

Bank Bailouts, International Linkages and Cooperation*

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Abstract

Financial institutions are increasingly linked internationally. As a result, financial crisis and government intervention have stronger effects beyond borders. We provide a model of international contagion allowing for bank bailouts. While a social planner trades off tax distortions, liquidation losses and intra- and inter-country income inequality, in the non-cooperative game between governments there are inefficiencies due to externalities, no burden sharing and free-riding. We show that, in absence of cooperation, stronger interbank linkages make government interests diverge, whereas cross-border asset holdings tend to align them. We analyze different forms of cooperation and their effects on global and national welfare.

Keywords: bailout, contagion, financial crisis, international institutional arrangements

JEL-Codes: F36, F42, G01, G28

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1 Introduction

Banking crises are a frequent phenomenon. 117 systemic banking crises have occurred in 93 countries since the late 1970s.¹ The fiscal burden attributed to banking and financial crises, stemming i.a. from direct government measures, is considerable as Reinhart and Rogoff (2009) find. Governments spent an estimated average of 12.8 percent of national GDP on interventions to restore financial stability (Honohan and Klingebiel (2000)). During the recent financial crisis pledged capital injections alone amounted to 2.2% of G-20 GDP.² Today, crises rarely remain local as a result of increased financial integration. Due to deepened international financial linkages, crisis can spread rapidly from one country's financial sector to the other.³ At the same time, with increased cross-border banking, domestic (non-bank) creditors are more often directly affected when a foreign bank gets into distress. Therefore, banking crises have become less of a domestic and more of an international issue, with governments in different countries responding. The recent crisis has made this clear and has shown that international conflicts of interest can occur in this context.⁴ There is an increasing perception that international coordination is needed and a high-level debate on how to improve global crisis management has developed.⁵

In this paper, we present the first theoretical model that studies banking crisis with real international spillover effects and government intervention in a formal framework. Extending Allen and Gale (2000), we model international contagion and allow for ex post government intervention in the form of bank bailouts. We explore inefficiencies from unilateral decision-making and show how cross-country bank liabilities affect government decisions. We analyze how different forms of cooperation can improve upon the non-cooperative outcome.

¹See Caprio and Klingebiel (2003).

²For more details, see IMF (2009).

³Degryse et al. (2009) find that the speed of propagation of contagion has increased over the past years and that a shock which affects the liabilities of one country may undermine the stability of the global financial system.

⁴Prominent examples are the AIG bailout, the case of Icesave, the freeze of Lehman assets by the German government or the resolution of Dexia and Fortis. Claessens (2009) investigates financial nationalism in the context of the recent financial crisis in detail. We elaborate on specific examples in Section 6.

⁵See e.g. De Larosiere Report (2009) and Claessens et al. (2010).

International contagion model In the model, contagion between two countries occurs through international balance sheet connections in the form of interbank deposits. Interbank deposits allow for international risk sharing of idiosyncratic liquidity shocks, but induce systemic risk. Crisis spreads from one representative bank (located in the *crisis country*) to the other (in the *affected country*) when the former goes bankrupt due to unexpected liquidity needs and interbank deposits cannot be repaid fully.

Facing a bankrupt bank, governments can intervene and prevent bankruptcy by injecting capital. The bailout directly affects depositors of the bank that is saved (increase in value of bank claims) and domestic households that have to finance the bailout through taxes (distortion of labor supply decision). Moreover, it benefits the foreign bank and its depositors, either because contagion is avoided (spillover effect on the affected country) or because its liquidation value is raised (spillover effect on the crisis country).

Given contagion, we determine the optimal continuation allocation by solving the social planner's problem. The social planner decides on intervention, taxes and contribution levels of the two countries. Therein, she trades off liquidation losses, tax distortions, and income inequality between early and late depositors of a bank as well as a tax smoothing and a consumption smoothing motive between countries. With equal Pareto weights, efficiency requires that the affected country finances a larger part of the bailout as the crisis country is always poorer in a crisis.

We study inefficiencies in government intervention when there is no cooperation. In the bailout game, the crisis country moves first and the affected country follows. We identify three sources of inefficiencies. First, externalities arise from the fact that governments maximize national welfare, but do not take spillover effects into account. A second inefficiency comes from no burden sharing, i.e. the inability of the governments, in the non-cooperative game, to share the cost of a bailout. Third, there is a free-riding problem related to the sequential nature of the game. A bailout by the affected country benefits the crisis country through increased returns on interbank deposits. The anticipation of the bailout may prevent the crisis country to intervene itself. The larger the interbank linkages, the bigger the incentives to free-ride.

To improve upon the equilibrium of the non-cooperative game, governments have to find a way to cooperate. Contributing to the current policy debate, we study three

different cooperation regimes. We compare them with respect to efficiency and discuss their Pareto properties.

Banks increasingly compete internationally for clients. We take this into account by introducing cross-border asset holdings of depositors and analyze their impact on government intervention. With domestic assets at stake abroad, governments partially internalize the spillover effects of their bailout decisions. We are also able to study differences in country size. Whether cross-country deposits tend to increase or decrease efficiency of the non-cooperative solution depends on the extent of cross-country deposits and potential asymmetries in country size. If country sizes are equal, cross-country deposits in general reduce inefficiencies.

Our model is able to capture several events of the recent crisis. It sheds light on the sources of diverging international interests resulting from the deeper integration of banking sectors. It shows when cooperation arrangements are particularly valuable to improve global welfare and when they are desirable from a national perspective. In that, it provides intuition for the factors driving international negotiations regarding a greater coordination, initiated in the aftermath of the crisis.

Related literature In our model, as in Allen and Gale (2000), contagion occurs as banks are linked through interbank deposits. Similarly, in Dasgupta (2004) systemic risk stems from balance sheet connections of banks.⁶ Alternatively, contagion can be modeled as an equilibrium phenomenon caused by reductions in available aggregate liquidity (see Diamond and Rajan (2005) or as being transmitted through informational spillovers (see Acharya and Yorulmazer (2008) and Chen (1999)).

Several papers have studied bailouts. In Gale and Vives (2002), a bailout is conducted through monetary policy, manipulating the price level. As in our paper, this is done to limit liquidation losses ex post. In Diamond and Rajan (2002) bailouts increase the aggregate liquidity available, which can lead to ambiguous effects on the stability of the banking system. Gorton and Huang (2004) consider an economy where agents inefficiently hoard liquidity. A government can reduce this problem by subsidizing distressed banks. Acharya and Yorulmazer (2007) show that regulators can have an incentive to bailout

⁶He applies the theory of global games, developed by Carlsson and van Damme (1993) and introduced to this setting by Morris and Shin (2003).

banks ex post if the number of banks failing is too large. Farhi and Tirole (2009) study strategic complementarities that arise when bailouts are done through non-targeted monetary policy. In recent work, Keister (2010) considers the effect of a bailout on financial fragility in a Diamond and Dybvig (1983) model.⁷

Some contributions put banking theory in an international perspective. Work in this area has focused on cooperation in respect of regulation (see Acharya (2003), Holthausen and Roende (2005), Dell’Ariccia and Marquez (2006) and Calzolari and Loranth (forthcoming)).

Only few papers treat international cooperation problems regarding the management of financial crisis. Agur (2009) studies the optimal institutional structure of a lender of last resort in an international framework with informational spillovers from intervention. While national authorities do not internalize the contagion effect, a central authority has limited signaling power. Therefore, the maximum welfare is achieved by central coordination. Freixas (2003) discusses externalities from bailouts in a multi-country setting. In a standard public goods model, he analyzes cooperation mechanisms which implement first-best bailouts. In an extension, Goodhart and Schoenmaker (2009) consider ex ante fiscal burden sharing rules. Our model goes beyond the public goods problem that the latter two papers study as we explicitly model international linkages and bailout decisions in a model of contagion.

The remainder of this paper is structured as follows. Section 2 introduces the model. Section 3 derives the solution to the social planner problem and the outcome of the non-cooperative game and draws comparisons between the two. Section 4 studies efficiency properties of different cooperation regimes. Section 5 extends the model by allowing for private cross-country deposits. Section 6 relates events of the recent financial crisis to our model. Section 7 concludes.

2 Model

Our model builds on Allen and Gale (2000). We use their basic framework to model interbank linkages and contagion. We extend the analysis to an international setting

⁷A related problem is the question of the lender of last resort. See among others Freixas et al. (2000), Rochet and Vives (2004), Goodhart and Huang (2005) and Kahn and Santos (2005).

with two countries and allow for bailouts in case of bankruptcy. Moreover, we introduce a production sector operating at date $t = 1$, employing labor whose income can be taxed by the government in order to finance interventions.

2.1 Setup

There are three time periods indexed by $t = 0, 1, 2$ and a continuum of ex ante identical agents of measure two. Each agent is endowed with one unit of a single consumption good at date $t = 0$. It serves as numéraire and can be invested in two different assets, a short asset and a long asset. The short asset represents a storage technology. For each unit invested at date t , it pays out one unit at date $t + 1$. Investment in the long asset takes place at date $t = 0$. It gives a higher payoff $R > 1$ at date $t = 2$, but can be liquidated at date $t = 1$ only at a loss. For one unit invested at date $t = 0$, $r < 1$ units can be recovered. At date $t = 1$, each agent decides on her supply of labor to the perfectly competitive production sector. Each unit of labor produces 1 unit of the consumption good. Consumers have Diamond-Dybvig preferences. With probability λ , an agent only values consumption at date $t = 1$ (early type), while with probability $1 - \lambda$ she is of the late type and values consumption only at date $t = 2$. Individual preferences are given by:

$$U(c_1, c_2) = \begin{cases} u(c_1) & \text{with probability } \lambda \\ u(c_2) & \text{with probability } 1 - \lambda, \end{cases} \quad (1)$$

where u is assumed to be increasing, strictly concave and twice continuously differentiable. Consumption of an agent of type i c_i is composed of three different elements: the return from the investment d_i , labor income n , and disutility of work expressed in consumption terms.⁸ That is:

$$c_i = d_i + \eta \left(n - \frac{n^2}{\kappa} - \frac{\kappa}{4} \right). \quad (2)$$

Disutility of work is quadratic, with shape parameters κ and η . Due to our assumptions on the utility function, the labor supply decision of the agent is independent of her type and we can drop the subscript i .⁹ The first-order condition for labor for both types of

⁸The latter two are not part of the standard Allen and Gale (2000) framework, which does not feature a labor sector. Introducing the labor sector allows us to micro-found convex costs for the financing of government expenditures. Our setup is similar to Cooper et al. (2008).

⁹Late consumers, who only consume at date $t = 2$, store their labor income from date 1 to date 2. The disutility of labor, although conceptually arising at date $t = 1$, unfolds only at $t = 2$.

consumers is:

$$\frac{du(c_i)}{dn} = u'(c_i)\eta \left(1 - \frac{2n}{\kappa}\right) = 0. \quad (3)$$

Therefore, each agent optimally supplies $n = \kappa/2$ units of labor. With the last term of Equation 2, we normalize the utility contribution of labor for the optimal labor supply to 0.¹⁰

By the law of large numbers, the probability λ of being an early consumer is also the fraction of early consumers in the economy. We assume that the population is divided into two groups of consumers (Group A and Group B), each of mass one. Within each group, the fraction of early consumers is stochastic. Across groups it is perfectly negatively correlated. There are two possible states of nature S_1 and S_2 , which are summarized in Table 1, where λ_H denotes a high fraction of early consumers and $0 < \lambda_L < \lambda_H < 1$. Both states occur with equal probability. Groups of consumers are thus identical in expectation and aggregate demand for liquidity is the same in both states.

Table 1: Liquidity shocks

	Group A	Group B	Probability
State S_1	λ_H	λ_L	0.5
State S_2	λ_L	λ_H	0.5

2.2 Optimal risk sharing and the first-best allocation

As there is no aggregate uncertainty, the first-best allocation implies perfect risk-sharing and is independent of the state. In order to determine the first-best solution, we consider a social planner that has perfect information. The planner chooses per-capita investment

¹⁰Due to the normalization, the date-0 investment decision of the bank is independent of the expected labor income and the bank's optimization problem that we consider later can be formulated as standard in the literature.

at date $t = 0$ so as to maximize overall expected utility treating all consumers alike:

$$\begin{aligned} \max_{\{d_1, d_2\}} \quad & \bar{\lambda}u(c_1(d_1)) + (1 - \bar{\lambda})u(c_2(d_2)) \\ \text{s.t.} \quad & x + y \leq 1, \\ & \bar{\lambda}d_1 \leq y, \\ & (1 - \bar{\lambda})d_2 \leq Rx, \end{aligned} \tag{4}$$

where x and y are the per capita amounts invested in the long and the short asset, respectively, and $\bar{\lambda} = \frac{\lambda_H + \lambda_L}{2}$. The three inequalities represent the resource constraints at date $t = 0$, $t = 1$ and $t = 2$. The social planner anticipates the optimal labor supply by the agents. Substituting $n = \kappa/2$ yields the standard objective function, which is independent of labor income: $\bar{\lambda}u(d_1) + (1 - \bar{\lambda})u(d_2)$. The unique solution to the problem is characterized by the condition:

$$u'(\bar{d}_1) = Ru'(\bar{d}_2), \tag{5}$$

where \bar{d}_1 and \bar{d}_2 denote consumption of early and late consumers in the first-best allocation.

It is optimal to provide date-1 consumption by investing in the short asset and date-2 consumption by investing in the long asset. Thus, at the optimum, all three resource constraints bind, and $\bar{d}_1 = y/\bar{\lambda}$ as well as $\bar{d}_2 = R(1 - y)/(1 - \bar{\lambda})$.

2.3 Decentralization and interbank deposits

The first-best allocation can be decentralized as an equilibrium with competitive banks and an interbank market. Assume that there are two representative banks, Bank A and Bank B. All consumers of Group A deposit in Bank A and all consumers of Group B deposit in Bank B. Banks offer demand deposit contracts to depositors and other banks, which independently of the state, promise a payment \bar{d}_1 upon withdrawal at date $t = 1$ per unit invested. A late withdrawer receives a pro rata share of the bank assets remaining at date $t = 2$, which in equilibrium is \bar{d}_2 . If the bank cannot serve all withdrawals at date $t = 1$, it is bankrupt and is liquidated. All depositors, i.e. also banks, receive the same

pro rata share of its liquidation value. Late consumers store the return and consume at date $t = 2$. As there is no sequential service constraint, no expectation driven bank runs occur and therefore, in equilibrium, there are no bankruptcies.¹¹ Consumers invest all their funds in their respective bank as this provides them with liquidity insurance. Moreover, each bank puts some of its funds in the other bank. The optimal interbank deposits, which implement the first-best allocation, are symmetric across banks. We assume that these equal just the minimum amount necessary to implement the first-best allocation, which implies $z = y(1 - \frac{\lambda y}{\lambda})$. In principle, however, deposits could be larger.¹² The value of the claims that a bank holds is given by $\bar{d}_1 z$ at date $t = 1$ and $\bar{d}_2 z$ at date $t = 2$, respectively.

2.4 Contagion

Following Allen and Gale (2000), we introduce the possibility of bank runs and contagion. The decentralized first-best allocation is fragile in so far as a perturbation can lead to bankruptcy of all banks in the system. We perturb the banking system by introducing a third state that is assigned a zero probability at date $t = 0$.¹³

As expectations remain unchanged, contracts and investment decisions at date $t = 0$ are the same as before. If state S_1 or S_2 occurs, allocations at date $t = 1$ and $t = 2$ are first-best. However, if the third state \bar{S} occurs, aggregate liquidity needs are higher than expected. As illustrated in Table 2, there is an additional fraction ϵ of early consumers in Bank A. Thus, the continuation equilibrium is different. As we are interested in

¹¹Note that our model does not feature multiple equilibria as e.g. in Diamond and Dybvig (1983) and Cooper and Ross (1998). Therefore, in our model, government intervention is not based on an equilibrium selection motive.

¹²If the interbank deposits were larger, contagion would take place for a larger set of parameters. Furthermore, given contagion, adverse spillover effects would be bigger.

¹³This assumption is a departure from rational expectations as agents do not correctly anticipate the possibility of a system-wide liquidity shortage. When the possibility of bankruptcy and contagion is not anticipated, moral hazard problems do not arise. The role of moral hazard in the context of financial crises and government intervention is the research focus of a growing literature (see e.g. Hellmann et al. (2000), Cooper and Ross (2002), Gale and Vives (2002), Cordella and Yeyati (2003), Farhi and Tirole (2009) and Keister (2010)). If introduced in our model, moral hazard should affect the trade-off between no intervention and a bailout by adding an additional cost to the latter. Our focus in this paper is, however, on the potential inefficiencies arising in the ex post decision process after a crisis occurred. The differences between the social planner solution and the outcome of the sequential game, driven by the three inefficiencies which we identify in our analysis, are in principle orthogonal to the problem of moral hazard; in particular, as long as countries and their ex ante expectations of being the crisis or the affected country are symmetric. Shutting down potential effects from ex ante expectations on the real investment allocation allows us to study in detail ex post intervention given bankruptcy and contagion.

the consequences of bankruptcy and potential contagion, we focus our analysis in the following on this state.

Table 2: Liquidity shocks with perturbation

	Bank A	Bank B	Probability
State S_1	λ_H	λ_L	0.5
State S_2	λ_L	λ_H	0.5
State \bar{S}	$\bar{\lambda} + \epsilon$	$\bar{\lambda}$	0

In state \bar{S} , the short assets of Bank A are not enough to satisfy its date-1 liquidity needs $\bar{d}_1(\bar{\lambda} + \epsilon)$ as the optimal investment decision at date $t = 0$ implies $y = \bar{d}_1\bar{\lambda}$. We assume that the following condition holds:

$$\frac{R}{r} \geq \frac{\bar{d}_2}{\bar{d}_1}.^{14} \quad (6)$$

This implies that liquidation of the long asset is the least attractive option to create additional liquidity. Hence, facing the additional fraction of early withdrawers, Bank A calls in its interbank claims before starting to liquidate the long asset. This, in turn, entails that also Bank B withdraws its interbank claims early as it faces more liquidity needs than it can satisfy with its short asset. Interbank claims, which are of the same size, cancel out.

Bank A is bankrupt if it had to liquidate so much of the long asset in order to satisfy liquidity needs of early consumers that late consumers would receive a payoff smaller than \bar{d}_1 . Anticipating this, late consumers decide to withdraw their funds early and a bank run occurs. The condition for bankruptcy of Bank A is:

$$\epsilon\bar{d}_1 > r \left((1 - y) - \frac{(1 - \bar{\lambda} - \epsilon)\bar{d}_1}{R} \right). \quad (7)$$

The term on the left hand side represents the additional liquidity needs that cannot be satisfied by the short asset. In order to prevent a run, Bank A must keep $\frac{(1 - \bar{\lambda} - \epsilon)\bar{d}_1}{R}$ units of the long asset such that the residual claim at date $t = 2$ per depositor is at

¹⁴This condition always holds for sufficiently low values of r . The right-hand side of the inequality depends on the parameter values of the model, but not on r as the first-best allocation is independent of the liquidation value of the long asset.

least \bar{d}_1 . The remaining units of the long asset can be liquidated, yielding a return of $r \left((1 - y) - \frac{(1 - \bar{\lambda} - \epsilon)\bar{d}_1}{R} \right)$ at date $t = 1$.

Bankruptcy of Bank A has an impact on the other bank through the interbank deposits. Bank B, similar to Bank A's private depositors, only receives a pro rata share of Bank A's liquidation value, which is smaller than the value of the actual claims as liquidation is costly. Whether these losses are sufficient to cause bankruptcy of Bank B depends on how much liquidity it can make available at date $t = 1$ without triggering a run. The bankruptcy condition for Bank B is:

$$z(\bar{d}_1 - q^A) > r \left((1 - y) - \frac{(1 - \bar{\lambda})\bar{d}_1}{R} \right), \quad (8)$$

where q^A represents the per unit liquidation value of Bank A. This value is affected by bankruptcy of Bank B. If both banks go bankrupt, each bank's liquidation value q^j is given by the value of the short asset plus the liquidation value of the long asset, hence $q^j = \bar{q} = y + (1 - y)r$, $\forall j \in \{A, B\}$. If the bank hit by contagion (Bank B) does not go bankrupt but fully repays the interbank deposit claims, the liquidation value of Bank A is raised. For bankruptcy of Bank B, Condition 8 has to hold in the case where Bank B fully repays Bank A's claims, hence when $q^A = \hat{q} = \frac{y + r(1 - y) + z\bar{d}_1}{1 + z}$. This is a sufficient condition as it implies that bankruptcy also takes place for any liquidation value below this upper bound. In the following, assume that the two bankruptcy conditions given by Equations 7 and 8 hold, i.e. liquidity shocks are sufficiently large to cause bankruptcy of Bank A and the loss on interbank deposits is sufficiently large so that Bank B goes bankrupt whenever there is bankruptcy of Bank A.

2.5 Government intervention

The two representative banks operate in an international setting with two countries. Bank A is located in Country A, Bank B in Country B. Banks are linked internationally through interbank deposits as described before. Country A is the source of the crisis hit by the liquidity demand shock. Country B is subject to contagion. We call Country A therefore the *crisis country* and Country B the *affected country*. Each country has a government that maximizes welfare of its population and decides whether to intervene when faced

with potential bankruptcy of its domestic bank. In order to finance an intervention, it taxes the labor income of domestic agents at date $t = 1$. It operates under a balanced budget.¹⁵

Abstracting from spillover effects, consider the decision problem of a government in a banking crisis. At date $t = 1$, it chooses between two actions.¹⁶ Firstly, it can decide not to intervene at all. Given our assumptions, this leads immediately to bankruptcy and liquidation of its bank. Each depositor receives a pro rata share q of the liquidated bank. If there is no intervention, which we denote by the subscript n , the welfare level V of the country is given by:

$$V_n = u(q). \quad (9)$$

Secondly, in order to prevent bankruptcy, the government can bail out its bankrupt bank. The costs of a bailout are derived from bankruptcy conditions 7 and 8, respectively. For a bailout a government has to supply at least the additional liquidity that the bank needs in order to prevent a bank run. If the bailout sum is larger than this minimal amount, the bank liquidates less long assets and late consumers get a payoff that exceeds \bar{d}_1 . Let b denote the payoff that late depositors receive when the bank is bailed out, gap the additional unexpected liquidity needs that occur in state \bar{S} and λ the fraction of early depositors that the bank faces. Then the general formula for the cost of a bailout is given by:

$$G(b) = gap - r \left((1 - y) - \frac{(1 - \lambda)b}{R} \right). \quad (10)$$

We distinguish different bailout levels by the amount b that late depositors receive. Define a partial bailout as the case where the minimum amount of liquidity is provided to avoid a bank run, i.e. all consumers of the bank receive \bar{d}_1 .¹⁷ In a full bailout, in contrast, so much liquidity is provided that late depositors receive the return they expected ex ante,

¹⁵If we allowed the government to borrow, it would have to raise taxes in the future to pay back its debt. The possibility to smooth taxes over time can reduce distortions. However, as long as raising funds is costly, the main trade-off remains unaffected.

¹⁶Note that we do not consider the option of stopping convertibility. Stopping convertibility would avoid a bank run at no direct costs. However, a fraction ϵ of early consumers would not be able to withdraw, which would reduce their consumption to zero. In a related problem, Ennis and Keister (2009) show that such a policy is not ex post credible.

¹⁷A deposit insurance would guarantee the same payoffs to consumers. However, there is a difference between a deposit insurance and a partial bailout. In case of a deposit insurance, the bank goes bankrupt and the government pays the difference between the liquidation value of the bank and the guaranteed deposits. A partial bailout is less costly as the provided funds avoid the early liquidation of some fraction of the long assets. For each unit of liquidity that the government provides, R/r funds are recovered.

i.e. $b = \bar{d}_2$. The optimal bailout is, in general, different from the two discussed above. It trades off the losses from liquidation with the costs of providing government funds.

We assume that consumers observe the bailout and know the tax rate $\tau \geq 0$, which the government imposes in order to finance the intervention. With this information, agents decide on how much labor to supply. Thus, taxes distort the agents' labor supply decision. The labor supply function is now given by:

$$n(\tau) = (1 - \tau) \frac{\kappa}{2}. \quad (11)$$

This implies the following Laffer curve for the government:

$$G = \tau n(\tau) = \frac{\kappa}{2} \tau (1 - \tau), \quad (12)$$

where G is government income raised. The tax rate $\tau^{max} = \frac{1}{2}$ yields the maximum tax income $G^{max} = \frac{\kappa}{8}$. As long as this upper bound is sufficiently high, each country can finance a domestic bailout. To facilitate notation, let $\tau(G)$ denote the tax rate that the government has to set in order to collect G . Furthermore, define $Z(G(b))$ as the total utility loss in terms of consumption due to distortionary taxation. As discussed before, for $\tau = 0$ this effect is normalized to 0 and $Z(0) = 0$. For $\tau > 0$, it is strictly positive and given by:

$$\begin{aligned} Z(G(b)) &= -\eta \left[n(\tau(G(b)))(1 - \tau(G(b))) - \frac{n(\tau(G(b)))^2}{\kappa} - \frac{\kappa}{4} \right] \\ &= \frac{\kappa \eta}{4} \left[1 - \left(\frac{1}{2} + \sqrt{\frac{1}{4} - \frac{2}{\kappa} G(b)} \right)^2 \right], \end{aligned} \quad (13)$$

where we substituted, in the second line of the expression, the optimal labor supply and tax rate, which are functions of $G(b)$.¹⁸

Due to equal taxation, each consumer incurs the same utility loss from taxation $Z(G(b)) > 0$, which increases with b . Welfare in the economy is then:

$$V_{bo}(b) = \lambda u(\bar{d}_1 - Z(G(b))) + (1 - \lambda) u(b - Z(G(b))). \quad (14)$$

¹⁸For our analysis, it would be sufficient for $Z(G(b))$ to be increasing, convex and twice continuously differentiable. We assume a specific functional form for illustrative purposes.

For the optimal bailout level b , the first-order conditions imply:

$$\frac{\lambda u'(c_1)}{(1-\lambda)u'(c_2)} = \frac{1 - Z'(G(b))G'(b)}{Z'(G(b))G'(b)}. \quad (15)$$

We assume that when governments are indifferent between intervention and no intervention, they do not intervene. Hence, a government chooses a bailout iff $V_{bo}(b^*) > V_n$, where b^* is the solution to 15. Whether no intervention or a bailout yields higher welfare depends crucially on the curvatures of the utility function and the function $Z(\cdot)$, as well as the return on the long asset R and the liquidation value r . They all affect the trade-off a government faces between tax distortions, liquidation losses and consumption inequality.

There is a notable difference between a partial bailout where just enough liquidity is provided to avoid bankruptcy and a bailout where liquidity is provided beyond this minimum amount. From Equations 9 and 14 a necessary condition for a partial bailout to be optimal is $\bar{d}_1 - Z(G(\bar{d}_1)) > q$. A partial bailout, if chosen by the government, thus implies a Pareto improvement. Any liquidity that is provided beyond $b = \bar{d}_1$ benefits late depositors only, while all depositors face a higher tax rate and thus a stronger distortion of their labor supply decision. This can be optimal, because the utility gain from increasing late depositors' payoffs may be larger than the utility loss of early depositors. However, moving from partial bailout to any other bailout level never implies a Pareto improvement.

Proposition 1 *(i) If chosen by the government, a partial bailout is a Pareto improvement relative to no bailout.*

(ii) Any additional liquidity provided beyond the amount required for a partial bailout redistributes resources among agents, but is not a Pareto improvement.

Proof. Omitted. ■

2.6 International spillover effects from intervention

Each government has the power to intervene in the banking sector within its jurisdiction only. Besides the direct effect of a bailout on domestic welfare, there is a spillover effect on the welfare of the other country as banks are connected through interbank deposits. The spillover effects of a bailout are asymmetric and differ between the crisis country and the affected country. A bailout of Bank A avoids contagion and fully protects Bank B.

In contrast, a bailout of Bank B increases the liquidation value of Bank A as interbank deposits are fully repaid.

As the source of bankruptcy differs between banks, bailout costs, in general, are not the same. While bankruptcy of Bank A is caused by unexpected liquidity needs creating a maturity mismatch, the reason for bankruptcy of Bank B lies in a real loss on assets.¹⁹ Equation 10 is valid for both countries, but *gap* and λ differ. From Equation 7, the explicit bailout cost for Bank A is:

$$G^A(b) = \epsilon \bar{d}_1 - r \left((1 - y) - \frac{(1 - \bar{\lambda} - \epsilon)b}{R} \right). \quad (16)$$

For Bank B, we have from Equation 8 that:

$$G^B(b) = z(\bar{d}_1 - \hat{q}) - r \left((1 - y) - \frac{(1 - \bar{\lambda})b}{R} \right). \quad (17)$$

In Country A, more individuals than expected want to consume early. This has two effects. First, by providing funds, the government in Country A can implicitly collect some return on the long asset due to the smaller fraction of late depositors. Given the same gap, a bailout is therefore cheaper in this country.²⁰ Second, the relative weight given to the two types of consumers differs. As it faces more early consumers, the government of Country A puts more weight on their welfare than the government of Country B. Due to asymmetric bailout costs and different fractions of early and late depositors, the optimal decision between no intervention and bailout as well as the choice of b typically differ between governments.

¹⁹To gain intuition for this difference, consider the case where a government can raise non-discriminatory lump-sum taxes. Then, Country A always prefers a bailout over no intervention as there is a pure liquidity problem. Bank B faces real losses in assets. Therefore, a bailout of Bank B is desirable if the liquidation loss that can be avoided exceeds resources that have to be provided for the bailout. It can be shown that this is always the case. The free-riding problem, nonetheless, remains. Country A does not necessarily prefer to do a bailout of Bank A over letting Country B bail out its bank. These results on lump-sum taxation are derived in a supplementary Appendix available upon request.

²⁰As the share of late consumers is smaller than expected, the government in Country A could raise the return of late consumers beyond the expected level \bar{d}_2 by providing funds. If the government could provide the funds conditional on becoming a residual claimant of the bank, it could collect the residual value of the bank after it has paid \bar{d}_2 to all late consumers, thereby potentially increasing efficiency. By restricting $b \in [\bar{d}_1, \bar{d}_2]$ in the optimization problem of the government, we do not consider this case. Note that for sufficiently large values of r the optimal b always lies within this interval.

Welfare levels We denote welfare of country j by V_{s^A, s^B}^j , where the first subscript stands for Country A's intervention decision, while the second subscript captures the action of Country B. We subsume the pair of actions taken by both countries by a . The general welfare function can then be formulated as:

$$V_a^j = \lambda^j u(c_1^j(a)) + (1 - \lambda^j) u(c_2^j(a, b)) \quad (18)$$

If neither country intervenes, all agents receive a pro rata share of the liquidation value of the bank. Each consumer, no matter in which bank she deposited her endowment, obtains \bar{q} :

$$V_{n,n}^A = V_{n,n}^B = u(\bar{q}). \quad (19)$$

If the government of the crisis country decides to bailout its domestic bank, then contagion is avoided and the bank in Country B remains unaffected by the crisis in Country A. Welfare of Country A from bailing out its bank is:

$$V_{bo,n}^A(b) = (\bar{\lambda} + \epsilon) u(\bar{d}_1 - Z(G^A(b))) + (1 - \bar{\lambda} - \epsilon) u(b - Z(G^A(b))). \quad (20)$$

As there is no contagion and Bank B remains unaffected, there is no scope for intervention and Country B's welfare attains the maximum:

$$V_{bo,n}^B = \bar{\lambda} u(\bar{d}_1) + (1 - \bar{\lambda}) u(\bar{d}_2). \quad (21)$$

If, however, Country A does not intervene, there is contagion and the government of Country B has to decide whether or not to intervene. If Country B does a bailout, Country A's welfare is raised as Bank A's liquidation value increases to $\hat{q} > \bar{q}$:

$$V_{n,bo}^A = u(\hat{q}). \quad (22)$$

If it does a bailout, the welfare level of the affected country is:²¹

$$V_{n,bo}^B(b) = \bar{\lambda} u(\bar{d}_1 - Z(G^B(b))) + (1 - \bar{\lambda}) u(b - Z(G^B(b))). \quad (23)$$

²¹Note that the bailout of Bank B has a positive feedback effect. Because Bank B does not go bankrupt, it can fully repay Bank A's claims. This in turn, raises the liquidation value of Bank A, of which Bank B receives a pro rata share.

3 Equilibrium

In this section we derive the optimal continuation allocation chosen by a social planner as well as the solution to the non-cooperative sequential game played between the two governments.²² We then discuss inefficiencies arising in the decentralized game as compared to the optimal continuation allocation. Finally, we report mappings between the two.

3.1 Social planner

The optimal continuation allocation is the solution to the problem of a benevolent social planner. Respecting the terms of the deposit contracts, she decides whether and where a bailout should be done. In case a bailout is optimal, she further decides on the bailout level b and the country-specific contributions X^A and X^B such that the sum of the contributions equals the funds required for the bailout $G^j(b)$, where j denotes the country whose bank is bailed out. We restrict contribution levels to be non-negative.²³ The social planner solves the following maximization problem:

$$\begin{aligned} \max_{a \in \{(n,n), (n,bo), (bo,n)\}, X^A, X^B} \quad & V = \sum_{j \in \{A,B\}} \Theta^j V_a^j(d_1^j(a), d_2^j(a, b), Z(X^j)) \quad (24) \\ \text{s.t.} \quad & G^j(b) = X^A + X^B, \\ & X^A \geq 0, \\ & X^B \geq 0, \end{aligned}$$

where Θ^j is the Pareto weight attributed to country j . The first-order conditions with respect to the contribution levels imply:

$$\frac{\Theta^A}{\Theta^B} = \frac{\bar{\lambda} u'(d_1^B(a) - Z(X^B)) + (1 - \bar{\lambda}) u'(d_2^B(a, b) - Z(X^B))}{(\bar{\lambda} + \epsilon) u'(d_1^A(a) - Z(X^A)) + (1 - \bar{\lambda} - \epsilon) u'(d_2^A(a, b) - Z(X^A))} \frac{Z'(X^B)}{Z'(X^A)}. \quad (25)$$

²²The solution to the game with simultaneous moves is in a supplementary Appendix available upon request.

²³Note that when there is a bailout of Bank A, optimality requires contribution levels to be non-negative. If there is a bailout of Bank B, it could, however, be optimal to have a positive cash transfer from taxpayers in Country B to consumers in County A.

Similar to a government with a single bank before, the social planner trades off tax distortions, early liquidation losses and income inequality between early and late depositors of the same bank. Furthermore, she faces two additional trade-offs. First, the disutilities from labor taxation, which in our setup are independent of individual income levels and convex in taxes raised, prescribe an equalization of contribution levels between countries, i.e. tax smoothing. Second, given the concave utility function, differences in income levels between countries, resulting from asymmetric effects of the banking crisis, make income equalization between countries desirable. The latter implies an additional consumption smoothing motive in the two country setting.

In general, all three possible sets of actions can be optimal.²⁴ As, given a banking crisis, Country A is always poorer than Country B, the following result regarding contribution levels holds:

Proposition 2 *Suppose countries have equal welfare weights. Then, the contribution to a bailout of the affected country X^B is larger than the contribution of the crisis country X^A .*

Proof. See the Appendix. ■

Note that, by setting contribution levels, the social planner has the ability to induce discriminatory taxes. This is an additional instrument in the international context, which is not available in a national crisis. Imagine a closed economy model where the two countries integrate. Then the maximization problem of the government that hosts both Banks A and B is the same as the problem of the social planner stated above with the exception that the government cannot set discriminatory taxes, i.e. there is perfect tax smoothing. As consumers have different payoffs in case of a banking crisis, discriminatory taxation, however, is desirable. The social planner thus attains a higher welfare level relative to the case of full integration.

²⁴Bailout costs differ between banks. Therefore, although a bailout of Bank A prevents contagion and raises Country B's welfare to the maximum, a bailout of Bank A does not necessarily dominate a bailout of Bank B. If $G^A(b)$ is sufficiently large, it can be optimal to save Bank B only. Therefore, without any restrictions on parameters, any of the three possible combinations of government actions can be optimal.

3.2 Non-cooperative bailout game

When there is no coordination, each government decides on its own whether and how to intervene. Strategic interaction arises as the welfare of one country depends on the action chosen by the other country. Note that the welfare level of each country, however, is independent of the liquidity provision to the foreign bank. Therefore, b is not a strategic variable.

The sets of strategies of the governments in Country A and Country B are given by $S_A = S_B = \{n, bo\}$. We consider subgame-perfect Nash equilibria (SPNE) in pure strategies of the game with sequential moves. The crisis country moves first and the affected country follows.²⁵ There are three possible equilibria of the sequential game:

- Proposition 3** (i) $a^* = (n, n)$ is a SPNE iff $V_{n,bo}^B \leq V_{n,n}^B$ and $V_{bo,n}^A \leq V_{n,n}^A$.
(ii) $a^* = (n, bo)$ is a SPNE iff $V_{n,bo}^B > V_{n,n}^B$ and $V_{bo,n}^A \leq V_{n,bo}^A$.
(iii) $a^* = (bo, n)$ is a SPNE iff $V_{bo,n}^A > V_{n,bo}^A$ or $V_{n,bo}^B \leq V_{n,n}^B$ and $V_{bo,n}^A > V_{n,n}^A$.

Proof. Note that from Equations 19 to 23 it follows that $V_{n,bo}^A > V_{n,n}^A$ and $V_{bo,n}^B > V_{n,n}^B$ as well as $V_{bo,n}^B > V_{n,bo}^B$ and $V_{bo,n}^B = V_{bo,bo}^B$. ■

3.3 Inefficiencies in the bailout game

When comparing decisions in the sequential game with those taken by the social planner, three sources of inefficiencies can be identified. The first source of inefficiency, *externalities*, is due to the fact that there are spillovers in both directions that are not taken into account by governments in the non-cooperative game. While a bailout of Bank A completely prevents all adverse effects on Bank B, a bailout of Bank B increases the liquidation value of Bank A. As governments only care about their own consumers, which coincide with domestic depositors, they undervalue the benefits of a bailout from the point of view of a social planner.

The second source of inefficiency is *no burden sharing*. Distortions of the labor supply decision of agents from taxation are convex in tax revenues. Therefore, the inability of

²⁵We consider a two-stage leader-follower game. An alternative would be to analyze the problem in a dynamic game with step-wise contributions. Admati and Perry (1991) study a contribution game related to our problem and show that even in such an extended setup, inefficiency may persist. See also Gale (2001) for a more general treatment of such games.

the governments in the non-cooperative game to spread bailout costs over both tax bases increases financing costs and results in an inefficiency.

The third source of inefficiency, *free-riding*, arises due to the sequential structure of the game. The crisis country may not bail out its domestic bank because it knows that then the affected country will do a bailout. As the first mover it can free ride on the bailout carried out by Country B.

Proposition 4 *An anticipated bailout in the affected country lowers the incentives for a bailout in the crisis country.*

Proof. Country A does a bailout iff $V_{bo,n}^A > V_{n,\cdot}^A$.

$$V_{n,\cdot}^A = \begin{cases} V_{n,n}^A & \text{if Country B does not intervene} \\ V_{n,bo}^A & \text{if Country B does a bailout,} \end{cases}$$

and $V_{n,bo}^A > V_{n,n}^A$. ■

If the crisis country does not intervene, a bailout by the affected country raises the welfare of the former. Therefore, the net benefit for Country A of bailing out its bank is smaller when a bailout of Bank B is anticipated.

The strength of the effect of a bailout in the affected country on the crisis country is determined by the size of the interbank deposits.

Proposition 5 (i) *The incentives for a bailout in the affected country decrease in the interbank deposits z .*

(ii) *If a bailout by the affected country is anticipated, then the incentives for a bailout in the crisis country decrease in the interbank deposits z .*

Proof.

$$\begin{aligned} \text{(i)} \quad \frac{\partial G^B(b)}{\partial z} = \bar{d}_1 - q^A < 0 &\Rightarrow \frac{\partial(V_{n,bo}^B - V_{n,n}^B)}{\partial z} > 0. \\ \text{(ii)} \quad \frac{\partial \hat{q}}{\partial z} = \frac{\bar{d}_1 - (y+r)(1-y)}{(1+z)^2} > 0 &\Rightarrow \frac{\partial(V_{bo,n}^A - V_{n,bo}^A)}{\partial z} < 0. \quad \blacksquare \end{aligned}$$

Equation 17 implies that with increasing interbank deposits z , a bailout in Country B becomes more costly as the loss from bankruptcy of Bank A grows. This makes it less attractive for the affected country to bail out its bank. The gains from a bailout of Bank B for Country A, however, are increasing in the interbank deposits z . Therefore, if, in

response to contagion, Country B bails out its bank, the incentive for Country A to do a bailout decreases in the interbank deposits.

3.4 Mappings between optimal continuation allocation and equilibrium of the bailout game

We can characterize possible mappings from outcomes of the non-cooperative game to the optimal continuation allocation and vice versa. Proposition 6 states which actions can be the solution to the social planner's problem for a given equilibrium of the sequential game.

Proposition 6 *Suppose countries have equal welfare weights.*

- (i) *If the SPNE is $a^* = (bo, n)$, then $a' = (bo, n)$.*
- (ii) *If the SPNE is $a^* = (n, bo)$, then $a' \in \{(bo, n), (n, bo)\}$.*
- (iii) *If the SPNE is $a^* = (n, n)$, then $a' \in \{(bo, n), (n, bo), (n, n)\}$.*

Proof. See the Appendix. ■

If (bo, n) is the SPNE, the social planner chooses the same outcome. However, when (n, bo) is the SPNE, a bailout of Bank A or Bank B can be optimal. Finally, all actions can be optimal when (n, n) is the SPNE.

Next, we consider the reverse mapping, i.e. we ask, which actions can be the equilibrium of the sequential game if the social planner finds a certain sequence of actions a' optimal:

Corollary 1 *Suppose countries have equal welfare weights.*

- (i) *If $a' = (bo, n)$, then $a^* \in \{(bo, n), (n, bo), (n, n)\}$.*
- (ii) *If $a' = (n, bo)$, then $a^* \in \{(n, bo), (n, n)\}$.*
- (iii) *If $a' = (n, n)$, then $a^* = (n, n)$.*

Proof. Follows directly from Proposition 6. ■

First, if the social planner finds a bailout of Bank A optimal, then all three outcomes are possible equilibria of the sequential game and $a^* \in \{(n, n); (n, bo); (bo, n)\}$. Second, if the social planner does not find a bailout of Bank A optimal, then it follows that

the government in Country A itself does not choose to bailout its domestic bank either. Finally, if the social planner finds that no country should intervene, then $a^* = (n, n)$ is also the SPNE of the sequential game. This follows from the fact that for the social planner to choose no intervention, we must have $u(\bar{q}) > V_{bo,n}^A$ and $u(\bar{q}) > V_{n,bo}^B$, a situation in which neither Country A nor Country B choose a bailout.

Actions taken in the sequential game only differ from the choice of the social planner if a bailout is optimal and Country A does not bail out its bank. All distortions are towards too little intervention. Either no bailout is chosen when one would be optimal or there is a bailout of Bank B when global efficiency requires a bailout of Bank A.

4 Cooperation regimes

To improve upon the equilibrium of the non-cooperative game, governments have to find a way to cooperate. We study three different types of cooperation regimes and their efficiency and Pareto properties to shed light on this issue.

4.1 Different types of cooperation

The first option is a central authority with mandating and fiscal power, i.e. it can mandate actions to be taken and set contribution levels. As we abstract from any additional frictions that a central authority may encounter, it corresponds to the social planner discussed above.²⁶ Second, we consider a central authority with mandating but without fiscal power. It can prescribe actions, but cannot implement burden sharing. Third, we allow governments to negotiate ex post over a contract. Such a contract fixes a bailout to be implemented by one government and a corresponding transfer payment by the other government. We now give a brief description of options 2 and 3.

²⁶The correspondence would no longer hold if we took certain problems of social choice into account. There could, for example, be asymmetric information between national governments and the central authority with respect to the cost of a bailout.

Central authority with mandating power The objective function of the central authority is the weighted sum of national welfare levels. It solves the following problem:

$$\max_{a \in \{(n,n), (n,bo), (bo,n)\}} V = \sum_{j \in \{A,B\}} \Theta^j V_a^j(d_1^j(a), d_2^j(a,b), Z(G^j)). \quad (26)$$

It is not necessary for the central authority to have the power to mandate b as governments automatically choose the optimal bailout level given no burden sharing.

Contracts Contracts specify actions to be taken and a burden sharing rule. We model the negotiation process between governments via Nash bargaining with symmetric negotiation power. The Nash bargaining problem is as follows:

$$\max_{a \in \{(n,bo), (bo,n)\}, X^A, X^B} (V^A(a,b,X^A) - V^B(a^*)) (V^B(a,b,X^B) - V^B(a^*)) \quad (27)$$

s.t. $G(a,b) = X^A + X^B$.

If a contract is signed, it implies a Pareto improvement compared to the non-cooperative benchmark. One necessary condition for a Pareto improvement is that the actions \tilde{a} that are prescribed by the contract differ from the equilibrium actions a^* of the bailout game. A contract cannot be an agreement on burden sharing or a different set of actions alone as the participation constraint of one government would be violated. A second necessary condition is that the country where the bailout does not take place helps finance the bailout.²⁷ This follows from the fact that governments maximize domestic welfare in the bailout game. A change in actions without compensation must therefore reduce domestic welfare of at least one country. There are two different outcomes of the non-cooperative game that allow for contracts to be signed:

Proposition 7 (i) If $a^* = (n,n)$ and contracts allow for a Pareto improvement, then $\tilde{a} = (n,bo)$ with $\tilde{X}^A > 0$ or $\tilde{a} = (bo,n)$ with $\tilde{X}^B > 0$.

(ii) If $a^* = (n,bo)$ and contracts allow for a Pareto improvement, then $\tilde{a} = (bo,n)$ with $\tilde{X}^B > 0$.

²⁷In this context another type of free-riding arises. If Country B anticipates a bailout by Country A, it will not agree on any kind of burden sharing. Therefore, as Country A cannot commit not to bail out its bank, it has to bear the full cost of the bailout. This is the case even though consumers in Country B are richer than consumers in Country A and might benefit substantially from the intervention.

Proof. Omitted. ■

Case (i) captures situations where neither country intervenes, but welfare can be improved by a bailout. In order for Country B to agree on bailing out its domestic bank, Country A has to subsidize the bailout and vice versa. In Case (ii), Country B would bailout its bank without any cooperation between countries. However, each country's welfare can be increased if Bank A instead of Bank B is bailed out and Country B subsidizes the bailout.

When solving the Nash problem, the marginal utilities of depositors of one country are weighted by the other country's Nash factor, which in general does not equal the welfare weight attributed by the social planner. In addition, redistribution is costly. Moving away from the optimal continuation allocation and choosing $\tilde{X}^j \neq X^j$ reduces the surplus from the change in actions. Therefore, the set of actions \tilde{a} and the burden sharing rule in form of \tilde{X}^A, \tilde{X}^B usually do not coincide with the optimal choice taken by the social planner.

4.2 Pareto properties

Central authority with mandating power The allocation under a central authority with mandating power can differ from the one of the bailout game only with respect to the actions. As it has no fiscal power, which would allow for burden sharing, a central authority with mandating power can only increase welfare at the expense of one country:

Proposition 8 *Suppose no country is indifferent between a bailout and no intervention. Then, a central authority with mandating power cannot induce a Pareto improvement.*

Proof. See the Appendix. ■

Central authority with mandating and fiscal power The decision of a central authority with mandating and fiscal power implies that the welfare of one country increases, while the other country can experience a gain or a loss relative to the outcome of the non-cooperative game. Consider the case where the actions taken in the non-cooperative game coincide with the choice of the central authority. Then, the central authority only modifies the cost that each country has to bear and b . In this case, no Pareto improve-

ment is possible, as b does not affect the welfare of the country that does not conduct the bailout and the country that does the bailout chooses the optimal bailout level anyway.

When the central authority does not only introduce burden sharing, but also mandates actions different from the ones taken in the bailout game, this can, but does not need to imply welfare improvements for both countries. While for Country A results are ambiguous, Country B gains whenever the central authority mandates a bailout where, without cooperation, no bailout would take place.²⁸

A central authority with mandating and fiscal power implements the optimal continuation allocation chosen by the social planner. A general ranking between contracts and a central authority with mandating power, with respect to efficiency, is not possible. A central authority with mandating power cannot alleviate the inefficiency due to no burden sharing, but fully internalizes spillover effects. While contracts allow for some form of burden sharing, they require a Pareto improvement and realized gains are limited by costly redistribution. Neither contracts nor a central authority with mandating power guarantee the implementation of the efficient actions.

5 Cross-country deposits and country sizes

Banks today compete more and more for clients across borders. As a result, international financial linkages have not only increased due to stronger international balance sheet connections of banks, but also through larger cross-border asset holdings of depositors. Related to this, the relative size of the banking sector differs across countries. Motivated by these facts, we extend the model to include cross-country deposits, which, at the same time, allows us to capture differences in country size.²⁹ We study their effects on government decisions.

²⁸Further details are in a supplementary Appendix available upon request.

²⁹Whenever some claims in a financial institution are held by foreigners, incentives of a government to support this institution are affected. Narrowly interpreted, cross-country deposits in our model correspond to bank deposits held by foreign natural persons. Our analysis of cross-country deposits, however, captures a problem relevant for a wide range of cross-border financial assets.

5.1 Extended model setup

Now, banks compete for customers in both countries who can decide freely on where to deposit their endowment.³⁰ Let α (β) denote the fraction of depositors of Bank A (Bank B) that live in Country A (Country B) and let $1 - \alpha$ ($1 - \beta$) denote the fraction of agents that are depositors of Bank A (Bank B) and live in Country B (Country A). Banks remain of equal size, each hosting a unit mass of deposits. We assume that the liquidity shock ϵ hits a bank. Therefore, the bailout cost G is independent of the distribution of depositors. How easily a bailout can be financed depends, however, on the tax base of a country, hence its size. The smaller the population, the higher the tax rate required to raise a fixed amount of funds. Countries differ in size if $\alpha \neq \beta$. The population of Country A is $P^A = \alpha + (1 - \beta)$. In Country B, it is $P^B = \beta + (1 - \alpha)$. With asymmetric country sizes, the distortion of the labor supply decision becomes country-specific and depends on the population size P^j . Substituting for the tax rate $\tau(G, P^j)$, we obtain the following expression:

$$Z^j(G, P^j) = \frac{\kappa\eta}{4} \left[1 - \left(\frac{1}{2} + \sqrt{\frac{1}{4} - \frac{2}{P^j\kappa}G} \right)^2 \right]. \quad (28)$$

The derivation of the payoffs of the bailout game with cross-country deposits is straightforward. For Country A the general form is:

$$V_a^A = \alpha [\lambda u(c_1^A(a)) + (1 - \lambda)u(c_2^A(a, b))] + (1 - \alpha) [\lambda u(c_1^B(a)) + (1 - \lambda)u(c_2^B(a, b))], \quad (29)$$

where c_i^j now represents the consumption of a type i depositor who invested in bank j . Welfare in Country B can be represented analogously. The equilibrium with cross-country deposits is pinned down by the same conditions as derived for the baseline model in Proposition 3.

³⁰We assume that each agent deposits its entire endowment either abroad or at home. Ex ante agents are indifferent where to deposit their endowments. This allows us to vary exogeneously the distribution of cross-country deposits. In principle, cross-country deposits could be endogenized. For example, a diversification motive would arise if investments were risky and risk was not perfectly correlated across countries.

5.2 Cross-country deposits, country sizes and inefficiencies

If the social planner gives equal weight to every consumer and country sizes are equal, cross-country deposits do not change her problem. Differences in country size, in contrast, have an effect as they imply different tax bases and thus country-specific tax distortions.

The bailout game is affected by the introduction of cross-country deposits in two ways. Firstly, governments now take the spillover effects into account because they care about domestic consumers that invested abroad. As a consequence, the amount of liquidity b that is provided by a government enters the welfare function of the other country. Nevertheless, as before, b is not a strategic variable.³¹ Secondly, as liquidity shocks are attributed to a bank rather than a country, the fraction of early and late depositors of each country in State \bar{S} is altered. The fraction of early depositors in Country A is reduced and is only $\alpha(\lambda + \epsilon) + (1 - \beta)\lambda$, while the equivalent fraction in Country B is increased and is $(1 - \alpha)(\lambda + \epsilon) + \beta\lambda$. As a consequence the welfare weights that are attributed to early and late consumers are modified.

Due to the fact that not all domestic agents deposit in the domestic bank, an additional unit of liquidity has a lower marginal contribution to national welfare and b^* will be lower than without cross-country deposits:

Proposition 9 *For a given country size, the smaller the fraction of domestically held deposits, the lower the optimal bailout level b^* chosen by a government.*

Proof. See the Appendix. ■

As the incentives for bailouts change, the extent of the free-riding problem changes as well. For equally sized countries, the effects can be summarized in the following proposition:

Proposition 10 *Suppose countries are of equal size ($\alpha = \beta$). Then, an increase in the fraction of deposits abroad*

(i) decreases the incentives for a bailout in the affected country.

(ii) increases the incentives for a bailout in the crisis country if it anticipates no bailout in the affected country.

³¹Consider two possible cases. i) If Country A does a bailout, then intervention by Country B is not necessary. ii) If Country B does a bailout, there is no strategic effect as it is the second mover.

Proof. See the Appendix. ■

The incentives for Country B to bail out its bank decrease with the fraction of deposits abroad $1 - \alpha$. This is because the direct effect of a bailout on consumers who have invested in Bank B is larger than its indirect effect on consumers who hold deposits in Bank A. While a partial bailout raises payoffs by $\bar{d}_1 - \bar{q}$ for Bank-B depositors, the increase for Bank-A depositors is only a fraction of that, $\hat{q} - \bar{q} = \frac{z}{1+z}(\bar{d}_1 - \bar{q})$. If $b^* > \bar{d}_1$, late consumers that invested domestically benefit more than early consumers, while the additional liquidity support does not impact the payoff to consumers that invested abroad.

If Country A anticipates that Country B will not bail out its bank, then the incentives of Country A to finance a bailout increase with the fraction of domestic deposits abroad. A bailout in Country A prevents contagion and has therefore a large effect on the payoffs to Country A consumers that invested in Bank B. With a growing fraction of domestic endowments deposited abroad, the benefits from a bailout increase for Country A. Note that a partial bailout in Country A is enough to guarantee that late depositors that invested in Bank B receive the originally promised amount \bar{d}_2 . Therefore, in order to provide the same welfare for domestic depositors, costs are lower when some of them are depositors of Bank B, i.e. $1 - \beta > 0$.

Differences in country size, which corresponds to the tax base, imply different levels of distortions from taxation to raise the same revenue. This can best be illustrated by considering the special case where half of the consumers in each country deposit abroad:

Proposition 11 *Suppose in each country half of the depositors invest abroad ($\alpha = 1 - \beta$).*

Then, holding b constant,

- (i) the incentives for a bailout in the crisis country increase with its size (α),*
- (ii) the incentives for a bailout in the affected country increase with its size (β).*

Proof. See the Appendix. ■

If α is smaller than β , the crisis country is smaller than the affected country. As a consequence, to cover the same cost of intervention, the necessary tax rate is higher and therefore distortions from taxation are larger in the crisis country than in the affected country. For the crisis country a bailout is relatively less attractive. In addition, if $1 - \alpha \geq \beta > 1 - \beta \geq \alpha$, the liabilities of the domestic bank are mainly held by foreigners.

For sufficiently small deposit holdings abroad ($1 - \beta$), this reduces the incentives for the crisis country to bailout its domestic bank even further and aggravates the free-riding problem.

6 Policy

The model presented in this paper captures the essence of international conflicts of interest, arising in a financial crisis, when banks are linked internationally through their balance sheets and attract clients across borders. We draw parallels between our model and real events, and relate our results to current policy issues.

In September 2008, the US treasury decided against a bailout of Lehman Brothers. This triggered world-wide financial distress through international linkages. Faced with a deepening financial crisis, governments in different countries had to decide subsequently on how to deal with failing financial institutions within their jurisdictions.

One day after Lehman's filing for bankruptcy, the US Federal Reserve Bank created a secured credit facility of up to \$85 billion to support AIG, an insurance corporation with large international balance sheet connections. The bailout, the cost of which in large part eventually accrues to US taxpayers, benefitted foreign financial institutions substantially. Without this measure, these would have suffered severe losses, which might have made government intervention in other countries necessary. While there was the notion of financial contributions by foreign countries, other governments eventually did not help finance the bailout. Our model suggests that, anticipating that the US would support AIG anyway, they were free-riding on the bailout.

The effective loss incurred by depositors rather than by financial counterparties was the cause for international dispute in the case of Icesave. Mainly UK and Dutch consumers invested their savings in Icesave accounts located in Iceland. When Icesave went bankrupt, the Icelandic government did not compensate all creditors of the bank for their losses, but only absorbed losses of their own nationals. The fiscal burden of providing deposit insurance to all depositors would have been very high due to the large size of Icesave liabilities relative to the Icelandic tax base.³²

³²Our model does not allow for government intervention that discriminates depositors by their nation. In work in progress, we consider this possibility in the context of ring-fencing of assets.

These incidences during the crisis have shown that governments, when deciding on how to deal with their banks in distress, mainly care about their own nationals, while an intervention can have large welfare effects on foreigners, too. This has the potential to create international conflict.³³ Accordingly, there is an increased perception that international coordination is needed and a world-wide debate on how to improve global crisis management in respect of international decision making as well as burden sharing has developed.

In this paper, we consider bailouts as a means to limit adverse effects of banking crisis. In reality measures applied for these purposes include deposit insurance, recapitalization schemes and resolution policies. Currently, there are efforts towards international policy coordination regarding each of these measures.³⁴ At the same time, there are discussions on structural reforms of the banking system so as to avoid conflicts in the first place, for example by restricting the cross-border operations of banks. In line with our model, reducing international interbank exposures helps limit international contagion. However, it comes at the cost of reduced sharing of idiosyncratic liquidity risks. Moreover, supervisory responsibilities with respect to internationally operating banks could be modified. Our model shows how interests can diverge when national authorities do not have the full intervention rights over their nationals' assets. Policies that force banks to operate abroad as subsidiaries instead of branches aim at reducing this problem.

As a response to the recent events, some new institutions have been created. The memorandum of understanding of the Nordic-Baltic Stability Group from August 2010 states that its members will not only share information, but also the burden in case of crisis. The group resembles the central authority with fiscal power discussed in this paper. As our model shows, Pareto improvements are not guaranteed under this cooperation regime, which may cause commitment problems.

The European Systemic Risk Board (ESRB), a new EU financial institution, has been established, amongst other things, to issue recommendations to the member states

³³The UK draw on the Anti-terrorism, Crime and Security Act from 2001 in order to freeze Icelandic assets in the UK in the course of the dispute centered around Icesave.

³⁴For bank resolution, see e.g. IMF (2010), Claessens et al. (2010) as well as European Commission (2010) on bank resolutions funds. For an overview on international deposit insurance, see Demirguc-Kunt et al. (2008). Regarding the EU, there have been amendments to the EU Deposit Guarantee Schemes Directive. Among others there is a new requirement for member states to ensure that deposit guarantee schemes co-operate with each other. Burden sharing has been addressed by Ecofin (2010).

on how to deal with banks in distress. So far, the ESRB has reputational power only. Our model may help explain why: a central authority with mandating power always makes one country worse off if deviating from the noncooperative solution. The ESRB would correspond to this cooperation regime if its recommendations were legally binding.

For cooperation regimes to work effectively, international enforcement mechanisms are crucial. Without them, ex ante agreements may become obsolete once a crisis occurs and the distribution of adverse effects across countries is determined. Within the EU such mechanisms may exist.³⁵ At a global level, establishing an institution that makes future cooperation credible is inherently difficult as ongoing G20 discussions indicate.

7 Conclusions

With globalization international linkages between financial systems have strengthened. Therefore, the effects of financial crisis and government intervention in one country on other countries have increased. While international cooperation in bank regulation has been considered extensively by the literature, cooperation in respect of crisis management has not. In this paper, we provide a model of international contagion with government intervention in response to banking crisis. It allows us to study formally how the interests of countries diverge during crisis when financial institutions are linked through their balance sheets and consumers invest abroad. We study the inefficiencies that arise when governments do not cooperate and show how different forms of cooperation affect welfare ex post.

The model fosters an understanding of conflicts of interest inherent in decision-making powers in place that can arise in a banking crisis and shows when these are particularly severe. Our analysis of cooperation can help guide institutional reforms and provide intuition for the motivations of governments.

Our framework may be suitable to study other aspects of government intervention. While we consider fiscal bailouts, banks could also be supported through intervention by central banks, which would entail different trade-offs. Also, other forms of intervention with international spillovers, such as ring-fencing of assets, could be analyzed.

³⁵Dewatripont et al. (2010) suggest that the EU competition authority should coordinate between national resolution authorities by taking full advantage of its state aid control mandate.

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Appendix

Proof of Proposition 2

Proof.

(1) Interior solution, where Condition 25 holds. For the case (n,bo), we have:

$$1 = \frac{\bar{\lambda}u'(\bar{d}_1 - Z(X^B)) + (1 - \bar{\lambda})u'(b - Z(X^B))}{u'(\hat{q} - Z(X^A))} \frac{Z'(X^B)}{Z'(X^A)}.$$

Suppose that $X^A \geq X^B$. Then:

$$\begin{aligned} & \bar{\lambda}u'(\bar{d}_1 - Z(X^B)) + (1 - \bar{\lambda})u'(b - Z(X^B)) \geq u'(\hat{q} - Z(X^A)) \\ \Rightarrow & u'(\bar{d}_1 - Z(X^B)) \geq u'(\hat{q} - Z(X^A)) \\ \Rightarrow & Z(X^B) - Z(X^A) \geq \bar{d}_1 - \hat{q} > 0 \\ \Rightarrow & X^B > X^A, \end{aligned}$$

which is a contradiction. Therefore, $X^B > X^A$.

For the case (bo,n), we have:

$$1 = \frac{\bar{\lambda}u'(\bar{d}_1 - Z(X^B)) + (1 - \bar{\lambda})u'(\bar{d}_2 - Z(X^B))}{(\bar{\lambda} + \epsilon)u'(\bar{d}_1 - Z(X^A)) + (1 - \bar{\lambda} - \epsilon)u'(b - Z(X^A))} \frac{Z'(X^B)}{Z'(X^A)}.$$

Suppose that $X^A \geq X^B$. Then, given $b \in [\bar{d}_1, \bar{d}_2]$:

$$\begin{aligned}
& \lambda u'(\bar{d}_1 - Z(X^B)) + (1 - \bar{\lambda})u'(\bar{d}_2 - Z(X^B)) \geq (\bar{\lambda} + \epsilon)u'(\bar{d}_1 - Z(X^A)) \\
& + (1 - \bar{\lambda} - \epsilon)u'(b - Z(X^A)) \\
& \Rightarrow \lambda u'(\bar{d}_1 - Z(X^B)) + (1 - \bar{\lambda})u'(\bar{d}_2 - Z(X^B)) \geq (\bar{\lambda} + \epsilon)u'(\bar{d}_1 - Z(X^A)) \\
& + (1 - \bar{\lambda} - \epsilon)u'(\bar{d}_2 - Z(X^A)) \\
& \Rightarrow \bar{\lambda}[u'(\bar{d}_1 - Z(X^B)) - u'(\bar{d}_1 - Z(X^A))] + (1 - \bar{\lambda})[u'(\bar{d}_2 - Z(X^B)) - u'(\bar{d}_2 - Z(X^A))] \\
& \geq \epsilon[u'(\bar{d}_1 - Z(X^A)) - u'(\bar{d}_2 - Z(X^A))] > 0 \\
& \Rightarrow Z(X^B) > Z(X^A) \Rightarrow X^B > X^A,
\end{aligned}$$

which is a contradiction. Therefore, $X^B > X^A$.

(2) Corner solutions. There are two possible corner solutions:

- (i) $X^B = 0$ and $X^A = G$,
- (ii) $X^A = 0$ and $X^B = G$.

We show that only (ii) can be optimal. Suppose (i), i.e. $X^B = 0$ and $X^A = G$. Then, the FOC of the social planner with respect to X^A is:

$$\begin{aligned}
\frac{\partial V}{\partial X^A} &= -[(\bar{\lambda} + \epsilon)u'(d_1^A(a) - Z(G)) + (1 - \bar{\lambda} - \epsilon)u'(d_2^A(a, b) - Z(G))]Z'(G) \\
&+ [\bar{\lambda}u'(d_1^B(a)) + (1 - \bar{\lambda})u'(d_2^B(a, b))]Z'(0).
\end{aligned}$$

This can be rearranged to:

$$\begin{aligned}
\frac{\partial V}{\partial X^A} &= \bar{\lambda}[u'(d_1^B(a))Z'(0) - u'(d_1^A(a) - Z(G))Z'(G)] \\
&+ (1 - \bar{\lambda} - \epsilon)[u'(d_2^B(a, b))Z'(0) - u'(d_2^A(a, b) - Z(G))Z'(G)] \\
&+ \epsilon[u'(d_2^B(a, b))Z'(0) - u'(d_1^A(a) - Z(G))Z'(G)] < 0,
\end{aligned}$$

as $u(\cdot)$ is concave, $Z(\cdot)$ is convex, $d_2^B \geq d_2^A \geq d_1^A$ and $d_1^B \geq d_1^A$. Welfare could be improved by decreasing X_A and increasing X_B . ■

Proof of Proposition 6

Proof.

(i) $a^* = (bo, n)$ iff $V_{bo,n}^A > V_{n,bo}^A$ or $V_{n,bo}^B \leq V_{n,n}^B$ and $V_{bo,n}^A > V_{n,n}^A$.

If $V_{bo,n}^A > V_{n,bo}^A$, using $V_{bo,n}^B > V_{n,bo}^B \Rightarrow V_{bo,n}^A + V_{bo,n}^B > V_{n,bo}^A + V_{n,bo}^B$.

Using $V_{n,bo}^A > V_{n,n}^A$ and $V_{bo,n}^B > V_{n,n}^B \Rightarrow V_{bo,n}^A + V_{bo,n}^B > V_{n,n}^A + V_{n,n}^B$.

If $V_{bo,n}^A > V_{n,n}^A$ and $V_{n,n}^B \geq V_{n,bo}^B \Rightarrow V_{bo,n}^A + V_{n,n}^B > V_{n,n}^A + V_{n,bo}^B$.

Using $V_{bo,n}^B - V_{n,n}^B > V_{n,bo}^A - V_{n,n}^A \Rightarrow V_{bo,n}^A + V_{bo,n}^B > V_{n,bo}^A + V_{n,bo}^B$,

and as $V_{bo,n}^B > V_{n,n}^B \Rightarrow V_{bo,n}^A + V_{bo,n}^B > V_{n,n}^A + V_{n,n}^B$.

(ii) $a^* = (n, bo)$ iff $V_{n,bo}^B > V_{n,n}^B$ and $V_{bo,n}^A \leq V_{n,bo}^A$.

As $V_{n,bo}^A > V_{n,n}^A \Rightarrow V_{n,bo}^A + V_{n,bo}^B > V_{n,n}^A + V_{n,n}^B$.

(iii) $a^* = (n, n)$ iff $V_{n,bo}^B \leq V_{n,n}^B$ and $V_{bo,n}^A \leq V_{n,n}^A$.

$\Rightarrow V_{bo,n}^A \leq V_{n,n}^A < V_{n,bo}^A$ and $V_{n,bo}^B \leq V_{n,n}^B < V_{bo,n}^B$. ■

Proof of Proposition 8

Proof. Suppose $a^* = \{(n, n); (n, bo)\}$ and $a' = (bo, n)$. Then, Country A is made worse off as $V_{bo,n}^A < \max\{V_{n,bo}^A; V_{n,n}^A\}$. Suppose $a^* = (n, n)$ and $a' = (n, bo)$. Then, Country B is made worse off as $V_{n,bo}^B < V_{n,n}^B$. ■

Proof of Proposition 9

Proof. With constant country size implying $\bar{P} - \alpha = (1 - \beta)$, the first-order condition of Country A for b implies:

$$\frac{\alpha(\bar{\lambda} + \epsilon)u'(\bar{d}_1 - Z(G(b))) + (\bar{P} - \alpha)[\bar{\lambda}u'(\bar{d}_1 - Z(G(b))) + (1 - \bar{\lambda})u'(\bar{d}_2 - Z(G(b)))]}{\alpha(1 - \bar{\lambda} - \epsilon)u'(b - Z(G(b)))} = \frac{1 - Z'(G(b))G'(b)}{Z'(G(b))G'(b)}.$$

The derivative of the left hand side (LHS) with respect to α , holding b constant, implies:

$$\begin{aligned} & \frac{\partial LHS}{\partial \alpha} \Big|_{b=\bar{b}} [\alpha(1 - \bar{\lambda} - \epsilon)u'(b - Z(G(b)))]^2 \\ &= -[\bar{\lambda}u'(\bar{d}_1 - Z(G(b))) + (1 - \bar{\lambda})u'(\bar{d}_2 - Z(G(b)))]\alpha(1 - \bar{\lambda} - \epsilon)u'(b - Z(G(b))) \\ & - (\bar{P} - \alpha)[\bar{\lambda}u'(\bar{d}_1 - Z(G(b))) + (1 - \bar{\lambda})u'(\bar{d}_2 - Z(G(b)))](1 - \bar{\lambda} - \epsilon)u'(b - Z(G(b))) < 0. \end{aligned}$$

The derivative of the LHS with respect to b implies:

$$\begin{aligned} & \frac{\partial LHS}{\partial b} [\alpha(1 - \bar{\lambda} - \epsilon)u'(b - Z(G(b)))]^2 \\ &= -\alpha(\bar{\lambda} + \epsilon)u'(\bar{d}_1 - Z(G(b))) + (\bar{P} - \alpha)[\bar{\lambda}u'(\bar{d}_1 - Z(G(b))) + (1 - \bar{\lambda})u'(\bar{d}_2 - Z(G(b)))] \\ & \alpha(1 - \bar{\lambda} - \epsilon)u''(b - Z(G(b))) > 0. \end{aligned}$$

The derivative of $Z'(G(b))G'(b)$ with respect to b is:

$$\frac{\partial Z'(G(b))G'(b)}{b} = Z''(G(b))(G'(b))^2 + Z'(G(b))G''(b) = Z''(G(b))(G'(b))^2 > 0.$$

Therefore, the derivative of the right hand side (RHS) with respect to b is negative. The statements on the derivatives above imply that an increase in α leads to a higher bailout level b being chosen by the government in Country A. The proof for Country B is analogous. ■

Proof of Proposition 10

Proof.

(i) Country B does a bailout iff $V_{n,bo}^B > V_{n,n}^B$. Now:

$$\begin{aligned} & \frac{\partial(V_{n,bo}^B - V_{n,n}^B)}{\partial(1 - \alpha)} \Big|_{b=\bar{b}} \\ &= u(\hat{q} - Z(G(b))) - [\bar{\lambda}u(\bar{d}_1 - Z(G(b))) + (1 - \bar{\lambda})u(b - Z(G(b)))] < 0 \\ &\Rightarrow \forall \alpha, \alpha' \in [0, 1] \text{ and } \forall b \in [\bar{d}_1, \bar{d}_2] : \alpha' > \alpha \Leftrightarrow V_{n,bo}^B(b; \alpha') > V_{n,bo}^B(b; \alpha). \end{aligned}$$

Let $b = \arg \max V_{n,bo}^B(b; \alpha)$ and $b' = \arg \max V_{n,bo}^B(b; \alpha')$ with $\alpha' > \alpha$. Then, from optimal behavior of Country B and above: $V_{n,bo}^B(b'; \alpha') \geq V_{n,bo}^B(b; \alpha') > V_{n,bo}^B(b; \alpha)$.

(ii) Country A does a bailout iff $V_{bo,n}^A > V_{n,n}^A$. Now:

$$\begin{aligned} \frac{\partial(V_{bo,n}^A - V_{n,n}^A)}{\partial(1-\beta)} \Big|_{b=\bar{b}} &= - [(\bar{\lambda} + \epsilon)u(\bar{d}_1 - Z^A(G^A(b))) + (1 - \bar{\lambda} - \epsilon)u(b - Z^A(G^A(b)))] \\ &+ [\bar{\lambda}u(\bar{d}_1 - Z^A(G^A(b))) + (1 - \bar{\lambda})u(\bar{d}_2 - Z^A(G^A(b)))] > 0 \\ \Rightarrow \forall \beta, \beta' \in [0, 1] \text{ and } \forall b \in [\bar{d}_1, \bar{d}_2] : \beta > \beta' &\Leftrightarrow V_{bo,n}^A(b; \beta') > V_{bo,n}^A(b; \beta). \end{aligned}$$

Let $b = \arg \max V_{bo,n}^A(b; \beta)$ and $b' = \arg \max V_{bo,n}^A(b; \beta')$ with $\beta > \beta'$. Then, from optimal behavior of Country A and above: $V_{bo,n}^A(b'; \beta') \geq V_{bo,n}^A(b; \beta') > V_{bo,n}^A(b; \beta)$. ■

Proof of Proposition 11

Proof.

(i) Country A: $\frac{\partial Z^A}{\partial \alpha} \Big|_{b=\bar{b}} = -\frac{G\eta}{4\alpha^2} \left(\frac{1}{2} + \sqrt{\frac{1}{4} - \frac{G}{\alpha\kappa}} \right) \left(\frac{1}{4} - \frac{G}{\alpha\kappa} \right)^{-\frac{1}{2}} < 0$.

Country A does a bailout if:

$$\frac{V_{bo,n}^A}{V_{n,n}^A} = \frac{(2\bar{\lambda} + \epsilon)u(\bar{d}_1 - Z(G^A(b))) + (1 - \bar{\lambda} - \epsilon)u(b - Z(G^A(b))) + (1 - \bar{\lambda})u(\bar{d}_2 - Z(G^A(b)))}{2u(\bar{q})} > 1,$$

$$\text{and } \frac{V_{bo,n}^A}{V_{n,bo}^A} = \frac{(2\bar{\lambda} + \epsilon)u(\bar{d}_1 - Z(G^A(b))) + (1 - \bar{\lambda} - \epsilon)u(b - Z(G^A(b))) + (1 - \bar{\lambda})u(\bar{d}_2 - Z(G^A(b)))}{u(\hat{q}) + \bar{\lambda}u(\bar{d}_1) + (1 - \bar{\lambda})u(b)} > 1.$$

$$\begin{aligned} \text{Now: } \frac{\partial(V_{bo,n}^A/V_{n,n}^A)}{\partial Z} \Big|_{b=\bar{b}} &= -\frac{(2\bar{\lambda} + \epsilon)u'(\bar{d}_1 - Z(G^A(b))) + (1 - \bar{\lambda} - \epsilon)u'(b - Z(G^A(b))) + (1 - \bar{\lambda})u'(\bar{d}_2 - Z(G^A(b)))}{2u(\bar{q})} < 0 \\ \Rightarrow \frac{\partial(V_{bo,n}^A/V_{n,n}^A)}{\partial \alpha} \Big|_{b=\bar{b}} > 0, \text{ and } \frac{\partial(V_{bo,n}^A/V_{n,bo}^A)}{\partial Z} \Big|_{b=\bar{b}} &= -\frac{(2\bar{\lambda} + \epsilon)u'(\bar{d}_1 - Z(G^A(b))) + (1 - \bar{\lambda} - \epsilon)u'(b - Z(G^A(b))) + (1 - \bar{\lambda})u'(\bar{d}_2 - Z(G^A(b)))}{u(\hat{q}) + \bar{\lambda}u(\bar{d}_1) + (1 - \bar{\lambda})u(b)} < 0. \\ \Rightarrow \frac{\partial(V_{bo,n}^A/V_{n,bo}^A)}{\partial \alpha} \Big|_{b=\bar{b}} > 0. \end{aligned}$$

(ii) Country B: $\frac{\partial Z^B}{\partial \beta} \Big|_{b=\bar{b}} = -\frac{G\eta}{4\beta^2} \left(\frac{1}{2} + \sqrt{\frac{1}{4} - \frac{G}{\beta\kappa}} \right) \left(\frac{1}{4} - \frac{G}{\beta\kappa} \right)^{-\frac{1}{2}} < 0$. Country B does a

bailout if: $\frac{V_{n,bo}^B}{V_{n,n}^B} = \frac{u(\hat{q} - Z(G^B(b))) + \bar{\lambda}u(\bar{d}_1 - Z(G^B(b))) + (1 - \bar{\lambda})u(b - Z(G^B(b)))}{2u(\bar{q})} > 1$.

$$\begin{aligned} \text{Now: } \frac{\partial(V_{n,bo}^B/V_{n,n}^B)}{\partial Z} \Big|_{b=\bar{b}} &= -\frac{u'(\hat{q} - Z(G^B(b))) + \bar{\lambda}u'(\bar{d}_1 - Z(G^B(b))) + (1 - \bar{\lambda})u'(b - Z(G^B(b)))}{2u(\bar{q})} < 0 \\ \Rightarrow \frac{\partial(V_{n,bo}^B/V_{n,n}^B)}{\partial \beta} \Big|_{b=\bar{b}} > 0. \quad \blacksquare \end{aligned}$$