one standard deviation persistent ($\rho_{cp} = 0.8$) cost-push shock. The solid black line shows the baseline QT trajectory.

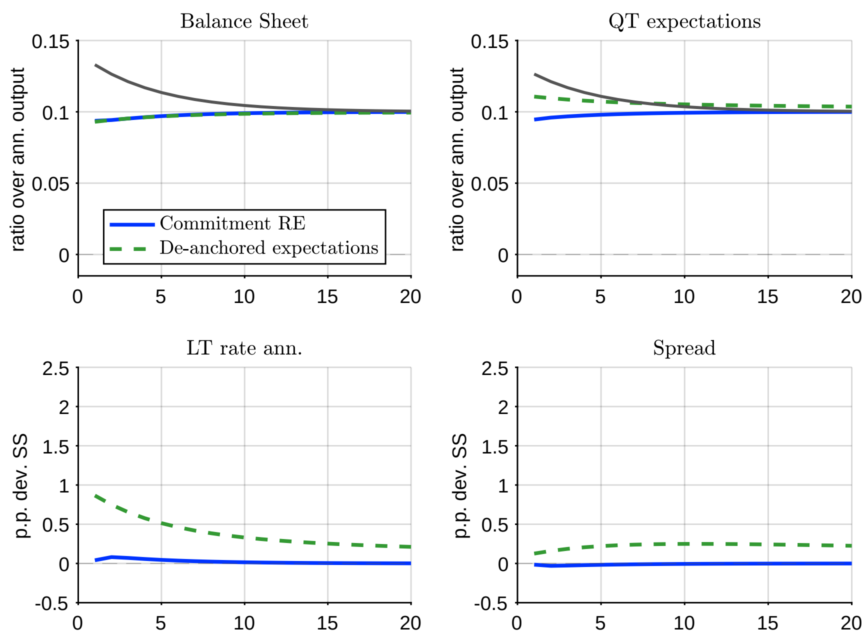
The ideal approach for a central bank would be to determine the optimal combination of its monetary policy instruments.[25] Figure 8 presents the effects of jointly setting the interest rate and the balance sheet policy to solve for the optimal Ramsey problem. It reveals that optimal interest rate policy remains slightly below the level when set independently, especially under de-anchored expectations. The balance sheet contraction is also contained, settling closer to the steady-state value, thereby further mitigating inflation with a smaller impact on economic activity. Within this policy mix, the short-term interest rate is proving to be a more powerful tool for controlling inflation. However, a measured application of QT can contribute to the moderation of inflationary pressures with a reduced impact on the output gap.[26]

[25] In this situation, as in the case of an optimal interest rate policy, the central bank is not constrained by the IS equation.

[26] Additionally, optimally setting both instruments minimizes the disparities in the consumption between the two types of household, as depicted in Appendix C.4. Moreover, we find that the loss function is minimized more effectively with the optimal mix of instruments. Specifically, when comparing the loss in the case of the

Figure 9 provides a deeper analysis of how an optimal Ramsey mix of monetary policy tools operates. When the central bank concurrently optimizes both interest rate and balance sheet policies, as opposed to relying solely on an optimal QT, it necessitates a less pronounced increase in the long-term rate to achieve its objectives. Importantly, in this case, the expectations formation of the private sector about the QT trajectory is of relatively low importance. The intuition for this result is that, in this case, smaller adjustments in the long-term rate are required under a dual-instrument approach, irrespective of the prevailing expectation formation scenario. Consequently, any variations in the long-term rate are a direct consequence of changes in QT expectations rather than the central bank's actions per se.

Figure 9: Optimal monetary policy mix, QT expectations



Notes: The dynamics of QT expectations when both instruments are set optimally, in response to a one standard deviation persistent ($\rho_{cp} = 0.8$) cost-push shock. The solid black line shows the baseline QT trajectory.

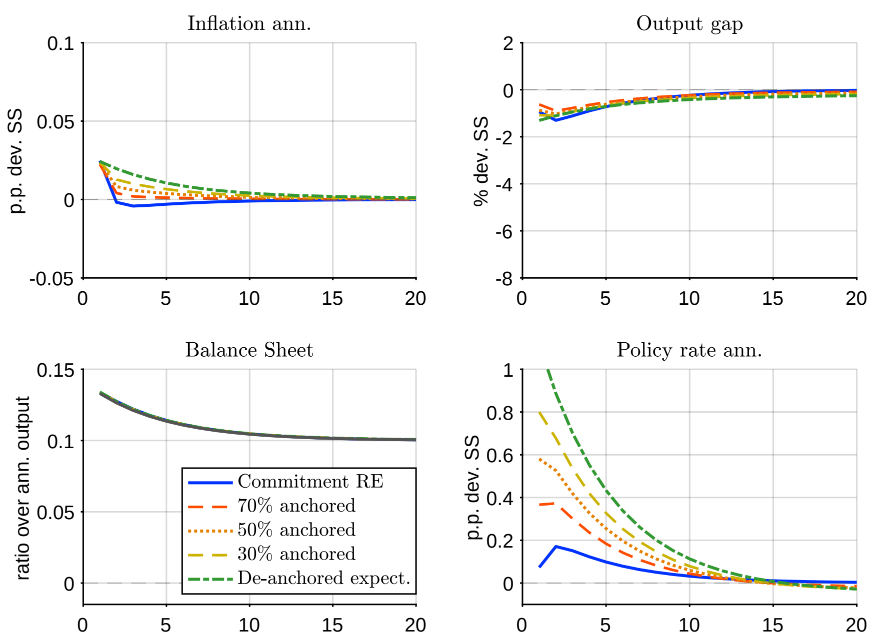
Overall, we emphasize the following three main takeaways about the optimal normalization policy when a central bank is facing positive price cost-push shocks, consistent with our analytical findings derived in Section 3. First, the interest rate stands out as the primary

optimal mix to that of the optimal interest rate policy only, the ratio is $\frac{L_{mix}}{L_{IR}} = 32\%$. Similarly, when comparing it to the loss in the case of the optimal balance sheet policy only, the ratio is $\frac{L_{mix}}{L_{QT}} = 26\%$.

instrument for macroeconomic stabilization, due to its higher efficacy in tempering inflationary pressures. Second, in scenarios in which QT is the sole policy tool employed, the anchoring of QT expectations becomes critical. Properly managed expectations allow the central bank to not only reduce inflation more effectively in the present but also to mitigate the adverse effects on the output gap throughout the policy horizon, thereby smoothing the economic adjustment process. Third, when both the interest rate and QT are used to minimize the loss function, the interest rate should take the lead in the policy response. The optimal strategy involves the interest rate actively responding to economic conditions while QT is more moderate, and declines less than if the balance sheet only is set so as to minimize the loss function.

4.3 The role of expectations

Figure 10: Optimal interest rate policy, with heterogeneous expectations

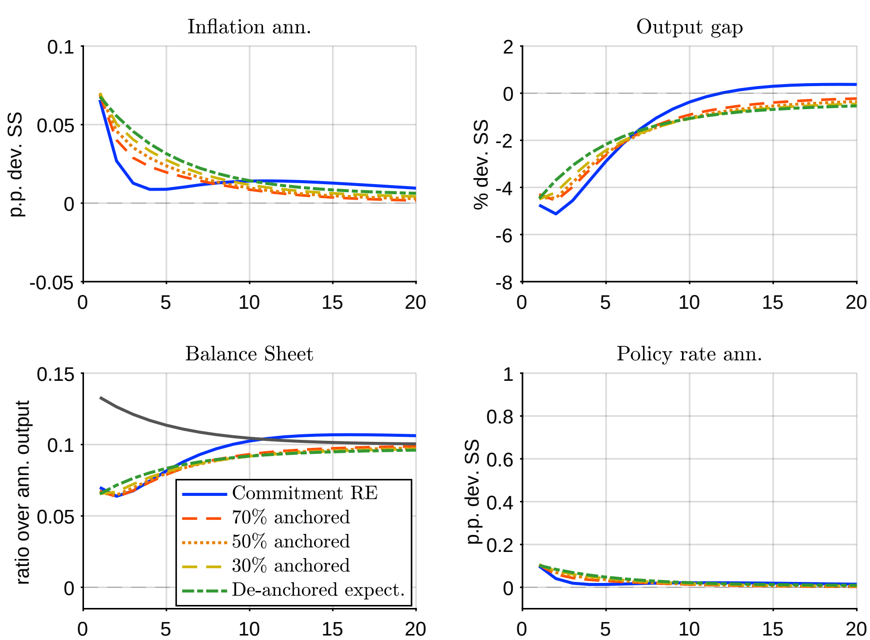


Notes: Using the interest rate only in response, in response to a one standard deviation persistent ($\rho_{cp} = 0.8$) cost-push shock. Dashed lines are different levels for the fraction of anchored (i.e., RE) agents. The solid black line shows the baseline QT trajectory.

Having detailed optimal monetary policy trajectories under our two extremes expectation scenarios (RE and de-anchored expectations), we now analyze in detail the importance of the expectation formation of the private sector. Using both survey and market data, central banks carefully monitor the risk of de-anchoring of agents' inflation expectations, recognizing that they play an essential role in shaping the path of monetary policy going forward. In this context, we examine how the central bank should implement its normalization strategy, for different shares of agents having anchored or de-anchored expectations.

Figure 10 - 11 - 12 report the results. The different trajectories depicted reflect varying proportions of agents having anchored expectations, each signifying a distinct level of confidence in the central bank's commitment. Figure 10 describes the implications for optimal interest rate policy. It highlights a crucial dynamic: the degree to which private agents' expectations are de-anchored significantly influences the path of interest rates. As we move closer to a scenario in which all agents are de-anchored, represented by the dotted green line, we see an upward shift in the recommended path for the interest rate. Conversely, a more anchored environment allows for a more tempered approach.

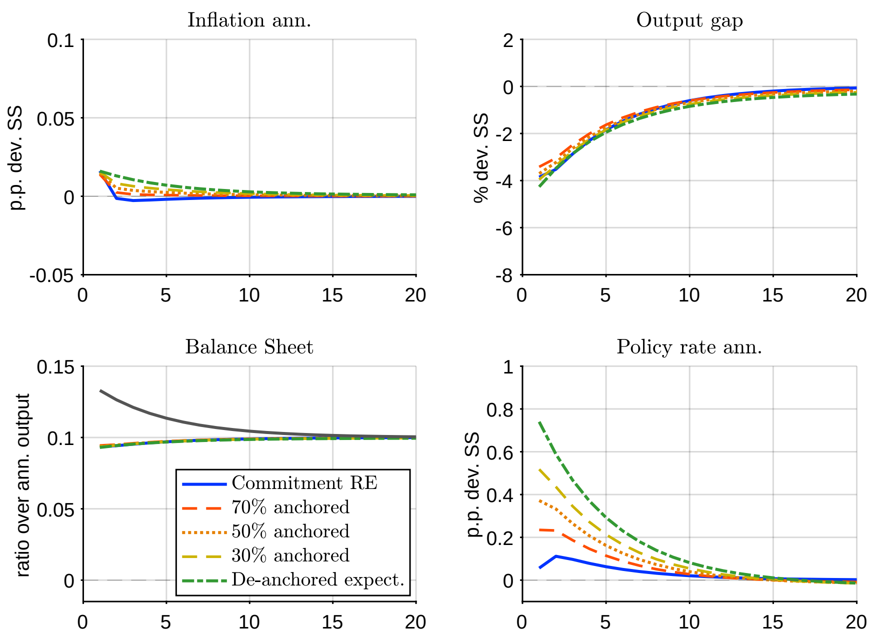
Figure 11: Optimal balance sheet policy, with heterogeneous expectations



Notes: Optimizing QT only in response to a one standard deviation persistent ($\rho_{cp} = 0.8$) cost-push shock, interest rates follow the baseline Taylor rule reacting to inflation. Dashed lines are different levels for the fraction of anchored (i.e., RE) agents. The solid black line shows the baseline QT trajectory.

Turning to the case of an optimal QT for a given interest rate rule in Figure 11, we find that the degree of anchored agents is essential for the central bank to effectively mitigate inflation and foster the recovery of the output gap, as optimal QT measures prove more effective when a larger fraction of agents hold anchored expectations. As the percentage of anchored agents decreases, the central bank must enact more pronounced balance sheet adjustments to stabilize inflation and the output gap, underscoring the challenges posed by de-anchored expectations.

Figure 12: Optimal monetary policy mix, with heterogeneous expectations

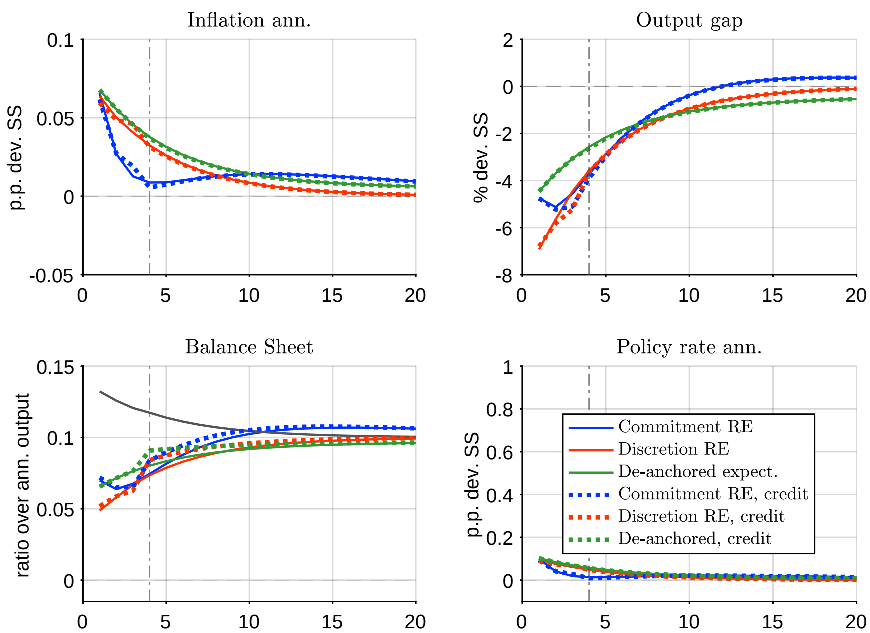


Notes: Optimizing QT and the interest rate, in response to a one standard deviation persistent ($\rho_{cp} = 0.8$) cost-push shock. Dashed lines are different levels for the fraction of anchored (i.e., RE) agents. The solid black line shows the baseline QT trajectory.

Lastly, Figure 12 presents the role of expectations for the optimal Ramsey monetary policy mix. This figure confirms that while expectations anchoring significantly affects the interest rate path, the QT policy path remains consistent across different expectation profiles. This invariance indicates that the central bank's QT strategy is robust to variations in the degree of expectations anchoring. In essence, whether a larger fraction of agents are fully anchored or not, the monetary policy adjustments that the central bank must implement do not vary, highlighting a clear distinction in the sensitivity of monetary policy instruments to the anchoring of expectations, under the optimal Ramsey policy mix in this model.

4.4 Facing multiple shocks, the effects of credit disturbances

Figure 13: Optimal QT policy, in response to a cosh-push and a credit shock



Notes: Optimizing QT only, in response to a one standard deviation persistent ($\rho_{cp} = 0.8$) cost-push shock, and a five standard deviation persistent ($\rho_\theta = 0.8$) credit shock. Interest rates follow the baseline Taylor rule reacting to inflation. The vertical dashed line represents the period when the credit shock hits. The solid colored lines are optimal policy when facing a cost-push only, while the dashed colored lines are when facing a cost-push and a credit shock. The solid black line shows the baseline QT trajectory.

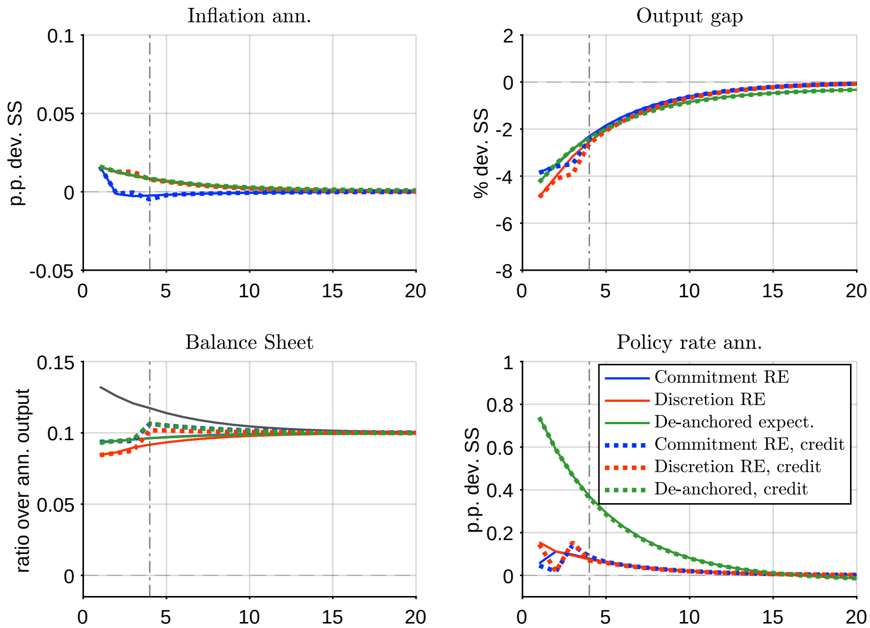
Balance sheet policies are also frequently used as a way to stabilize stressed financial markets. When addressing inflationary shocks during monetary policy normalization, policymakers may concurrently face shocks affecting the financial system.[27] How should policymakers respond optimally to such a combination of disturbances?

This section explores optimal policy responses to substantial credit shocks occurring one year after initial cost-push inflation shocks, depicted in Figure 13 and Figure 14. These credit shocks, represented by dashed lines, resemble a severe tightening of financial intermediaries' balance sheets, akin to a credit crunch.

Compared to a case with inflationary shocks only (depicted with solid lines), we find that large credit disturbances can push the central bank to slow down and even partially re-

[27]A recent example is the Federal Reserve's intervention in the wake of the failure of Silicon Valley Bank and Signature Bank.

Figure 14: Optimal monetary policy mix, in response to a cosh-push and a credit shock



Notes: Optimizing QT and the interest rate, in response to a one standard deviation persistent ($\rho_{cp} = 0.8$) cost-push shock, and a five standard deviation persistent ($\rho_{\theta} = 0.8$) credit shock. Interest rates follow a standard Taylor rule reacting to inflation. The vertical dashed line represents the period when the credit shock hits. The solid colored lines are optimal policy when facing a cost-push only, while the dashed colored lines are when facing a cost-push and a credit shock. The solid black line shows the baseline QT trajectory.

verse their normalization efforts. Results are consistent across the different specifications, commitment and discretion under RE, as well as with optimal policy under de-anchored expectations. Notably, as depicted in Figure 14, in which the central bank optimizes the use of both its policy instruments, the balance sheet becomes the primary tool for responding to the credit shock. Meanwhile, the trajectories of the policy rate remain broadly aligned with cases involving only a price cost-push shock. Therefore, this shows that, when confronted with such a combination of shocks and employing both policy instruments in an optimal way, the central bank's approach is to tailor each tool to respond to a specific shock: the policy rate is primarily focused on controlling inflationary pressures, while the balance sheet is deployed to stabilize financial markets in response to the credit shock.

5 Conclusion

This paper is motivated by the recent rate hikes implemented by major central banks, partly in response to supply shocks, and by the ongoing unwinding of their large balance sheets. Given the importance attached to expectations formation mechanisms in central banks' recent strategy review, this study aims to analyze their implications for the design of optimal normalization policy. Our key finding is that, when both monetary policy instruments (i.e. policy rate and asset holdings) are set optimally, the expectations formation mechanism matters largely for the policy rate while its importance drops for the decisions about unwinding of the balance sheet.

We design optimal normalization policy in the four equation model of Sims et al. (2023) under rational and de-anchored expectations assuming QT entails quadratic efficiency costs. Looking at how each monetary policy instrument (i.e. the interest rate and central bank asset holdings) can be set such as to follow an optimal rule, we have shown that both can succeed in taming inflation following persistent cost-push shocks. However, when only the central bank asset holdings are set optimally, inflation stabilization entails larger output gap losses. Hence, setting the policy rate optimally using a model consistent welfare criterion proves to be a superior strategy for the central bank.

The finding that the expectations formation mechanism matters substantially less for the optimal decision about the path of the asset holdings, when both instruments are set optimally, carries important implications for the design of QT of major central banks. Essentially, this result shows that communication about the future intentions regarding the path of their balance sheet is less restricted, in the sense that it does not seem to be subject to front loading or to affect agents' optimal decisions in a substantially different manner conditioning on the way they form their expectations.

We acknowledge however that our conclusions are drawn in a rather simplified framework and, as such, a richer model featuring additional transmission channels is deemed necessary. At the same time, this is a first attempt to incorporate state-of-the art techniques to compute optimal normalization policy in a model with bounded rationality.

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A Extending the Sims et al. (2023) with a quadratic efficiency cost

In this section, we present an extension of the original model by Sims et al. (2023) with a quadratic efficiency cost.[28]

The model makes several simplifying assumptions in this section to get allow the model to reduce to four equations in the log-linearized form. The quantitative implications of the four equation model are similar to more complicated models.

The economy is populated by the following agents: two types of households (parent and child, with respective shares of z and $1-z$), a representative financial intermediary, production firms, and a central bank. As detailed in Sims et al. (2023), the dynamics of the child's consumption in our model are in-line with the behavior of investment in Sims and Wu (2021).

A.1 Households

A.1.1 Parent, or patient household

A representative parent maximizes its discounted lifetime utility from consumption, C_t and labor L_t:

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_{s,t}^{1-\sigma} - 1}{1-\sigma} - \psi \frac{L_{s,t}^{1+\chi}}{1+\chi} \right] \tag{A.1}$$

with $\sigma > 0$ is the inverse elasticity of intertemporal substitution, $\chi \geq 0$ is the inverse Frisch elasticity, and $\psi > 0$ is a scaling parameter.

The parent's budget constraint is:

$$P_t C_t + S_t \leq W_t L_t + R_{t-1}^s S_{t-1} + P_t D_t + P_t D_t^{FI} + P_t T_t - P_t X_t^b - P_t X_t^{FI} \tag{A.2}$$

in which P_t is the nominal price of consumption. The parent earns income from W_t the nominal wage, D_t dividends from ownership in firms, D_t^{FI} dividends from ownership in financial intermediaries, and from lump-sum transfers from the tax authority T_t. It can save in nominal short-term bonds S_{t-1} which pays a gross interest rate R_{t-1}^s. Finally, it

[28] For simplicity, we present the model under rational expectations, as in the original paper by Sims et al. (2023). The inclusion of boundedly rational expectations are discussed in the main text, in Section 2.

makes transfers to the child X_t^b, as well as a transfer, X_t^{FI} to financial intermediaries, each period. The first-order conditions for C_t, L_t and S_t are:

$$\psi L_t^{\chi} = C_t^{-\sigma} w_t \tag{A.3}$$

$$\Lambda_{t-1,t} = \beta \left(\frac{C_t}{C_{t-1}} \right)^{-\sigma} \tag{A.4}$$

$$1 = R_t^s \mathbb{E}_t \Lambda_{t,t+1} \Pi_{t+1}^{-1} \tag{A.5}$$

with $\Lambda_{t-1,t}$ the stochastic discount factor of the parent, $w_t = \frac{W_t}{P_t}$ the real wage and $\Pi_t = \frac{P_t}{P_{t-1}}$ the gross inflation rate.

A.1.2 Child, or impatient household

The child does not supply labor, he thus only gets utility from consumption, $C_{b,t}$:

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_b^t \left[\frac{C_{s,t}^{1-\sigma} - 1}{1-\sigma} \right] \tag{A.6}$$

It is less patient than the parent, i.e. $\beta_b < \beta$.

The child can borrow/save using long-term bonds B_t. As in Woodford (2001), long-term bonds are modelled as perpetuities with geometrically decaying coupon payments. Coupon payments decay at rate denoted by $\kappa \in [0,1]$. Issuing agent a bond in time t leads to payments of 1, κ, κ^2, . . . in the following periods. Using decaying coupon bonds allows to only keep track of the total outstanding bonds, rather than individual issues. The new issuance of long-term bond is:

$$NB_t = B_t - \kappa B_{t-1} \tag{A.7}$$

Newly issued bonds trade at market price Q_t, such that the total value of the bond portfolio equals $Q_t B_t$. The gross return on the long bond is defined by:

$$R_t^b = \frac{1 + \kappa Q_t}{Q_{t-1}} \tag{A.8}$$

The child has the following budget constraint:

$$P_t C_{b,t} + B_{t-1} \leq Q_t \left(B_t - \kappa B_{t-1} \right) + P_t X_t^b \tag{A.9}$$

This means that the sum of the nominal value of consumption and coupon payments on outstanding debt cannot exceed the value of new bond issuances plus the nominal value of the transfer from the parent.

First-order conditions for the consumption of the child and the long-term bond give:

$$\Lambda_{b,t-1,t} = \beta_b \left(\frac{C_{b,t}}{C_{b,t-1}} \right)^{-\sigma} \tag{A.10}$$

$$1 = \mathbb{E}_t \Lambda_{b,t,t+1} R^b_{t+1} \Pi^{-1}_{t+1} \tag{A.11}$$

A.2 Financial Intermediaries

Unlike Gertler and Karadi (2011), and Sims and Wu (2021), Sims et al. (2023) assume for simplicity and tractability that the representative financial intermediary (FI) is born each period and exits the industry in the subsequent period. It receives an exogenous amount of net worth from the parent household, $P_t X_t^{FI}$:

$$P_t X_t^{FI} = P_t \bar{X}^{FI} + \kappa Q_t B^{FI}_{t-1} \tag{A.12}$$

It consists of two components: fixed amount of new equity $\bar{X}^{FI}$, and outstanding long-bonds inherited from past intermediaries $\kappa Q_t B^{FI}_{t-1}$.

The FI has the following balancing condition:

$$Q_t B_t^{FI} + RE_t^{FI} = S_t^{FI} + P_t X_t^{FI} \tag{A.13}$$

The left-hand side of this equations corresponds to the assets of the FI, i.e. long-term lending to the child $Q_t B_t^{FI}$ and central bank's reserves RE_t^{FI}. The right-hand side, the liabilities, is comprised of short-term deposits from the parent S_t^{FI} and the transfer $P_t X_t^{FI}$.

The FI pays interest, R_t^s, on short-term debt. It earns interest, R_t^{re}, on reserves, as well as a return on long-term bonds R^b_{t+1}. Upon exiting after period t, the FI's dividend to the parent household are therefore equal to:

$$P_{t+1} D^{FI}_{t+1} = \left(R^b_{t+1} - R^s_t \right) Q_t B_t^{FI} + \left(R_t^{re} - R_t^s \right) RE_t^{FI} + R_t^s P_t X_t^{FI} \tag{A.14}$$

The FI maximises expected dividends, which are discounted by the nominal stochastic discount factor of the parent $\Lambda_{t,t+1}$, subject to a risk-weighted leverage constraint in which long-term bonds receive a risk weight of unity, while reserves on account with the central bank have a risk weight of zero:

$$Q_t B_t^{FI} \leq \Theta_t P_t \bar{X}^{FI} \tag{A.15}$$

This states that the value of the long-term loans to the child cannot be larger than a multiple Θ_t of the value of its equity. Θ_t is viewed as a credit shock, and obeys an AR(1) process.

The first-order conditions of the FI with respect to its choice variables, the quantity of long bonds and reserves are:

$$\mathbb{E}_t \Lambda_{t,t+1} \Pi_{t+1}^{-1} \left(R_{t+1}^b - R_t^s \right) = \Omega_t \tag{A.16}$$

$$\mathbb{E}_t \Lambda_{t,t+1} \Pi_{t+1}^{-1} \left(R_t^{re} - R_t^s \right) = 0 \tag{A.17}$$

with Ω_t the multiplier on the leverage constraint. The binding constraint, i.e. $\Omega_t > 0$, generates excess returns between the expected return on long bonds and the cost of funds.

A.3 Production

The production side of the economy consists of three sectors: final output, retail output, and wholesale output. There is a representative final good firm and representative wholesale producer. There are a continuum of retailers, indexed by $f \in [0, 1]$.

Final output:

Each final output firm faces a downward-sloping demand function given by:

$$Y_t(f) = \left(\frac{P_t(f)}{P_t} \right)^{-\epsilon} Y_t \tag{A.18}$$

with $\epsilon > 1$ the elasticity of substitution.

This gives rise to an aggregate price index:

$$P_t = \left[\int_0^1 P_t(f)^{1-\epsilon} df \right]^{\frac{1}{1-\epsilon}} \tag{A.19}$$

Retail output:

Retailers purchase wholesale output at price $P_{m,t}$, which can be viewed as nominal marginal cost and repackage it for sale at $P_t(f)$. Retailers are subject to a Calvo (1983) pricing friction. Each period, there is a probability $1 - \phi$ that a retailer adjusts its price, with $\phi \in [0, 1]$.

When a retailer does adjust its prices, it picks a price to maximize the present value of expected profits, discounted by the stochastic discount factor of the parent household. Optimization results in an optimal reset price, $P_{*,t}$, that is common across updating retailers. Denote real marginal cost as $p_{m,t} = \frac{P_{m,t}}{P_t}$, the optimal reset price then satisfies:

$$P_{*,t} = \frac{\epsilon}{\epsilon - 1} \frac{X_{1,t}}{X_{2,t}} \tag{A.20}$$

$$X_{1,t} = P_t^{\epsilon} p_{m,t} Y_t + \phi \mathbb{E}_t \Lambda_{t,t+1} X_{1,t+1} \tag{A.21}$$

$$X_{2,t} = P_t^{\epsilon-1} Y_t + \phi \mathbb{E}_t \Lambda_{t,t+1} X_{2,t+1} \tag{A.22}$$

Wholesale output:

The wholesale firm produces output $Y_{m,t}$ using a production function which takes labor as input:

$$Y_{m,t} = A_t L_t \tag{A.23}$$

with A_t an exogenous productivity disturbance following a known stochastic process.

Real wages $w_t = \frac{W_t}{P_t}$ are determined by:

$$w_t = p_{m,t} A_t \tag{A.24}$$

A.4 The central bank

The central bank has two instruments, namely the policy rate, R_t^s, and its portfolio of long-term bonds issued by the child, B_t^{cb}. The policy rate evolves according to a simple Taylor rule subject to the ELB:

$$\ln R_t^s = \max\left[0\ ;\ \phi_\pi\left(\ln \Pi_t - \ln \Pi\right) + \phi_x\left(\ln Y_t - \ln Y_t^*\right) + s_r \varepsilon_{r,t}\right] \tag{A.25}$$

The central bank finances its portfolio of long-term bonds via the creation of reserves RE_t. Its balance sheet condition is:

$$Q_t B_t^{cb} = RE_t \tag{A.26}$$

QE is defined as the real value of the central bank's bond portfolio:[29]

$$QE_t = Q_t b_t^{cb} \tag{A.27}$$

where $b_t^{cb} = \frac{B_t^{cb}}{P_t}$.

In their original specification, Sims et al. (2023) assume that QE is costless. We follow Gertler and Karadi (2011), Karadi and Nakov (2021) and Kabaca et al. (2023) and assume that the central bank pays an efficiency cost, Γ_t, whenever it conducts QE or QT. In this case, the central bank earns a surplus on its asset holdings which is transferred lump-sum to the parent household and receives the following form, in real terms:

$$T_t = \frac{R_t^b Q_{t-1}}{\pi_t} b_{t-1}^{cb} - \frac{R_{t-1}}{\pi_t} re_{t-1} - \Gamma_t \tag{A.28}$$

where $re_t = RE_t / P_t$, with

$$\Gamma_t = \frac{\tau}{2}\left(Q_t b_t^{cb}\right)^2 \tag{A.29}$$

As in Kabaca et al. (2023), these costs reflect the notion that the central bank faces several distortions, such as political costs and other implementation constraints (e.g., costs of maintaining a large balance sheet or identifying preferred government sector markets) when purchasing or selling long-term government bonds. Given its quadratic structure, both QE and QT incur costs to the central bank. We think of QT as active and passive, in the sense that it coincides with active selling of central bank asset holdings and with no re-investments of the proceeds of maturing bonds.

[29] In our baseline calibration of the model, the QE policy follows a simple AR(1) process.

A.5 Aggregate conditions

Market clearing requires the following conditions:

$$RE_t = RE_t^{FI} \tag{A.30}$$

$$S_t = S_t^{FI} \tag{A.31}$$

$$S_t = S_t^{FI} \tag{A.32}$$

$$B_t = B_t^{FI} + B_t^{cb} \tag{A.33}$$

$$Y_t v_t^p = A_t L_t \tag{A.34}$$

A key feature of Sims et al. (2023) is that they assume that the transfer from parent to child, X_t^b, is time-varying, but that neither the parent nor the child behaves as though it can influence the value:

$$P_t X_t^b = (1 + \kappa Q_t) B_{t-1} \tag{A.35}$$

This assumption, referred to as the "full-bailout"[30] implies that, even though the child solves a dynamic problem and has a forward-looking Euler equation, its consumption is in fact static:

$$P_t C_{b,t} = Q_t B_t \tag{A.36}$$

This assumption by Sims et al. (2023) on the parent-child transfer allows one to eliminate a state variable and simplifies the system to four equations, although it is not crucial for the qualitative or quantitative properties of the model.

In our model, due to the introduction of the QE cost, the aggregate resource constraint of the economy changes. Considering the budget constraint of the parent and plugging in dividends from firm financial intermediary ownership as well as the expression for nominal remittances (A.28), we obtain:

[30]At each period, the parent pays off the child's debt. Sims et al. (2023) provide evidence that relaxing this assumption does not fundamentally alter the behavior of the model in response to shocks.

$$
\begin{aligned}
P_t C_t = & R^S_{t-1} S_{t-1} + P_t Y_t + \left(R^b_t - R^S_{t-1}\right) Q_{t-1} B^{FI}_{t-1} + \left(R^{RE}_{t-1} - R^S_{t-1}\right) RE^{FI}_{t-1} \\
& + R^S_{t-1} P_{t-1} X^{FI}_{t-1} + R^b_t Q_{t-1} B^{CB}_{t-1} - R^{RE}_{t-1} RE_{t-1} - P_t \Gamma_t - (1 + \kappa Q_t) B_{t-1} \\
& - Q_t B^{FI}_t - RE^{FI}_t
\end{aligned} \tag{A.37}
$$

Using the definition of the balance sheet of the financial intermediary (A.13), we can substitute out for $P_{t-1} X^{FI}_{t-1}$ in (A.37) above and then simplify to get:

$$
P_t C_t = P_t Y_t + R^b_t Q_{t-1} B_{t-1} - (1 + \kappa Q_t) B_{t-1} - P_t \Gamma_t - Q_t B^{FI}_t - RE^{FI}_t \tag{A.38}
$$

Accounting for the fact that long-term rates are defined as $R^b_t = \frac{1+\kappa Q_t}{Q_{t-1}}$ and using the bonds market clearing condition (A.33), the balance sheet of the central bank (A.26) and the fact that the full bailout condition (A.36), we arrive at:

$$
C_t + C_{b,t} = Y_t - \Gamma_t \tag{A.39}
$$

Using the definition of for QE_t in (A.27), we may rewrite the efficiency cost expression for Γ_t as follows:

$$
\Gamma_t = \frac{\tau}{2} QE^2_t \tag{A.40}
$$

This allows us to rewrite the resource constraint (A.39) as follows:

$$
Y_t = C_t + C_{b,t} + \frac{\tau}{2} QE^2_t \tag{A.41}
$$

A_t and Θ_t obey conventional log AR(1) processes. Potential output $Y_{*,t}$ is defined as the equilibrium level of output consistent with price flexibility (i.e. $\phi = 0$) and where the credit shock is constant, i.e. $\Theta_t = 0$. The natural rate of interest, R^f_t, is the gross real short-term interest rate consistent with this level of output. $X_t = \frac{Y_t}{Y_{*,t}}$ is the output gap.

A.6 The full log-linearized model

We write below all log-linearized conditions of the model, under rational expectations. The majority of these equations are similar to Sims et al. (2023), with the exception of the introduction of the QE cost. Lowercase variables denote log, i.e. percentage change, deviations from steady state. We use a “hat” when the corresponding level variable is already lower-case. Variables without a time subscript denote non-stochastic steady state values. The model is linearized around a steady state with zero trend inflation (i.e. $\Pi = 1$) where the leverage constraint on intermediaries binds.

$$\chi l_t = -\sigma c_t + \widehat{w}_t \tag{A.42}$$

$$\lambda_{t-1,t} = -\sigma \left(c_t - c_{t-1}\right) \tag{A.43}$$

$$0 = \mathbb{E}_t \lambda_{t,t+1} + r_t^s - \mathbb{E}_t \pi_{t+1} \tag{A.44}$$

$$\lambda_{b,t-1,t} = -\sigma \left(c_{b,t} - c_{b,t-1}\right) \tag{A.45}$$

$$r_t^b = \frac{\kappa}{R^b} q_t - q_{t-1} \tag{A.46}$$

$$0 = \mathbb{E}_t \lambda_{b,t,t+1} + \mathbb{E}_t r_{t+1}^b - \mathbb{E}_t \pi_{t+1} \tag{A.47}$$

$$q_t + \widehat{b}_t^{FI} = \theta_t \tag{A.48}$$

$$\left[Qb^{FI}(1-\kappa)\right] q_t + Qb^{FI}\widehat{b}_t^{FI} - \kappa Q b^{FI}\widehat{b}_{t-1}^{FI} + \kappa Q b^{FI}\pi_t + re \cdot \widehat{re}_t = s \cdot \widehat{s}_t \tag{A.49}$$

$$\mathbb{E}_t \lambda_{t,t+1} - \mathbb{E}_t \pi_{t+1} + \frac{R^b}{sp}\mathbb{E}_t r_{t+1}^b - \frac{R^s}{sp} r_t^s = \omega_t \tag{A.50}$$

$$r_t^{re} = r_t^s \tag{A.51}$$

$$\widehat{p}_{*,t} = \widehat{x}_{1,t} - \widehat{x}_{2,t} \tag{A.52}$$

$$\widehat{x}_{1,t} = (1-\phi\beta)\widehat{p}_{m,t} + (1-\phi\beta)y_t + \phi\beta\mathbb{E}_t\lambda_{t,t+1} + \epsilon\phi\beta\mathbb{E}_t\pi_{t+1} + \phi\beta\mathbb{E}_t\widehat{x}_{1,t+1} \tag{A.53}$$

$$\widehat{x}_{2,t} = (1-\phi\beta)y_t + \phi\beta\mathbb{E}_t\lambda_{t,t+1} + (\epsilon-1)\phi\beta\mathbb{E}_t\pi_{t+1} + \phi\beta\mathbb{E}_t\widehat{x}_{2,t+1} \tag{A.54}$$

$$\widehat{w}_t = \widehat{p}_{m,t} + a_t \tag{A.55}$$

$$(1-z)c_t + z c_{b,t} + \left(\frac{\tau Q E}{Y}\right) q e_t = y_t \tag{A.56}$$

$$\widehat{v}_t^p + y_t = a_t + l_t \tag{A.57}$$

$$\widehat{v}_t^p = 0 \tag{A.58}$$

$$\pi_t = \frac{1-\phi}{\phi}\widehat{p}_{*,t} \tag{A.59}$$

$$q_t + \widehat{b}_t^{cb} = \widehat{re}_t \tag{A.60}$$

$$\widehat{b}_t = \frac{b^{FI}}{b}\widehat{b}_t^{FI} + \frac{b^{cb}}{b}\widehat{b}_t^{cb} \tag{A.61}$$

$$c_{b,t} = q_t + \widehat{b}_t \tag{A.62}$$

$$a_t = \rho_a a_{t-1} + s_A \varepsilon_{A,t} \tag{A.63}$$

$$\theta_t = \rho_\theta \theta_{t-1} + s_\theta \varepsilon_{\theta,t} \tag{A.64}$$

$$r_t^s = \max\left[0\ ;\ \phi_\pi(\pi_t - \pi^T) + \phi_x(x_t - x^T) + s_r\varepsilon_{r,t}\right] \tag{A.65}$$

$$qe_t = \widehat{re}_t \tag{A.66}$$

$$x_t = y_t - y_t^* \tag{A.67}$$

A.7 Updated IS and Phillips Curve Equations

In this subsection we reduce the system of log-linearized equations to get to the four equations presented in the main model.

To obtain the *IS curve*. We start by combining first-order conditions on consumption of each type of households and interest rates (A.43)-(A.45) and (A.47) with the aggregate resource constraint (A.56):

$$y_t = \mathbb{E}_t y_{t+1} - \frac{1-z}{\sigma}\left(r_t^s - \mathbb{E}_t\pi_{t+1}\right) - \frac{z}{\sigma}\left(\mathbb{E}_t r_{t+1}^b - \mathbb{E}_t\pi_{t+1}\right) - \left(\frac{\tau QE}{Y}\right)\left[qe_{t+1} - qe_t\right] \tag{A.68}$$

Combining (A.60)-(A.62) with (A.66) and the binding leverage constraint (A.48) allows us to write the consumption of the child as a function of credit shocks and QE:

$$c_{b,t} = \frac{b^{FI}}{b}\left(q_t + \widehat{b}_t^{FI}\right) + \frac{b^{cb}}{b}qe_t = \bar{b}^{FI}\theta_t + \bar{b}^{cb}qe_t \tag{A.69}$$

with $\bar{b}^{FI} = b^{FI}/b$ and $\bar{b}^{cb} = b^{cb}/b$ the steady state fraction of total bonds held by financial intermediaries and the central bank, respectively.

Using the child's first-order condition we get:

$$\mathbb{E}_t r^b_{t+1} - \mathbb{E}_t \pi_{t+1} = \sigma \left[\mathbb{E}_t c_{b,t+1} - c_{b,t}\right] = \sigma \left[\bar{b}^{FI}\left(\mathbb{E}_t \theta_{t+1} - \theta_t\right) + \bar{b}^{cb}\left(\mathbb{E}_t qe_{t+1} - qe_t\right)\right] \tag{A.70}$$

Combining these two equations we get:

$$\begin{aligned} y_t = \mathbb{E}_t y_{t+1} - \frac{1-z}{\sigma}\left(r^s_t - \mathbb{E}_t \pi_{t+1}\right) - \left(z\bar{b}^{cb} + \frac{\tau QE}{Y}\right)\left(\mathbb{E}_t qe_{t+1} - qe_t\right) \\ - z\bar{b}^{FI}\left(\mathbb{E}_t \theta_{t+1} - \theta_t\right) \end{aligned} \tag{A.71}$$

This equation can also be written as a function of the interest rate spread:

$$y_t = \mathbb{E}_t y_{t+1} - \frac{1}{\sigma}\left(r^s_t - \mathbb{E}_t \pi_{t+1}\right) - \frac{z}{\sigma}\left(\mathbb{E}_t r^b_{t+1} - r^s_t\right) - \left(\frac{\tau QE}{Y}\right)\left[qe_{t+1} - qe_t\right] \tag{A.72}$$

Define the hypothetical natural rate of output, y^*_t, as the level of output consistent with flexible prices and no credit market shocks. That is, y^*_t is the level of output consistent with $\widehat{p}_{m,t} = \theta_t = qe_t = 0$, or:

$$y^*_t = \frac{(1+\chi)(1-z)}{\chi(1-z)+\sigma} a_t \tag{A.73}$$

The natural rate of interest, r^f_t, is the real rate consistent with the IS equation holding at the natural rate of output absent credit shocks. It can be expressed as:

$$r^f_t = \frac{\sigma}{1-z}\left(\mathbb{E}_t y^*_{t+1} - y^*_t\right) = \frac{\sigma\left(\rho_A - 1\right)}{1-z} y^*_t \tag{A.74}$$

Using (A.67), this allows to express the IS curve as a function of the output gap:

$$\begin{aligned} x_t = \mathbb{E}_t x_{t+1} - \frac{1-z}{\sigma}\left(r^s_t - \mathbb{E}_t \pi_{t+1} - r^f_t\right) - \left(z\bar{b}^{cb} + \frac{\tau QE}{Y}\right)\left(\mathbb{E}_t qe_{t+1} - qe_t\right) \\ - z\bar{b}^{FI}\left(\mathbb{E}_t \theta_{t+1} - \theta_t\right) \end{aligned} \tag{A.75}$$

The *Phillips curve* can be derived as follows. First, we can combine (A.52)-(A.54) such as to have an equation linking prices and marginal costs:

$$\pi_t = \gamma \widehat{p}_{m,t} + \beta \mathbb{E}_t \pi_{t+1} \tag{A.76}$$

with $\gamma = \frac{(1-\phi)(1-\phi\beta)}{\phi}$. Combining (A.42) with (A.55) and (A.57), making note of the fact that $\widehat{v}_t^p = 0$ around a zero inflation steady state, gives an expression for the marginal costs:

$$\widehat{p}_{m,t} = \chi y_t - (1+\chi)a_t + \sigma c_t \tag{A.77}$$

Using the aggregate resource constraint (A.56) allows us to write this as:

$$\widehat{p}_{m,t} = \frac{\chi(1-z)+\sigma}{1-z} y_t - (1+\chi)a_t - \frac{\sigma z}{1-z} c_{b,t} - \frac{\sigma}{(1-z)} \frac{\tau QE}{Y} qe_t \tag{A.78}$$

Using the definition for c_t^b derived in (A.69):

$$\widehat{p}_{m,t} = \frac{\chi(1-z)+\sigma}{1-z} y_t - (1+\chi)a_t - \frac{\sigma z}{1-z} \left[\bar{b}^{FI}\theta_t + \bar{b}^{cb} qe_t\right] - \frac{\sigma}{(1-z)} \frac{\tau QE}{Y} qe_t \tag{A.79}$$

This can be expressed as a function of output gap, using (A.73):

$$\widehat{p}_{m,t} = \frac{\chi(1-z)+\sigma}{1-z} x_t - \frac{\sigma z}{1-z} \left[\bar{b}^{FI}\theta_t + \bar{b}^{cb} qe_t\right] - \frac{\sigma}{(1-z)} \frac{\tau QE}{Y} qe_t \tag{A.80}$$

Plugging this equation into (A.76), and defining $\zeta = \frac{\chi(1-z)+\sigma}{1-z}$ gives the *Phillips curve* in this four equation model.

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \gamma \zeta x_t - \frac{\gamma\sigma}{1-z} \left(z\bar{b}^{cb} + \frac{\tau QE}{Y} \right) qe_t - \frac{\gamma\sigma z}{1-z} \bar{b}^{FI}\theta_t + cp_t \tag{A.81}$$

To this equation we add an ad-hoc price cost-push shock cp_t.

B Supplementary analytical details

This appendix provides additional details to complement Section 3. In this Appendix B.1, we provide a proof of Proposition 1. Appendix B.2 shows the impulse responses to a persistent restrictive policy shock for both monetary policy instruments, while Appendix B.3

presents the resulting trade-off matrices in the case optimal policy for both the interest rate and the balance sheet, in the case of heterogeneous expectations.

B.1 Proof of Proposition 1

Before formulating the minimization problem of the central bank under commitment in the presence of de-anchored expectations it is useful to work on inflation and output gap expectations. Exploiting the law of motion of de-anchored expectations (9), and iterating backwards we may write the law of motion of inflation expectations as follows:

$$\omega_t^{\pi} \approx \bar{g} \sum_{s=0}^{\infty} (1-\bar{g})^s \pi_{t-s-1} \tag{B.1}$$

The Phillips curve under de-anchored expectations reads as follows:

$$\pi_t = \beta\omega_t^{\pi} + \gamma\zeta x_t - \frac{\gamma\sigma}{1-z}\left(z\bar{b}^{cb} + \frac{\tau QE}{Y}\right) qe_t - \frac{\gamma\sigma z}{1-z}\bar{b}^{FI}\theta_t + cp_t \tag{B.2}$$

where we can plug in expression (B.1) for inflation expectations. The minimization problem of the central bank under commitment and de-anchored expectations reads as follows:[31]

$$\min_{\pi_t, x_t} \mathbb{E}_t \sum_{t=1}^{\infty} \Bigg[\frac{\lambda_\pi}{2}\pi_t^2 + \frac{\lambda}{2}x_t^2 - \xi_{\pi,t}\left(\pi_t - \beta\left(\bar{g}\sum_{s=0}^{\infty}(1-\bar{g})^s \pi_{t-s-1}\right) - \gamma\zeta x_t + \frac{\gamma\sigma}{1-z}\left(z\bar{b}^{cb} + \frac{\tau QE}{Y}\right) qe_t + \frac{\gamma\sigma z}{1-z}\bar{b}^{FI}\theta_t - cp_t\right)\Bigg] \tag{B.3}$$

where $\xi_{\pi,t}$ is the Lagrange multiplier on the Phillips curve. The FOC are summarized as:

$$\pi_t: \; \lambda_\pi \pi_t - \xi_{\pi,t} + \bar{g}\beta\mathbb{E}_t \sum_{s=0}^{\infty} \xi_{\pi,t+s+1}\beta^{s+1}(1-\bar{g})^s = 0 \tag{B.4}$$

$$x_t: \; \lambda_x x_t + \xi_{\pi,t}\gamma\zeta = 0 \tag{B.5}$$

[31] Note that the expectation operator in the minimization problem of the central bank corresponds to the expectations of the central bank itself. As mentioned in the main body of the text, we have assumed that the central bank is rational and has full knowledge of the actual law of motion of the economy and the way agents form their expectations. As a result, the expectations of the central bank are model-consistent.

where solving for $\xi_{\pi,t}$ in (B.5), we receive:

$$\xi_{\pi,t} = -\frac{\lambda_x}{\gamma\zeta} x_t, \quad \forall t. \tag{B.6}$$

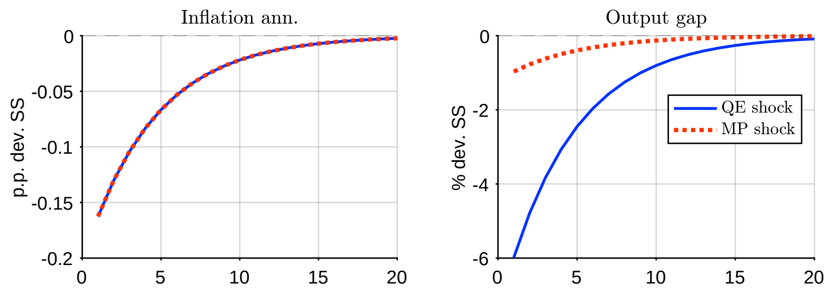
and substituting in (B.4) for the current and the future lagrange multipliers, we end up to the following expression:

$$\lambda_\pi \pi_t = -\frac{\lambda_x}{\gamma\zeta}\left(x_t - \bar{g}\beta^2 \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s (1-\bar{g})^s x_{t+s+1}\right) \tag{B.7}$$

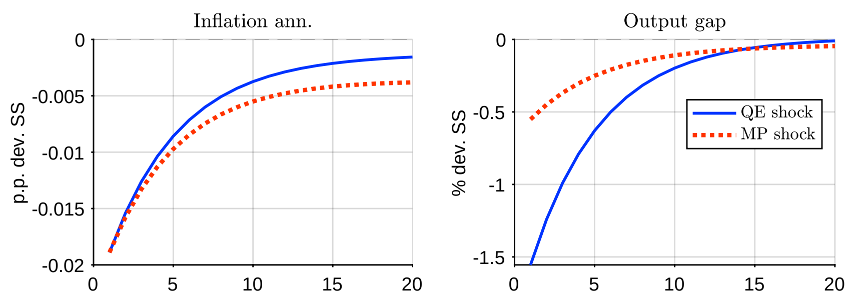
B.2 Impulse Responses to policy shocks

In this appendix, we present the impulse responses to a persistent restrictive policy shock for both monetary policy instruments. Figure 15 shows that, both in the anchored and de-anchored models, the interest rate shock brings inflation down for substantially lower output gap loss compared to the QT shock. The implied trade-off for both policy instruments is amplified under de-anchored expectations.

Figure 15: IRFs to a Monetary Policy and a QT shock in the RE and de-anchored models



(a) Rational expectations model



(b) De-anchored expectations model

Notes: Impulse response functions to a one standard deviation short-term interest rate shock, and a one standard deviation QT shock delivering the same initial impact on inflation. The top panel is the RE model, the bottom panel is the de-anchored model.

B.3 Trade-off matrices under heterogeneous expectations

The presence of both de-anchored and anchored agents leads to a role for past (i.e. lower triangular), current (i.e. main diagonal) and future output gap (i.e. upper triangular) values. This is depicted in Figure 16.

Figure 16: Trade-off matrices under heterogeneous expectations

(a) Optimal interest rate policy

(b) Optimal balance sheet policy

Notes: Trade-off matrices for optimal interest rate (left panel) $\mathcal{T}^{\varepsilon_r}$ and balance sheet (right panel) $\mathcal{T}^{\varepsilon_{qe}}$ polices, in a case in which 50% of agents are de-anchored. Each row is one period in the sequence, each line shows a relevant period of the output gap entering in the optimal policy rule. The intensity of the color corresponds to the magnitude of the trade-off.

C Supplementary details on the optimal policy simulations

This appendix presents supplementary material for our quantitative optimal policy analysis described in Section 4. Appendix C.1 derives our model-consistent loss function. In Appendix C.2, we provide additional details on the solution method we used to construct the optimal interest and balance sheet trajectories. Appendix C.3 explores how the gain parameter in the forecasting process of de-anchored agents influences our findings. Finally, C.4 shows the path of the consumption of the two types of households, in the case of optimal policy under commitment and rational expectations.

C.1 Derivation of the welfare criterion

We define aggregate welfare as the sum of the utility functions of the parent and the child:

$$W_t = V_t + V_{b,t} \tag{C.1}$$

We follow the approach of Rotemberg and Woodford (1997) and take a second-order approximation of the utility function of the parent and the child, respectively.

A second-order approximation of the utility function of the child reads as follows:

$$V_{b,t} = V_b + U_{C_b} C_b \left(c_{b,t} + \frac{1}{2}(1 + \frac{U_{C_b C_b} C_b}{U_{C_b}}) c_{b,t}^2 \right) \quad \text{(C.2)}$$

A second-order approximation of the utility function of the parent reads as follows:

$$V_t = V + U_C C \left(c_t + \frac{1}{2}(1 + \frac{U_{CC} C}{U_C}) c_t^2 \right) - U_L L \left(l_t + \frac{1}{2}(1 + \frac{U_{LL} L}{U_L}) l_t^2 \right) \quad \text{(C.3)}$$

where $U_{C_b} = U_C = C^{-\sigma}$, $U_{C_b C_b} = U_{CC} = -\sigma C^{-\sigma-1}$, $U_L = L^{\chi}$ and $U_{LL} = \chi L^{\chi-1}$. Log-linearizing the aggregate production function, we get that $y_t = \alpha_t + \hat{\upsilon}_t^p + l_t$.[32] Taking instead a second-order Taylor approximation of the production function around the zero steady-state inflation, we receive:

$$l_t = y_t + \frac{Y}{2} y_t^2 + \frac{1}{2}(\hat{\upsilon}_t^p)^2 - \frac{L}{2} l_t^2 + t.i.p. \quad \text{(C.4)}$$

where $t.i.p.$ stands for terms irrelevant of policy, such as the productivity shock in this case. Note that in this case $\hat{\upsilon}_t^p = log(\upsilon_t^p) - log(\upsilon^p)$. From ch. 6 of Woodford (2003) we can write, under a Calvo price setting mechanism:

$$y_t^2 = \epsilon var(p_{*,t}) \quad \text{(C.5)}$$

and, in turn, the variance of optimal relative prices reads as follows:

$$\sum_{t=0}^{\infty} \beta^t var(log(p_{*,t})) = \frac{1}{1-\phi\beta} \sum_{t=0}^{\infty} \beta^t \left[\frac{\phi}{1-\phi} \pi_t^2 \right] + t.i.p. + O\left(||\xi^3|| \right) \quad \text{(C.6)}$$

Substituting (C.4) in (C.3), we receive:

$$\begin{aligned} V_t = V + U_C C \left(c_t + \frac{1}{2}(1-\sigma) c_t^2 \right) - U_L L \left(y_t + \frac{Y}{2} y_t^2 + \frac{1}{2}(\tilde{\upsilon}_t^p)^2 - \frac{L}{2} l_t^2 \right) \\ - \frac{1}{2} U_L L (1+\chi) l_t^2 + t.i.p. \end{aligned} \quad \text{(C.7)}$$

[32] Note that we are log-linearizing around the zero steady-state inflation, which implies that price dispersion is irrelevant up to first order.

Note that from equations (C.5) and (C.6), we have:

$$y_t^2 = \frac{\epsilon\phi}{(1-\phi\beta)(1-\phi)}\pi_t^2 + t.i.p. + O\left(||\xi^3||\right) \tag{C.8}$$

where $O\left(||\xi^3||\right)$ captures terms of order higher than two. Substituting the above expression in (C.7), we receive:

$$\begin{aligned} V_t = V + U_C C\left(c_t + \frac{1}{2}(1-\sigma)c_t^2\right) - U_L L \frac{Y\epsilon\phi}{2(1-\phi\beta)(1-\phi)}\pi_t^2 \\ - U_L L\left(y_t + \frac{1}{2}(\tilde{v}_t^p)^2 - \frac{L}{2}l_t^2\right) - \frac{1}{2}U_L L(1+\chi)l_t^2 + t.i.p. + O\left(||\xi^3||\right) \end{aligned} \tag{C.9}$$

Note that from the first order condition of the Parent's household problem with respect to labor supply (and after normalizing steady state real wage to one), we have that $\psi U_L = U_C$. We may thus rewrite (C.9) as follows:

$$\begin{aligned} V_t = V + U_C Y\Bigg\{\left(\frac{C}{Y}c_t + \frac{C}{2Y}(1-\sigma)c_t^2\right) - \frac{Y\epsilon\phi}{2\psi(1-\phi\beta)(1-\phi)}\pi_t^2 \\ - \frac{1}{\psi}\left(y_t + \frac{1}{2}(\tilde{v}_t^p)^2 - \frac{L}{2}y_t^2\right) - \frac{1}{2\psi}(1+\chi)y_t^2\Bigg\} + t.i.p. + O\left(||\xi^3||\right) \end{aligned} \tag{C.10}$$

where we have used the square of the expression (C.4) to substitute out for l_t^2. We now normalize steady state output to one. Gathering terms and setting $\psi = 1$, we can write:

$$\begin{aligned} V_t = V + U_C Y\Bigg\{\left(\frac{C}{Y}c_t + \frac{C}{2Y}(1-\sigma)c_t^2\right) - \frac{Y\epsilon\phi}{2(1-\phi\beta)(1-\phi)}\pi_t^2 \\ - y_t - \frac{1}{2}(\tilde{v}_t^p)^2 - \frac{\chi}{2}y_t^2\Bigg\} + t.i.p. + O\left(||\xi^3||\right) \end{aligned} \tag{C.11}$$

Taking a second order approximation of the resource constraint (A.41), we can write:

$$y_t - \frac{C}{Y}c_t - \frac{C_b}{Y}c_{b,t} = -\frac{Y}{2}y_t^2 + C\frac{C}{2Y}c_t^2 + C_b\frac{C_b}{2Y}c_{b,t}^2 + \tau\frac{b^{cb}}{Y}qe_t + b^{cb}E\frac{\tau b^{cb}}{2Y}qe_t^2 \tag{C.12}$$

Returning now to the welfare of the child, we can rewrite it as follows, using the marginal utility of consumption and the marginal disutility from labor at the steady state:

$$V_{b,t} = V_b + U_{C_b} Y \left(\frac{C_b}{Y} c_{b,t} + \frac{C_b}{2Y}(1-\sigma) c_{b,t}^2 \right) \tag{C.13}$$

Assuming that at the steady state $U_C = U_{C_b}$, and plugging (C.11) and (C.13) in (C.1), we receive:

$$\begin{aligned} W_t = W - U_C Y \Bigg\{ & \frac{C}{Y} \left(\frac{C - 1 + \sigma}{2} \right) c_t^2 + \frac{C_b}{Y} \left(\frac{C_b - 1 + \sigma}{2} \right) c_{b,t}^2 + \frac{Y \epsilon \phi}{2(1-\phi\beta)(1-\phi)} \pi_t^2 \\ & + \frac{\chi - Y}{2} y_t^2 + \frac{1}{2} (\tilde{v}_t^p)^2 + \tau \frac{Qb^{cb}}{Y} qe_t + \tau Qb^{cb} \frac{Qb^{cb}}{2Y} qe_t^2 \Bigg\} + t.i.p. + O\left(||\xi^3||\right) \end{aligned} \tag{C.14}$$

Clearly, by using the calibration of Sims et al. (2023), where $\sigma = \chi = 1$ and accounting for a normalization of output at the steady state, $Y = 1$, we can simplify further to get:

$$\begin{aligned} W_t = W - U_C Y \Bigg\{ & \frac{C}{Y} \left(\frac{C}{2} \right) c_t^2 + \frac{C_b}{Y} \left(\frac{C_b}{2} \right) c_{b,t}^2 + \frac{Y \epsilon \phi}{2(1-\phi\beta)(1-\phi)} \pi_t^2 \\ & + \frac{1}{2} (\tilde{v}_t^p)^2 + \tau \frac{Qb^{cb}}{Y} qe_t + \tau Qb^{cb} \frac{Qb^{cb}}{2Y} qe_t^2 \Bigg\} + t.i.p. + O\left(||\xi^3||\right) \end{aligned} \tag{C.15}$$

Note that the linear term, qe_t, in the loss function above arises because in our extension of the model to allow for quadratic efficiency costs of QE/QT, the steady state is inefficient, since $QE \neq 0$ at the steady state. As shown in Benigno and Woodford (2005), linear terms show up in the welfare criterion.

Following Sims et al. (2023), we can write the weights in the loss function as:

$$\omega_C = \frac{C}{Y} \left(\frac{C}{2} \right) = \frac{0.67}{1} \left(\frac{0.67}{2} \right) = 0.2244$$

$$\omega_{C_b} = \frac{C_b}{Y} \left(\frac{C_b}{2} \right) = \frac{0.33}{1} \left(\frac{0.33}{2} \right) = 0.0545$$

$$\omega_\pi = \frac{Y \epsilon \phi}{2(1-\phi\beta)(1-\phi)} = 65.0246$$

$$\omega_{qe} = \tau Qb^{cb}\frac{Qb^{cb}}{2Y} = 5e-05$$

C.2 Computational details

When solving for optimal policy, we use a method similar to de Groot et al. (2021), McKay and Wolf (2022) and Hebden and Winkler (2021) and construct sequence-space linear-quadratic policy problems. Doing so, we use IRFs to policy shocks from a specified model, and a baseline (which is here the response to a cost-push shock in the same model as the one used for the IRFs). Similar solution methods can be found in McKay and Wolf (2023) and Barnichon and Mesters (2023) using both IRFs from VARs and Local Projections.

As regularly discussed in the literature (e.g. Fernández-Villaverde et al., 2016 and Auclert et al., 2021), by certainty equivalence, the first-order perturbation solution of models with aggregate risk is identical to the solution of the model in linearized perfect-foresight. Under perfect foresight, each variable can be written in the sequence space as:

$$Z = Z^B + \mathcal{A}^{z,\varepsilon_y}\,\varepsilon_y$$

with Z^B the baseline path over all periods, and $\mathcal{A}^{z,\varepsilon_y}$ the matrix that collects the impulse responses of the variable of interest over all periods, to a sequence of contemporaneous and news policy shocks ε_y, under a baseline policy rule.[33]

Identifying the optimal trajectory of the desired policy instrument requires solving for a sequence of policy deviations that satisfy the first-order conditions of the optimal policy problem, as defined in a sequence space representation:[34]

$$\begin{aligned}\min_{\{\Pi,C,C_b,QE,\varepsilon_y\}} \mathrm{E}_0\Big[&\frac{1}{2}\Pi'\Omega_\Pi\Pi + C'\Omega_C C + C_b'\Omega_{C,b}C_b + QE'\Omega_{QE}QE\\ &+ \Xi^{\Pi'}\left(-\Pi + \Pi^B + \mathcal{A}^{\Pi,\varepsilon_y}\varepsilon_y\right) + \Xi^{C'}\left(-C + C^B + \mathcal{A}^{C,\varepsilon_y}\varepsilon_y\right)\\ &+ \Xi^{C_b'}\left(-C_b + C_b^B + \mathcal{A}^{C_b,\varepsilon_y}\varepsilon_y\right) + \Xi^{QE'}\left(-QE + QE^B + \mathcal{A}^{QE}\varepsilon_y\right)\Big]\end{aligned}$$

[33] We expose the solution for one policy instrument, but it is straightforward to extend it to allow for multiple instruments.

[34] We omit the linear qe_t term from the loss function in our analysis, considering its impact to be marginal. The inclusion of this term would introduce only a minor adjustment (a "shifter") to the optimal policy rule, which would be insignificant for the scope of our results.

$\Xi \equiv \text{diag}\left(1, \beta, \ldots, \beta^T\right) \otimes \Lambda$ with Λ the Lagrange multipliers, and $\Omega \equiv \text{diag}\left(1, \beta, \ldots, \beta^T\right) \otimes \omega$ the weights associated with each term in the loss function.

The first-order conditions are:

$$\Omega_\Pi \; \Pi = \Xi^\Pi$$

$$\Omega_C \; C = \Xi^C$$

$$\Omega_{C_b} \; C_b = \Xi^{C_b}$$

$$\Omega_{QE} \; QE = \Xi^{QE}$$

$$\mathcal{A}^{\Pi,\varepsilon_y\prime} \; \Xi^\Pi + \mathcal{A}^{C,\varepsilon_y\prime} \; \Xi^C + \mathcal{A}^{C_b,\varepsilon_y\prime} \; \Xi^{C_b} + \mathcal{A}^{QE,\varepsilon_y\prime} \; \Xi^{QE} = 0$$

Combining them gives:

$$\mathcal{A}^{\Pi,\varepsilon_y,\prime} \; (\Omega_\Pi \; \Pi) + \mathcal{A}^{C,\varepsilon_y,\prime} \; (\Omega_C \; C) + \mathcal{A}^{C_b,\varepsilon_y,\prime} \; (\Omega_{C_b} \; C_b) + \mathcal{A}^{QE,\varepsilon_y,\prime} \; (\Omega_{QE} \; QE) = 0$$

Substituting for the law of motion of all endogenous variables in the loss function:

$$\begin{aligned} &\mathcal{A}^{\Pi,\varepsilon_y\prime}\left(\Omega_\Pi \; \left(\Pi^B + \mathcal{A}^{\Pi,\varepsilon_y}\varepsilon_y\right)\right) + \mathcal{A}^{C,\varepsilon_y\prime}\left(\Omega_C \; \left(C^B + \mathcal{A}^{C,\varepsilon_y}\varepsilon_y\right)\right) + \\ &\mathcal{A}^{C_b,\varepsilon_y\prime}\left(\Omega_{C_b}\left(C_b^B + \mathcal{A}^{C_b,\varepsilon_y}\varepsilon_y\right)\right) + \mathcal{A}^{QE,\varepsilon_y\prime}\left(\Omega_{QE}\left(QE^B + \mathcal{A}^{QE,\varepsilon_y}\varepsilon_y\right)\right) = 0 \end{aligned}$$

We can then solve for the optimal sequence of policy shocks $\tilde{\varepsilon_y}$. For simplicity, stacking all loss function variables into Z_l and all weights into Ω gives:

$$\tilde{\varepsilon_y} = -\left(\mathcal{A}^{Z_l,\varepsilon_y\prime}\Omega\mathcal{A}^{Z_l,\varepsilon_y}\right)^{-1}\left(\mathcal{A}^{Z_l,\varepsilon_y\prime}\Omega \, Z_l^B\right) \tag{C.16}$$

Optimal deviations $\tilde{\varepsilon_y}$ to the baseline path of the policy instrument y is set to offset as well as possible (in a weighted least-squares sense) the deviations of the policy targets incurred by the exogenous shocks.

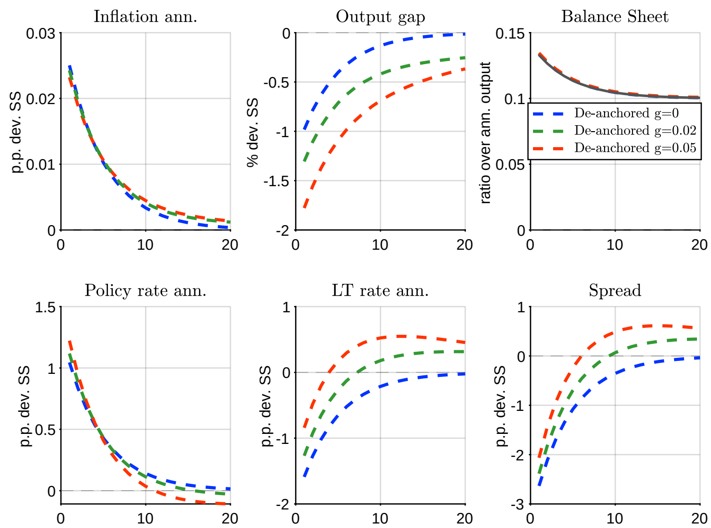
C.3 Role of the gain

The impact of varying the gain parameter in the forecasting rule for de-anchored agents is an important aspect of our model. Unlike the endogenous gain approach in Gáti (2023) or Carvalho et al. (2023), we calibrate this parameter at a fixed value. But what would be the impacts of a different gain? Figures 17 and 18 presents the outcomes for different gain calibrations. We compare our baseline calibration with a gain of $\bar{g} = 0.02$, to a higher gain of $\bar{g} = 0.05$, which close to the estimated value in the sample pre-1999 in Eusepi et al. (2020), and also consider the scenario with a zero gain $\bar{g} = 0$.

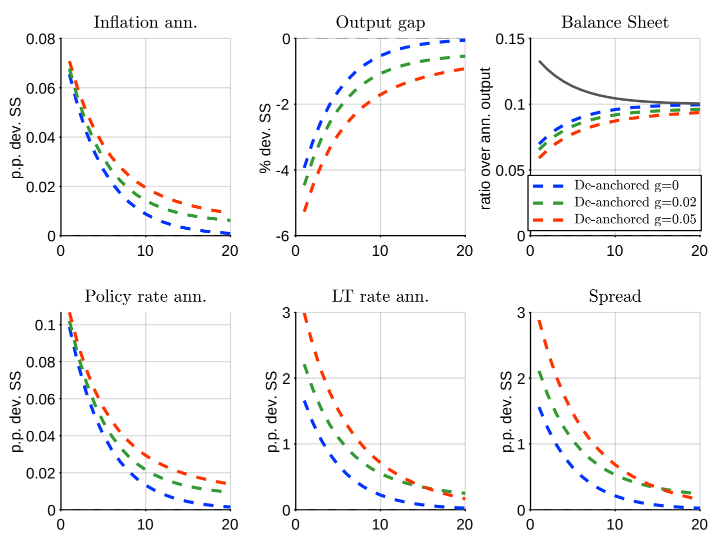
As the gain parameter increases, indicating a greater influence of recent forecast errors on the forecasts of agents with de-anchored expectations, there is a notable amplification in macroeconomic fluctuations. Inflation and the output gap deviate further from their targets, leading the central bank to react more vigorously.

More specifically, policy rate and balance sheet adjustments are becoming more pronounced, as the central bank seeks to counter increased volatility and maintain economic stability. The long-term interest rate and therefore the spread also reflect this dynamic, adjusting in line with the degree of de-anchoring of expectations. Overall, central bank optimal policy instruments are calibrated more aggressively to mitigate the destabilizing effects of higher gain values.

Figure 17: Optimal interest rate and balance sheet policies, in response to a cost-push shock, with different gains



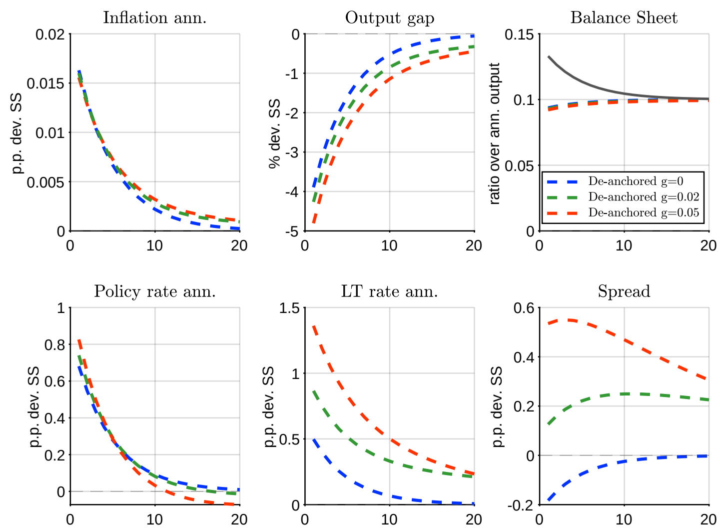
(a) Optimal interest rate policy



(b) Optimal QT policy, for a given interest rate rule

Notes: Optimizing the interest rate only (top panel) or QT only (bottom panel) in response to a one standard deviation persistent ($\rho_{cp} = 0.8$) cost-push shock. In the case of optimal QT only, interest rates follow the baseline Taylor rule reacting to inflation only. Calibration of the loss function follows the welfare criterion. Each line shows a different level for the gain parameter. The solid black line shows the baseline QT trajectory.

Figure 18: Optimal monetary policy mix, in response to a cost-push shock, with different gains

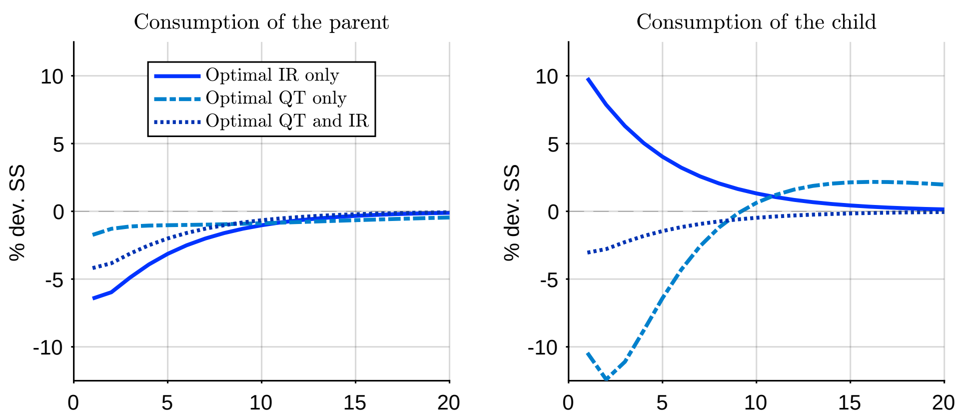


Notes: Optimizing both instruments in response to a one standard deviation persistent ($\rho_{cp} = 0.8$) cost-push shock, interest rates follow the baseline Taylor rule reacting to inflation only. Calibration of the loss function follows the welfare criterion. Each line shows a different level for the gain parameter. The solid black line shows the baseline QT trajectory.

C.4 The role of each monetary policy instrument on the optimal consumption paths

Figure 19 depicts the consumption patterns of the "parent" (patient) and "child" (impatient) households under various optimal combinations of monetary policy instrument. The key observation is that optimally setting both the interest rate and the balance sheet minimizes consumption disparities between the two household types. When only the interest rate is optimized, the "parent" is more negatively affected, as it is more sensitive to changes in the short-term interest rate. Conversely, optimizing only the balance sheet negatively impacts the "child" household's consumption, due to the full bailout assumption linking it directly to balance sheet reductions.

Figure 19: Responses of the consumption of the two types of households



Notes: Optimal trajectories of the consumption of the two types of households, the "parent" and the "child" under commitment and RE, in response to a persistent ($\rho_{cp} = 0.8$) cost-push shock. Calibration of the loss function follows the welfare criterion.

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