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* Views expressed are those of the authors and do not necessarily reflect official positions of De Nederlandsche Bank.

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Empirical evidence on the Euler equation for investment in the US*

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Abstract

Is the typical specification of the Euler equation for investment employed in DSGE models consistent with aggregate macro data? Using state-of-the-art econometric methods that are robust to weak instruments and exploit information in possible structural changes, the answer is yes. Unfortunately, however, there is very little information about the values of the parameters in aggregate data because investment is unresponsive to changes in capital utilization and the real interest rate. Bayesian estimation using fully-specified DSGE models is more accurate likely due to informative priors and cross-equation restrictions.

Keywords: Investment, Adjustment costs, Weak identification.

JEL classification: C2, E22.

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1 Introduction

An important component of the demand side of standard Dynamic Stochastic General Equilibrium (DSGE) models is aggregate investment. The seminal contribution by Christiano et al. (2005) proposes an investment-adjustment cost model coupled with the assumption of variable capital utilization to capture the inertial response of aggregate investment to monetary policy shocks. This specification for investment behavior has become standard in the DGSE literature, and the implied Euler equation for investment features in most DSGE models used for policy analysis. The key structural parameters of this investment block of modern DSGE models are: the investment adjustment cost parameter that denotes the inverse of the elasticity of investment with respect to the shadow price of capital, the elasticity of the capital utilization cost function, and the persistence of the investment-specific technology shock.[1]

However, estimates of these three key parameters differ greatly across papers in the literature. The next Section shows that different parameter values entail a very different response of investment, and hence of output, to various macroeconomic shocks. Not surprisingly, this then translates to different implications regarding the main drivers of business cycle fluctuations. It seems important, therefore, to investigate why the estimates of the key parameters vary greatly and how accurately they can be estimated using aggregate macro data. This is what we do in this paper in a single-equation limited-information framework. More specifically, we ask the following two questions.

First, is the typical specification of the Euler equation for investment employed in DSGE models together with an autoregressive investment-specific shock consistent with aggregate macro data? We test the bare minimum implications of this model with state-of-the-art generalized method of moments (GMM) tests, and we find that the answer is yes. In other words, there is no evidence against this specification that can be found in aggregate macro

[1]The results we report in the paper are for the most standard investment equation that appears in DSGE models. We also studied a specification with capital adjustment costs and the results were very similar. Appendix E contains results for capital adjustment costs.

data. This is a positive message that we add to the literature. We are not aware of any paper in the literature that checks the consistency of the implied Euler equation for investment with aggregate data as a single equation, rather than through the lens of Bayesian estimation of a full DSGE model. We use recently developed econometric methods in Magnusson and Mavroeidis (2014) and in Mikusheva (2021) to deal with weak identification. The former incorporates subsample information in the data arising from structural changes in the economy such as policy regime shifts. These methods also serve as parameter stability tests that are fully robust to weak instruments, and, hence, they provide reliable evidence on the stability of the parameters over the sample. The latter explores information of all potential instruments available for inference using a split-sample technique. Moreover, using the common assumption of variable capital utilization to derive our estimated equation allows us to use observable variables, such as the real interest rate and capacity utilization that is fully consistent with the investment block currently used in modern DSGE modeling, instead of proxies for unobservable variables, such as the return on capital or the Tobin's marginal Q. Finally, the single-equation approach is robust to potential misspecification in other equations of the system.[2]

The second question is: how much can we learn from aggregate time series data about the values of the three key investment Euler equation parameters? Unfortunately, the answer to this question is not much when using single-equation limited-information methods and available instruments. Those parameters are generally very poorly identified, that is, there is very little information about these parameters in aggregate data. Using typical lagged values of the endogenous variables as instruments, the confidence sets contain almost the entire parameter space. This motivates us to consider external sets of instruments, such as oil

[2] In the literature, there exists alternative approaches to dealing with misspecification in DSGE models. Sargent (1989) and Ireland (2004) introduce errors in the measurement equations of the state-space model. Del Negro and Schorfheide (2004) use prior distributions for structural VARs that are centered at the DSGE model-implied cross-equation restrictions, generating a continuum of empirical models referred to as DSGE-VARs (see also Del Negro and Schorfheide, 2009). More recently, Inoue et al. (2020) propose a method for detecting and identifying misspecification in structural models and show that DSGE models can be severely misspecified.

prices, financial uncertainty measures, government expenditure shocks and monetary policy shocks; however, we find that they barely improve identifying the main structural parameters. Comparing our results to those in Magnusson and Mavroeidis (2014), structural change is not as informative for the identification of the Euler equation for investment as it is for the NKPC. This is in line with the results in Ascari et al. (2021), suggesting that policy regime shifts have had more impact on nominal variables than on real variables over our sample.

To further understand the above results, we estimate a single-equation semi-structural model parameterized in terms of the slope coefficients of the investment equation with respect to the capital utilization rate and the real interest rate. We show that, when the persistence of the investment-specific technology shock is large, the slopes of the semi-structural parameters are not well-identified because the change in the capital utilization rate and the real interest rate is poorly forecastable. In contrast, when the persistence of the investment-specific technology shock is low, these slope parameters are well-identified and near zero, suggesting that investment is unresponsive to the real interest rate[3] and to capital utilization. However, because the mapping from the semi-structural to the structural parameters is ill-posed near zero, the structural parameters are weakly identified - small changes in the slope coefficients generate large changes in the capital utilization and investment adjustment cost parameters.

Then, we present the implications of weak identification of structural parameters in terms of possible ranges of the impulse response functions and of the forecast error variance decompositions using the standard medium-scale DSGE model of Justiniano et al. (2010) (JPT, henceforth). Our analysis shows that the results are most sensitive to the value of the investment adjustment cost parameter and the persistence of the investment-specific technology shock. Hence, pinning down these two parameters is key and more important than identifying a value for the elasticity of capital utilization.

The previous weak identification finding contrasts with the results commonly reported in the DSGE literature, leading us to investigate how DSGE models could achieve identifica-

[3]This is consistent with Keynes' old argument and early empirical results surveyed in Taylor (1999). See also the discussion in Mertens (2010) and Brault and Khan (2020).

tion of these parameters (again using the JPT model). There are two possible ways, either through the prior or through the joint model dynamics of the variables in the system together with the related cross-equation restrictions implied by rational expectations.[4] Both features differentiate the DSGE estimation from our method. While estimating a system of equations could help identification through cross-equation restrictions, possible misspecification in parts of the model could lead to biased estimates.

Our analysis suggests the following. Consistent with our GMM results, the elasticity of the capital utilization cost function is not identified by the data because the posterior mostly (or almost exclusively) relies on information coming from the prior. On the other hand, the investment adjustment cost parameter is identified by the cross-equation restrictions implied by the structure of the DSGE model when using the JPT model and data set. The persistence parameter of the investment-specific shock is always high and well-identified, even if we relax some of the cross-equation restrictions and use a loose prior. We conjecture that the model wants to match the very persistent dynamics of the observable macroeconomic time series.

Our paper could be of interest to any researchers using the standard investment cost specification proposed by Christiano et al. (2005) in their medium-scale DSGE models. The literature is immense and a small subset of influential papers are considered in Figure 1 below.[5] Our paper is also related to the literature that estimates individual equations employed in standard DSGE models using GMM methods and aggregate data, specifically, the New Keynesian Phillips Curve (Galí and Gertler, 1999), the Taylor rule (Clarida et al., 2000), and the Euler equation for consumption (Yogo, 2004). The present paper is the first to conduct a similar exercise for the other main component of the demand side of modern DSGE models:

[4] *"Communism of models gives rational expectations much of its empirical power and underlies the cross-equation restrictions that are used by rational expectations econometrics to identify and estimate parameters. A related perspective is that, within models that have unique rational expectations equilibria, the hypothesis of rational expectations makes agents' expectations disappear as objects to be specified by the model builder or to be estimated by the econometrician. Instead, they are equilibrium outcomes.[...] Identification is partially achieved by the rich set of cross-equation restrictions that the hypothesis of rational expectations imposes."*(Sargent, 2008, p. 194-195)

[5] In a recent paper, Foroni et al. (2022) shows that the identification of the investment adjustment cost parameter could be biased by time aggregation, that is, by aggregating monthly data into quarterly.

investment.

Finally, our contribution and results are complementary to the literature that estimates investment functions using microeconomic data. Closely related to our work are Groth and Khan (2010) and Eberly et al. (2012), who estimate the investment adjustment-cost model of Christiano et al. (2005) on 18 industries and on firm-level data, respectively.[6] Groth and Khan (2010) find that the adjustment cost parameter is very small in U.S. manufacturing industries, implying that investment is highly sensitive to the current shadow value of capital, in contrast to the aggregate estimates from well-known DSGE models reported in Figure 1, see Groth and Khan (2010, Table 5). Eberly et al. (2012) report a strong lagged-investment effect in their estimates, implying inertial dynamics of investment for the set of firms in their sample. We, instead, want to assess the ability to identify the parameters of the investment block of DSGE models from aggregate data. Our macroeconomic approach abstracts from heterogeneous adjustment costs or elasticities of investment, since this would require modelling the heterogeneity of firms at the micro level.[7]

The structure of the paper is as follows. Section 2 presents the theoretical specification and it investigates the implication of using different values for the parameters of the investment equation, as taken from influential papers in the DSGE literature. Section 3 describes the econometric methodology and Section 4 presents the data. Section 5 presents the empirical results. Section 6 discusses the identification of the key parameters of the investment equation in DSGE models, in light of our results. Section 7 concludes. Additional empirical results, data sources and econometric methods are reported in the online Appendix.

[6] There is a large literature mainly concerned with additional costs of external financing as a result of information asymmetries and agency costs which may increase the sensitivity of investment decisions to sources of internal finance such as cash flows for constrained firms (Fazzari et al., 1988, Bond and Meghir, 1994, and Kaplan and Zingales, 1997).

[7] This is a common problem of linking aggregate and firm level elasticities in the presence of heterogeneity due to the different constraints faced by firms (or households in the case of the Euler equation for consumption). Keane and Rogerson (2012), for example, make a similar point regarding the difficulty in reconciling the microeconomic and the macroeconomic estimates of the elasticity of labour supply.

2 The Euler equation of investment and DSGE models

The investment Euler equation used in our empirical investigation comes from the most standard specification used in medium-scale DSGE models. Since the seminal paper by Christiano et al. (2005), DSGE models commonly employ the assumption of investment adjustment costs (IAC) and variable capital utilization. The investment Euler equation is derived from the households' problem, assuming that households take investment decisions, own the capital stock and rent capital to the firms.[8] The representative household accumulates end-of-period t physical capital ($\hat{K}_{t+1}$) according to a standard capital accumulation equation

$$\hat{K}_{t+1} = \nu_t \left[1 - \mathcal{S}\left(\frac{I_t}{I_{t-1}}\right)\right] I_t + (1-\delta)\hat{K}_t, \tag{1}$$

where I_t is investment, δ is the depreciation rate, ν_t is the investment-specific technology shock, that is, a shock to the efficiency with which the final good can be transformed into physical capital, and the IAC function $\mathcal{S}(\cdot)$ is such that $\mathcal{S}(1) = \mathcal{S}'(1) = 0$ with $\kappa = \mathcal{S}''(1) > 0$. Here, κ, the adjustment cost parameter, denotes the inverse of the elasticity of investment with respect to the shadow price of capital. The log of the investment shock follows the autoregressive stochastic process $\log \nu_t = \rho \log \nu_{t-1} + \varepsilon_t^v$, where ρ is the autoregressive coefficient.[9]

Capital owning households choose the capital utilization rate, u_t, that transforms physical capital $\hat{K}_t$ into effective capital K_t, i.e., $K_t = u_t\hat{K}_t$, which is rented to intermediate goods

[8] This is mainly for convenience in the literature. A DSGE model yields isomorphic first-order conditions for the investment side of the model irrespective of whether investment decisions are taken by firms or by households. Appendix A describes the model in more details and shows the derivations.

[9] For the investment Euler equation with Capital Adjustment Cost (CAC), equation (1) is

$$\hat{K}_{t+1} = \nu_t I_t + (1-\delta)\hat{K}_t - D(\hat{K}_t, I_t),$$

where $D(\hat{K}_t, I_t) = \frac{\sigma}{2}\left(\frac{I_t}{\hat{K}_t} - \delta\right)^2 \hat{K}_t$, and $\sigma > 0$ governs the magnitude of adjustment costs to capital accumulation. We also perform the same econometric analysis for a specification using CAC instead of IAC. The results are similar to ones reported in Section 5, and, therefore, are placed in Appendix E.

producers at the rate r_t^k. Standard assumptions are: i) $\bar{u} = 1$ and $a(\bar{u}) = 0$, where a bar over a variable denotes its steady state value and $a(u)$ is a function that measures the cost of capital utilization per unit of physical capital; ii) the curvature of the function $a(u)$, given by $a''(u)/a'(u)$, measures the elasticity of capital utilization cost and it is such that $\zeta = a''(1)/a'(1) > 0$.

The log-linearized first-order condition yields the following dynamic equation for investment

$$\Delta\widetilde{i}_t = (\beta + \phi_q)\, E_t\Delta\widetilde{i}_{t+1} - \beta\phi_q E_t\Delta\widetilde{i}_{t+2} + \frac{1}{\kappa}\left[\phi_k\zeta E_t\widetilde{u}_{t+1} - \widetilde{r}_t^p + \widetilde{\nu}_t\right] - \frac{\phi_q}{\kappa}E_t\widetilde{\nu}_{t+1}, \tag{2}$$

where lowercase letters with a tilde denote the respective log deviations of the variables from their steady state, $\widetilde{r}_t^p$ denotes the log-deviation of the ex-ante real interest rate from steady state, β is the discount factor, and $\phi_q = (1-\delta)\beta$ and $\phi_k = 1 - \phi_q$. Intuitively, investment depends positively on expected capital utilization ($E_t\widetilde{u}_{t+1}$), negatively on the real interest rate ($\widetilde{r}_t^p$) and positively on the current value of the investment-specific shock ($\widetilde{\nu}_t$). The assumed specification of the adjustment cost makes investment depend on its own one-period lag ($\widetilde{i}_{t-1}$) and its expected leads ($E_t\widetilde{i}_{t+1}, E_t\widetilde{i}_{t+2}$).

Figure 1 shows that estimates of the elasticity of capital utilization cost (ζ) and the investment adjustment cost (κ) vary widely across various well-known papers in the literature that estimate medium-scale DSGE models. It is worth noting that such differences could be due to model specification alternatives, data sets and sample periods as well as differences in estimation techniques with some papers using full-information Bayesian estimation (e.g., JPT, 2010; SW, 2007) while others using limited-information impulse response matching (e.g., CEE, 2005; ACEL, 2011).

Notwithstanding these differences, to appreciate the implications of the different values of the parameters characterizing the investment equation we use the JPT model and look at the impulse response. Figure 2 shows how the impulse responses of output and investment

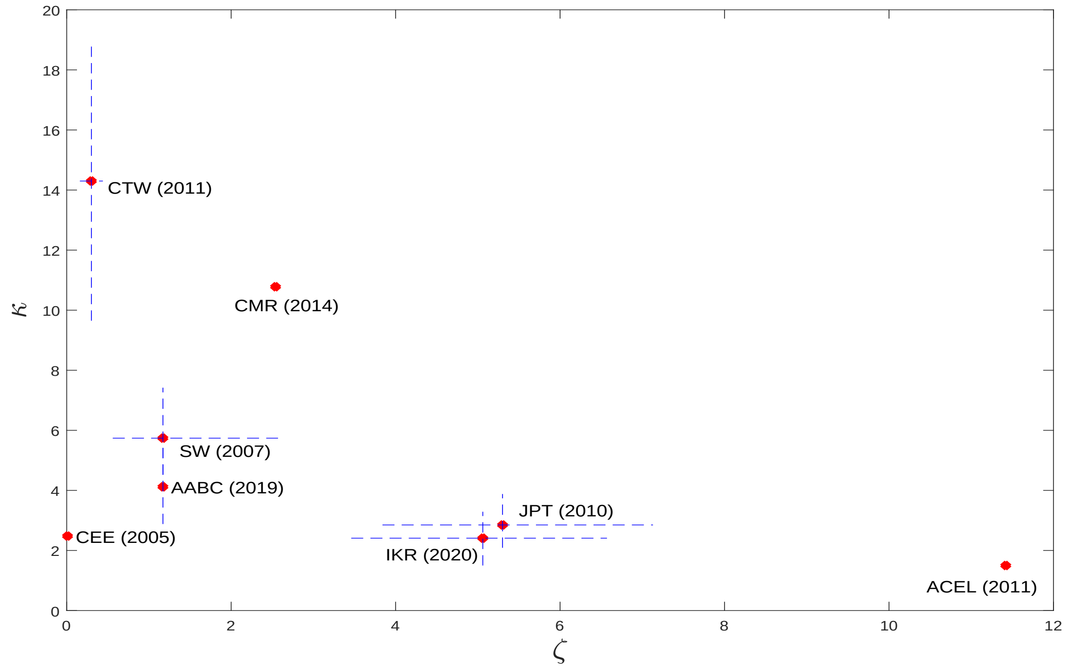

Figure 1: Estimates of the elasticity of capital utilization cost (ζ) and the investment adjustment cost (κ) including the 90% credible intervals from the literature: Christiano et al. - CEE(2005), Smets and Wouters - SW(2007), Justiniano et al. - JPT(2010), Altig et al. - ACEL(2011), Christiano et al. - CTW(2011), Christiano et al. - CMR(2014), Arias et al. - AABC(2020), Inoue et al. - IKR(2020).

to the seven structural shocks in JPT's model change when we keep all the parameters fixed at the JPT posterior median, while we vary the value of κ and ζ using the following four values from the literature (see Figure 1) : (i) Justiniano et al. - JPT(2010): $\kappa = 2.85$ and $\zeta = 5.30$; (ii) Christiano et al. - CTW(2011): $\kappa = 14.30$ and $\zeta = 0.30$; (iii) Altig et al. - ACEL(2011): $\kappa = 1.50$ and $\zeta = 11.42$; (iv) Christiano et al. - CEE(2005): $\kappa = 2.48$ and $\zeta = 0.01$.

It is evident that the response of investment to the various shocks varies substantially from a quantitative point of view in terms of impact effect, peak response and persistence. With the exception of the technology shock, a larger value of κ tends to dampen the response of investment. The different responses of investment then translate into different responses in output. As a consequence, these different calibrations have an impact on the variance decomposition of output growth, as shown in Table 1. Not surprisingly, as the value of κ increases, the importance of the investment-specific shock as a driver of the business cycle

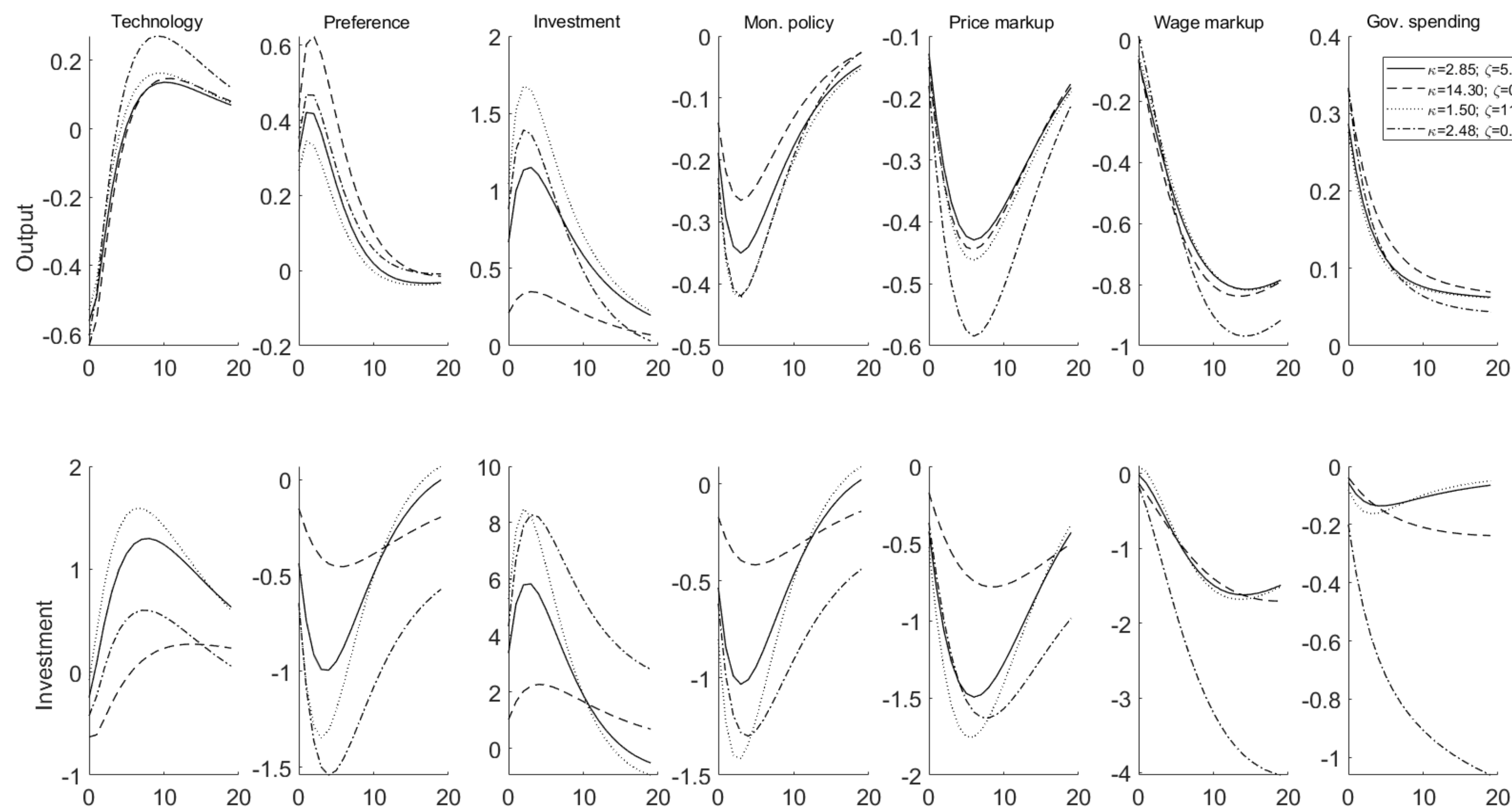

Figure 2: Impulse responses of output and investment to a one-standard deviation structural shock in JPT's model for alternative calibration of κ and ζ. The remaining parameters are set at the posterior median estimate of JPT.

decreases. The ACEL and the CTW calibrations, which exhibit the smallest and highest values of the investment adjustment cost parameter ($\kappa = 1.5$ and $\kappa = 14.3$, respectively), have the highest and lowest contributions of the investment-specific shock to the variance of output growth (69% and 8%, respectively) among the four different calibrations. Hence, the main result in JPT, for example, about the investment-specific shock being the major driver of the business cycle relies on a relatively low value for κ, as also discussed by JPT. The effects of different values for ζ are more difficult to grasp from this analysis, but more will be said about it below.

Table 1: Variance decomposition of output growth in JPT's model

Parameters	Tec	Pref	Inv	MP	PM	WM	Govt
JPT: $\kappa = 2.85, \zeta = 5.30$	0.21	0.10	0.49	0.04	0.03	0.05	0.07
CTW: $\kappa = 14.30, \zeta = 0.30$	0.31	0.29	0.08	0.03	0.04	0.10	0.14
ACEL: $\kappa = 1.50, \zeta = 11.42$	0.14	0.04	0.69	0.04	0.02	0.03	0.04
CEE: $\kappa = 2.48, \zeta = 0.01$	0.18	0.08	0.55	0.04	0.03	0.06	0.06

Notes: Justiniano et al. - JPT(2010), Christiano et al. - CTW(2011), Altig et al. - ACEL(2011), Christiano et al. - CEE(2005). Shocks: Technology (Tec), Preference (Pref), Investment (Inv), Monetary Policy (MP), Price markup (PM), Wage markup (WM), Government spending (Govt).

Another crucial parameter for investment fluctuations is the persistence of the investment-specific shock, ρ. Figure 3 shows the impulse responses of output and investment to an investment shock in JPT (2010) for grid values of ρ between zero and one - using their posterior median values for κ and ζ. Not surprisingly, the larger the value of ρ is, the larger and more persistent the responses of output and investment are. It is worth noting that in models different from JPT the other parameters may also differ and, therefore, the IRFs may end up being closer than they appear here. Nevertheless, given the relevance of these parameters for explaining investment behavior and business cycle fluctuations, it is important to understand the diversity of estimates reported in the literature and investigate whether GMM can pin down the value of these parameters more accurately.

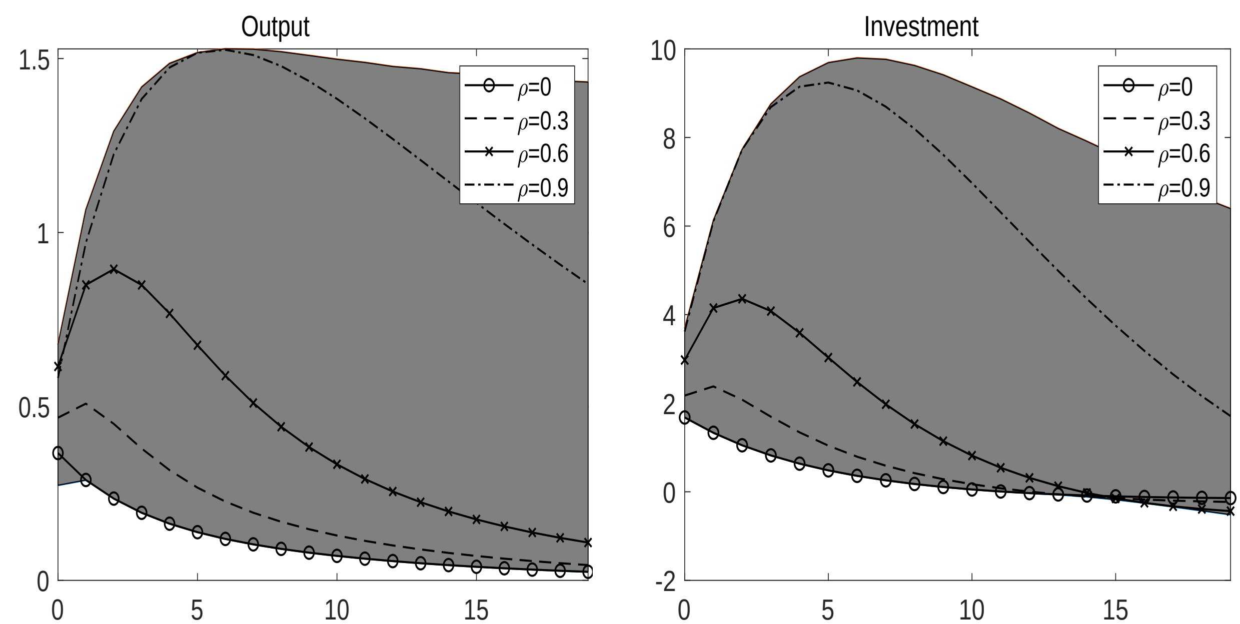

Figure 3: Impulse responses of output and investment to a one-standard deviation investment shock in JPT's model evaluated at the posterior median of all parameters except ρ. The shaded area shows the range of IRFs for $\rho \in [0, 1)$.

3 Econometric Methodology

This Section describes the methods we use to estimate the model given in equation (2). Without a complete specification of a DSGE model that includes equation (2), the expectations on its right-hand side are unobserved. We replace the expected future terms in (2) with their realized values and find valid instruments for them, which are typically predetermined variables. However, in this case, we first need to quasi-difference the equation to remove the autocorrelation in $\widetilde{\nu}_t$ that would otherwise rule out using predetermined variables as instruments. Specifically, removing expectations and quasi-differencing equation (2) yields

$$\begin{aligned}[1+\rho(\beta+\phi_q)]\Delta\widetilde{i}_t &= \rho\Delta\widetilde{i}_{t-1} + [\beta+\phi_q+\rho\beta\phi_q]\Delta\widetilde{i}_{t+1} - \beta\phi_q\Delta\widetilde{i}_{t+2} \\ &\quad + \frac{\phi_k\zeta}{\kappa}\widetilde{u}_{t+1} - \frac{\rho\phi_k\zeta}{\kappa}\widetilde{u}_t - \frac{1}{\kappa}\widetilde{r}^p_t + \rho\frac{1}{\kappa}\widetilde{r}^p_{t-1} + \epsilon_t, \end{aligned} \tag{3}$$

where ϵ_t is an error term defined in equation (CS 13) in Appendix A.1. The time series properties of the residual term ϵ_t are crucial for the selection of valid instruments. It can be gauged from equation (CS 13) that, under rational expectations, ϵ_t is a moving average process of order 2 that consists of current values of the investment adjustment cost shock ε^ν_t and current and future forecast errors of inflation, investment and capital utilization. Hence, it is orthogonal to any predetermined variables.[10]

We estimate the structural parameters in the baseline equation (3) using the generalized method of moments (GMM) framework proposed by Hansen and Singleton (1982), with orthogonality conditions obtained from the assumption that the residuals, ϵ_t, in the baseline equation (3) are uncorrelated with any predetermined variables Z_t. In our estimation, we set $\beta = 0.99$ and $\delta = 0.025$, and compute confidence sets for the remaining structural parameters $\theta = (\rho, \kappa, \zeta)$.

[10] Note that the error term ϵ_t will remain orthogonal to predetermined variables even if the investment specific shock $\widetilde{\nu}_t$ is only observed with a lag, i.e., at the beginning of period $t+1$, as can be gauged from the derivations in Appendix A.1. Moreover, since we observe u_t and r^p_t, we do not need to explicitly model shocks to capacity utilization and marginal utility. We thank an anonymous referee for raising those points.

Our econometric analysis relies solely on methods of inference that are robust to the presence of potential weak instruments, while allowing for heteroskedasticity and autocorrelation in the residuals. We estimate confidence sets based on the S test of Stock and Wright (2000). The S set is constructed as follows. We specify a grid of points within the parameter space. For each of these points, we test whether the identifying restrictions of the model hold using a Wald-type test (Stock and Wright, 2000, call this the S test). All the points in the grid that have not been rejected by a 10% level S test make up the 90% confidence S set.

In addition to the S sets, we also estimate confidence sets based on a test proposed by Magnusson and Mavroeidis (2014), the quasi-local level S (qLL-S) test. The qLL-S set combines the average information on the moment conditions over the sample, which is what the S set uses, with information on the validity of the moment conditions over subsamples. It can be thought of as using subsample information as additional instruments. This subsample information is relevant in two cases: (i) when the parameters of the model are unstable, or (ii) when the parameters of the model are constant but there is time variation in other parts of the economy, for example, monetary policy regime shifts. In case (i), the qLL-S test can be interpreted as a structural change test that is robust to weak identification; therefore a non-rejection is an indication of parameter stability. In case (ii), the qLL-S can have more power than the corresponding S test if the information that comes from structural change elsewhere in the economy is sufficiently strong. Hence, in either case, the qLL-S sets usefully complement the S sets.

The orthogonality condition $E_{t-1}(\epsilon_t) = 0$ in (3) implies that any predetermined variable could be used as an instrument. Therefore, the number of potential instruments is sizeable. However, the S and qLL-S sets may be unreliable if the number of instruments is large relative to the sample size. Therefore, we keep the number of instruments small when we compute S and qLL-S sets. This may be inefficient if the most informative instruments are excluded from the set of instruments, or if the information is spread over a large number of instruments. To address this possibility, we use a split-sample S set. This is an extension of a method recently proposed by Mikusheva (2021) to obtain reliable inference in linear instrumental variables models with time series data and a large number of possibly weak instruments. In Section B of the Appendix we present information about computation of the S, qLL-S and split-sample S tests.[11]

4 Data

We use quarterly aggregate time series data for the US over the period 1967q1 to 2019q4. We consider two proxies for Investment (I_t). One corresponds to Fixed Private Investment as in Smets and Wouters (2007) (SW, henceforth), while the other proxy is the sum of Gross Private Domestic Investment and Personal Consumption Expenditure on Durable Goods following JPT. Both investment measures are in real per capita terms and deflated using their respective implicit price deflators. For the nominal interest rate r_t, we use the quarterly average of the effective Federal Funds rate, and inflation π_t is obtained from the GDP deflator as $\pi_t = \log(P_t/P_{t-1})$. The ex-post real interest rate r_t^p is defined as $r_t^p = r_t - \pi_{t+1}$. For capital utilization (u_t), we use the Federal Reserve Board's time series on capacity utilization, which measures the intensity with which all factors of production are used in the industrial production sector (Christiano et al., 2005). When estimating equation (3), we use the log values of investment and capacity utilization, that is, $i_t = \log(I_t)$ and $u_t = \log(U_t)$.[12]

Other variables included in the set of predetermined/exogenous variables Z_t are Romer and Romer's monetary policy shock, Ramey and Zubairy's military news shock, oil price inflation and (financial) uncertainty measure VXO. Detailed description of the data, its sources and transformations are given in Appendix C.

[11] Mikusheva (2021) only explicitly discusses the case of linear moment conditions. It is straightforward to extend her proposal to nonlinear GMM, see Appendix B.2 for details.

[12] In equation (3) the variables appear with a tilde, that is, in log-deviations from steady state. In computing the tests, we collect all the steady state terms in the constant term included in that equation.

5 Results

This Section first presents the results of our baseline estimation. Then, we report results based on external instruments, followed by results obtained from combining both lagged endogenous and exogenous instruments together and employing the split-sample method proposed by Mikusheva (2021). Finally, we define a semi-structural model to estimate the slope coefficients of the investment equation with respect to the capital utilization rate and the real interest rate.

5.1 Baseline Estimation

We begin by investigating the baseline specification equation (3) using one lagged value of the variables that appear in the model as the set of instrumental variables, namely Δi_{t-1}, r^p_{t-2}, and u_{t-1}. We keep the number of instruments small to avoid problems associated with the use of many instruments, see Andrews and Stock (2007). We estimate the model using both the SW and the JPT definitions of investment discussed in Section 4. Three-dimensional confidence sets for the parameters $\theta = (\rho, \kappa, \zeta)$ are obtained by considering the ranges $[0, 1) \times (0, 20] \times (0, 10]$ in line with previous studies, see Figure 1. The results are reported in Figure 4.

Figures 4 (a) and (b) report 90% S sets for the parameters ρ, κ, ζ using SW and JPT investment proxies, respectively. In both cases, the S sets comprise almost the entire parameter space. The only part rejected by the data are small values of κ and ζ when $\rho < 1$. The good news from this result is that the investment equation is not rejected by the aggregate data. Nevertheless, the data is essentially uninformative over a very large part of the parameter space. The results remain essentially unchanged when considering two lags instead of one lag for each of the three instruments (see Figure S.1 in Appendix D). Additionally, the qLL-S sets reported in Figures 4 (c) and (d) are very similar to the S sets, indicating no presence of parameter instability or violation of the moment conditions in subsamples.

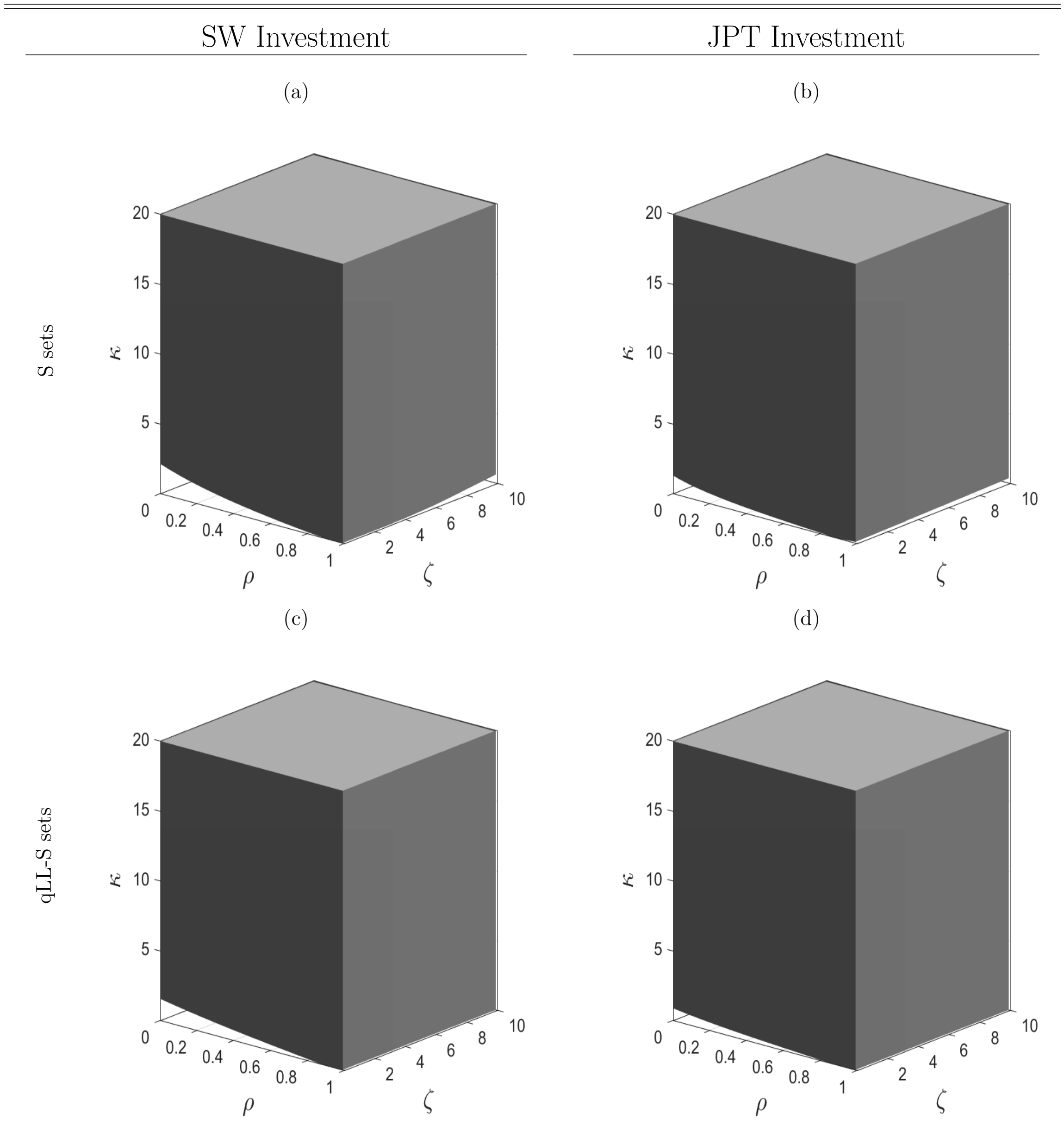

Figure 4: 90% S and qLL-S confidence sets for $\theta = (\rho, \kappa, \zeta)$ derived from the investment Euler equation model (3). Instruments: constant, Δi_{t-1}, r^p_{t-2}, u_{t-1}. The investment proxies are Fixed Private Investment (left column) and the sum of Gross Private Domestic Investment and Personal Consumption Expenditure on Durable Goods (right column). Newey and West (1987) HAC. Period: 1967Q1-2019Q4.

Hence, there is minimal information arising from the restrictions on the dynamics of the data to identify the investment equation. As a consequence of the lack of identification, all previous parameter estimates reported in Figure 1 are included in the confidence sets, that is, those estimates are potentially valid values of the true underlying structural parameters of the investment Euler equation (3).

5.2 External Instruments

Given the findings in Figure 4, we explore a more extensive set of information contained in contemporaneous external instruments. These instruments include (i) the monetary policy shock of Romer and Romer (2004), (ii) the military news shock of Ramey (2011, 2016) and updated by Ramey and Zubairy (2018), which captures news about changes in military spending, (iii) changes in the (log) oil price, and (iv) the (standardized) S&P 100 Volatility Index (VXO), which is a proxy for (financial) uncertainty shocks.[13] Some of the external instruments do not cover the entire period. We, therefore, use the longest available sample when estimating the confidence sets. Apart from oil price, the external instruments are in levels. We use the contemporaneous values as instruments.

To keep the number of instruments comparable to Figure 4, we report results in which r^p_{t-2} is replaced by an external instrument, or r^p_{t-2} and u_{t-1} are replaced by a pair of the external instruments. For completeness, we also report results using all of the external instruments together.[14] The resulting S and qLL-S confidence sets are reported in Figures 5 and 6 for the SW and JPT investment proxies, respectively. To facilitate comparison across cases, we report the baseline results at positions (a) and (i) in those figures.

In almost all cases, the results are very similar to Figure 4 which used only lagged

[13] Another related potential external instrument is overall macroeconomic uncertainty as studied by Jurado et al. (2015); however, Ludvigson et al. (2021) point out that macroeconomic uncertainty responds to business cycle fluctuations making it an endogenous variable, while they suggest that financial uncertainty is exogenous.

[14] In Appendix D, we report results in which the external instruments are added together with r^p_{t-2} and u_{t-1} in the set of instruments. The results, which are reported in Figures S.3 and S.4, are very similar to the ones found in Figures 5 and 6 in this section.

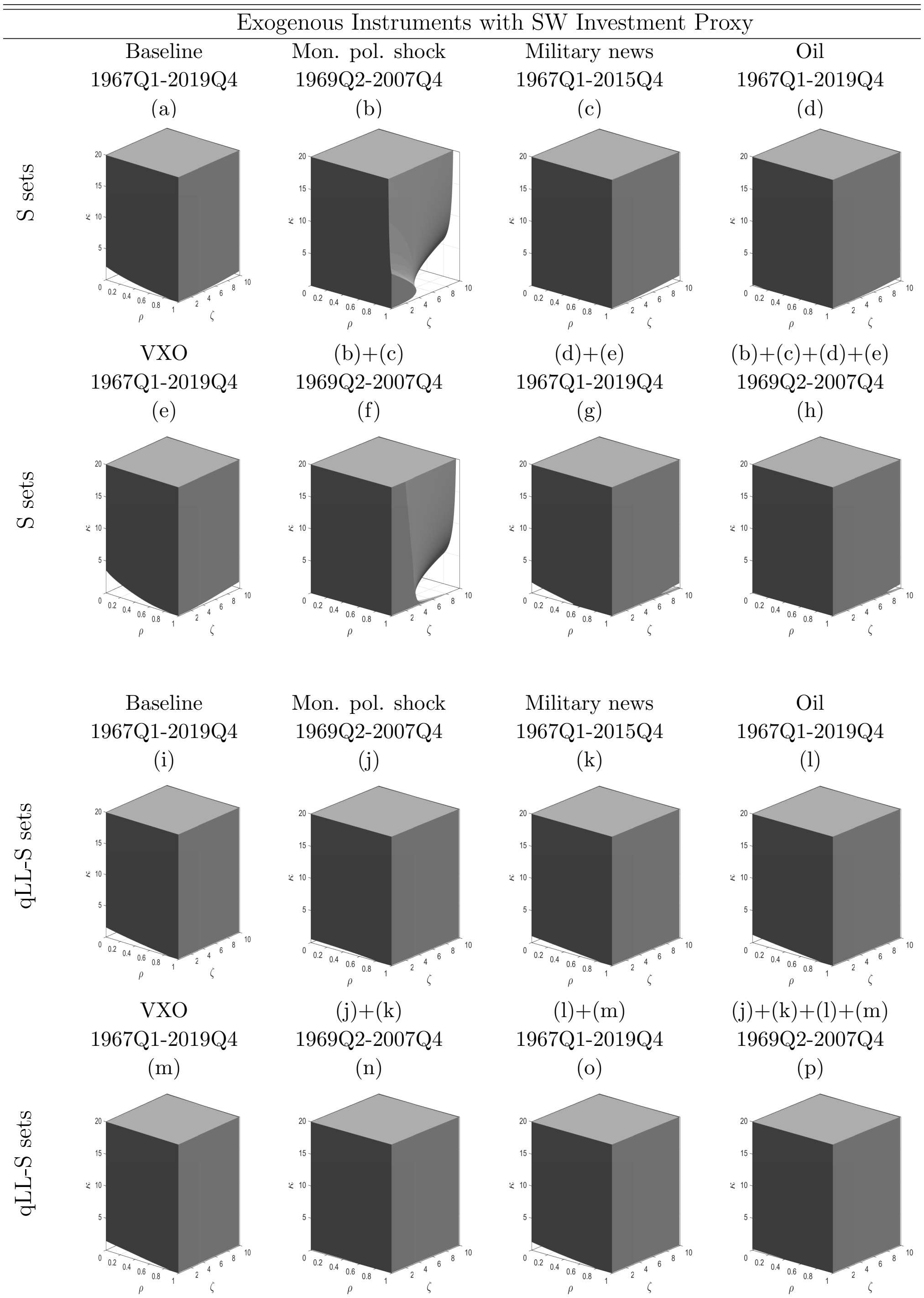

Figure 5: 90% S and qLL-S confidence sets for $\theta = (\rho, \kappa, \zeta)$ derived from the investment Euler equation model (3) using Fixed Private Investment as investment proxy. A constant and Δi_{t-1} are common instruments in all specifications while u_{t-1} enters only in specifications (a) to (e) and (i) to (m). The additional instrument(s) by specification is (are): Baseline r^p_{t-2}; Mon. pol. shock: Romer and Romer's (2004) monetary policy shock; Military news: Ramey and Zubairy's (2018) military news shock; Oil: growth rate of real oil price; VXO: financial uncertainty.

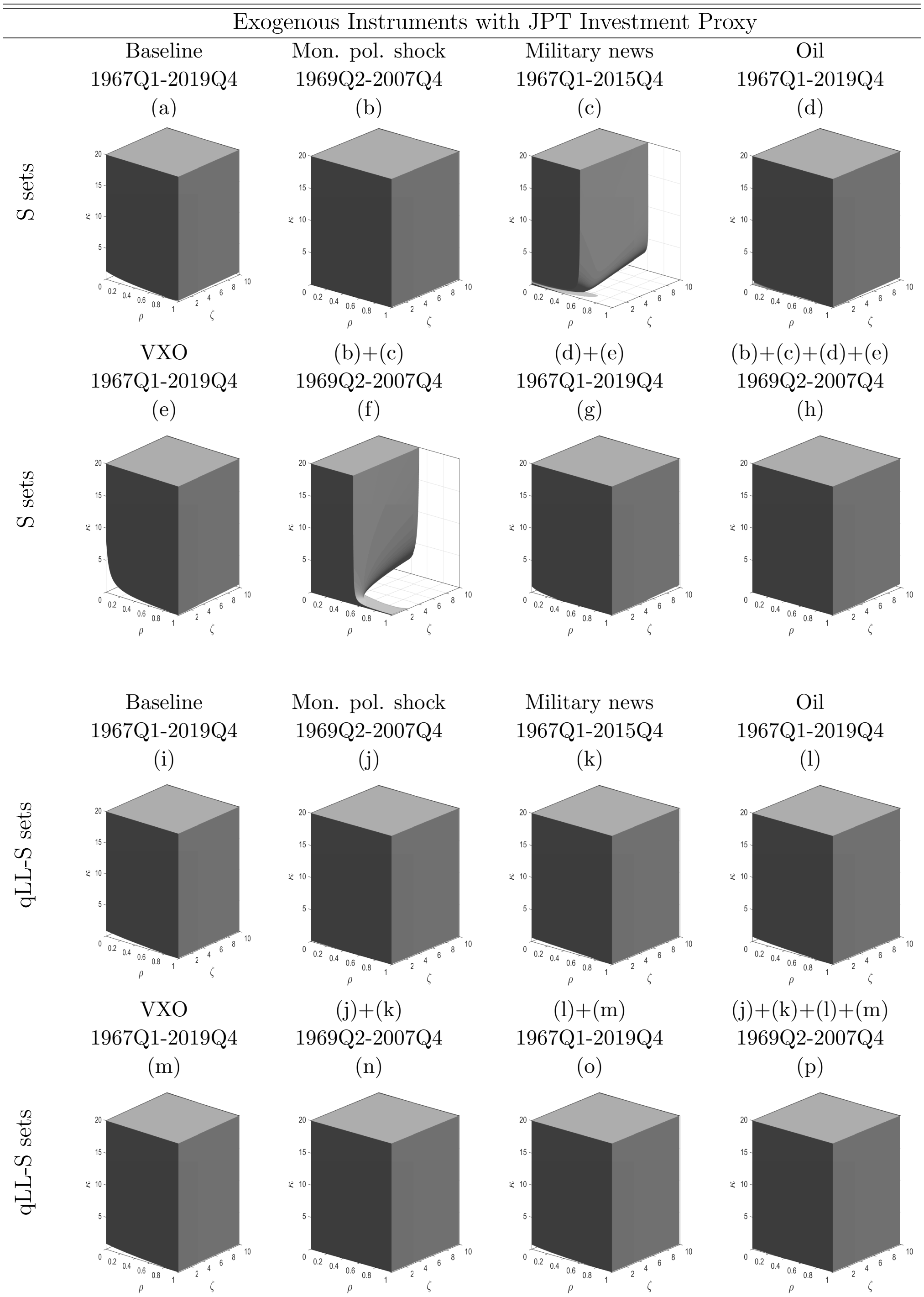

Figure 6: 90% S and qLL-S confidence sets for $\theta = (\rho, \kappa, \zeta)$ derived from the investment Euler equation model (3) using the sum of Gross Private Domestic Investment and Personal Consumption Expenditure on Durable Goods as investment proxy. A constant and Δi_{t-1} are common instruments in all specifications while u_{t-1} enters only in specifications (a) to (e) and (i) to (m). The additional instrument(s) by specification is (are): Baseline r^p_{t-2}; Mon. pol. shock: Romer and Romer's (2004) monetary policy shock; Military news: Ramey and Zubairy's (2018) military news shock; Oil: growth rate of real oil price; VXO: financial uncertainty.

variables as instruments. The specifications which include monetary policy shocks in the set of instruments for the SW investment measure, as shown in Figures 5 (b) and (f), result in a slight reduction of the S confidence sets. The reduction is somewhat more sizable for the specifications which include military news shocks in the set of instruments when using the JPT investment proxy. In Figures 6 (c) and (f) the resulting S sets are roughly 40% smaller than the rest of the S sets. Nevertheless, even in this case, the parameters κ and ζ remain very weakly identified. The main implication of using the military news shock is that values of $\rho > 0.6$ can be rejected, which contradicts the findings in SW and JPT.[15] Therefore, contemporaneous information from arguably exogenous instruments does not seem to help identify the parameters of the investment equation. Additionally, as before, there is no evidence of parameter instability or violations of the moment conditions over subsamples.

5.3 Combining all the Instruments

The S and qLL-S tests exploit information arising from only a handful of instruments at a time, even though equation (3) implies a large number of potential instruments, because they become unreliable when the number of instruments is large relative to the sample size. However, the use of many instruments could potentially sharpen our inference if information is spread thinly among them. To study this possibility, we compute split-sample S sets. Ideally, we would like to combine all the instruments that we have used so far in a single estimation, but, because of data limitations with the external instruments, we do two separate estimations instead. The first uses four lags of the three instruments Δi_t, r^p_{t-1} and u_t, which are available over our full sample period. The second estimation adds to the aforementioned lagged instruments the external instruments that are available over a shorter sample. In both cases, we use approximately the first half of the sample to estimate the first-stage regression coefficients and the second half to compute the test statistic, as explained in Appendix B.2.

Figures 7 (a) and (b) report the results of the split-sample S confidence sets for (ρ, κ, ζ)

[15]The 90% credible interval for ρ in SW and JPT is roughly between 0.60 and 0.80. The same interval is between 0.47 and 0.76 in Inoue et al. (2020).

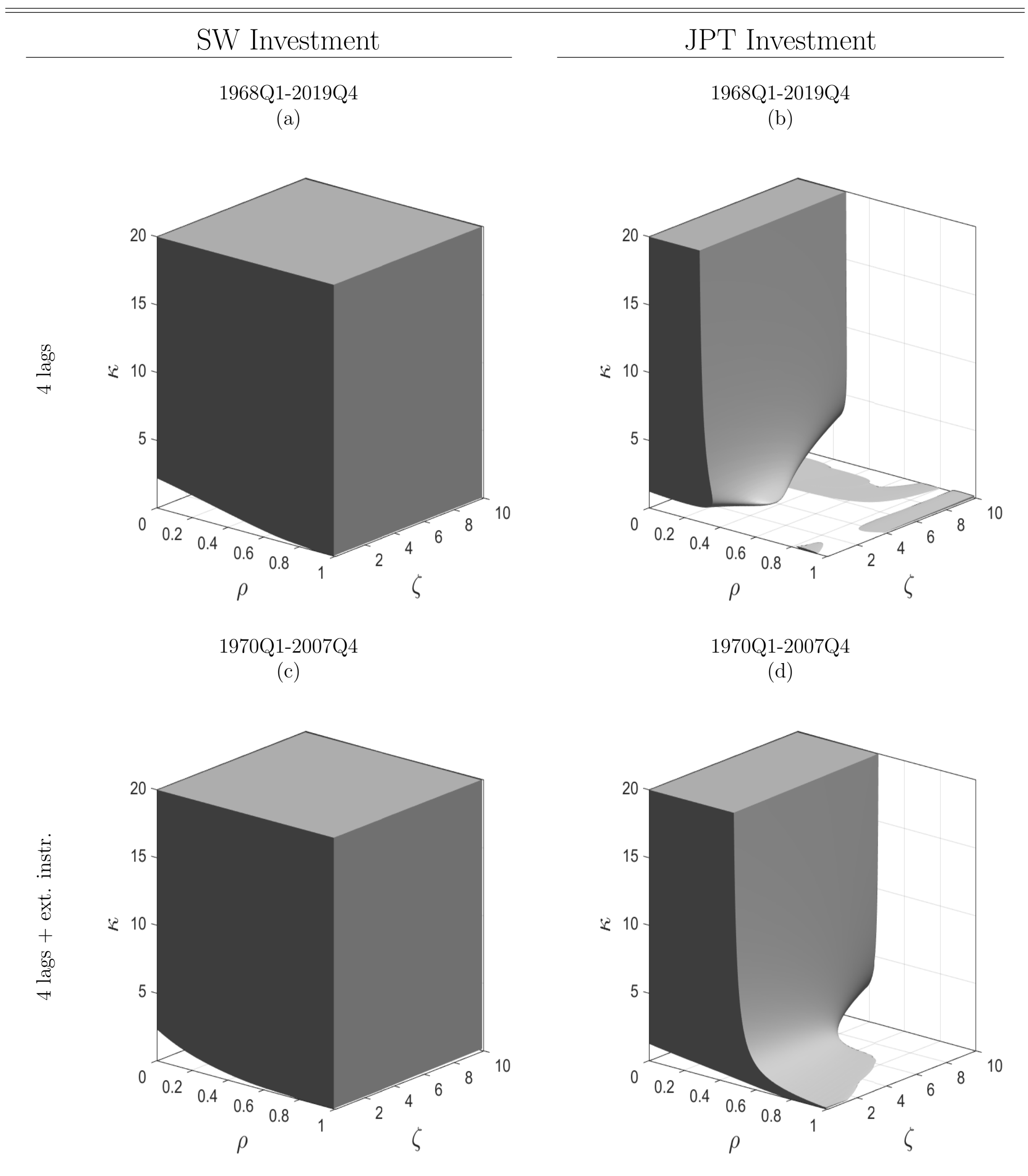

Figure 7: 90% Mikusheva split-sample S confidence sets for $\theta = (\rho, \kappa, \zeta)$ derived from the investment Euler equation model (3). Instruments - 4 lags: $\left\{\Delta i_{t-j}, r^p_{t-1-j}, u_{t-j}\right\}_{j=1}^{4}$. 4 lags + ext. inst.: additional instruments are Romer and Romer's (2004) monetary policy shock; Ramey and Zubairy's (2018) military news shock, the growth rate of real oil price, and the financial uncertainty (VXO). A constant is included in all specifications. The investment proxies are Fixed Private Investment (left column) and the sum of Gross Private Domestic Investment and Personal Consumption Expenditure on Durable Goods (right column). Newey and West (1987) HAC.

with the lagged instruments only, using the SW and JPT investement data, respectively. The results with the SW data are essentially the same as before: very weak identification of all three parameters. The results with the JPT data are somewhat more informative than before. The size of the split-sample S confidence set is less than a third of the S and qLL-S sets reported in Figures 4(b) and (d). However, the confidence sets contain most of the values of κ and ζ, so most of the information gains affect only the parameter ρ: values greater than 0.5 are effectively excluded from the split-sample confidence set. This is consistent with the results reported in Figure S.1 in Appendix D which compares the results using one and two lags of the instruments. The addition of more lags shrinks the JPT confidence sets but only in terms of ρ, and in the same direction as in Figure 7.

Figures 7 (c) and (d) perform the same exercise but adding the external instruments used in Figures 5 and 6, respectively. Note that the smaller sample size relative to Figures 7 (a) and (b) means that the results are not directly comparable - the combined larger instrument set can be more informative, but the smaller estimation sample makes inference less precise. Nevertheless, the pictures look very much in line with the results reported earlier. With SW investment data, the confidence set is entirely uninformative, exactly as in Figures 4 (a) and (c), and Figure 5 above. With JPT investment data, the split-sample S confidence set is about half the size of the sets in Figures 4 (b) and (d), with all the extra information affecting only the persistence parameter ρ. Interestingly, the confidence set in Figure 7 (d) is quite similar to the S set reported in Figure 6 (c) that uses military news as a single external instrument, suggesting that military news may be the most informative of all the instruments we have considered. Still, identification of κ and ζ remains very weak. Therefore, we conclude that the previous uninformative confidence sets were not due to the use of a limited selection of the available instruments, and that the investment equation is genuinely weakly identified.

5.4 Semi-structural Model

Finally, to better understand why the structural parameters are so poorly identified beyond the weak instruments issues discussed above, we study a semi-structural model. Note that we can re-write (3) as

$$\begin{aligned}
\left[1+\rho\left(\beta+\phi_q\right)\right]\Delta\widetilde{i}_t-\rho\Delta\widetilde{i}_{t-1}-\left[\beta+\phi_q+\rho\beta\phi_q\right]\Delta\widetilde{i}_{t+1}+\beta\phi_q\Delta\widetilde{i}_{t+2}\\
=\varphi\left(\widetilde{u}_{t+1}-\rho\widetilde{u}_t\right)-\phi\left(\widetilde{r}_t^p-\rho\widetilde{r}_{t-1}^p\right)+\epsilon_t,
\end{aligned} \tag{4}$$

where $\varphi=\frac{\phi_k\zeta}{\kappa}$ and $\phi=\frac{1}{\kappa}$ are reduced-form parameters, for which we consider the ranges $[0,10]$ and $[0,20]$, respectively.

For ease of exposition, we construct two-dimensional confidence sets for (φ,ϕ) for fixed values of ρ at 0.0, 0.6, 0.8, and 0.9. We estimate (4) using the same set of instruments and sample period as in Figure 4. Figure 8 plots the resulting two-dimensional confidence sets for (φ,ϕ).

The figure suggests that, when ρ is between 0 and 0.6, φ and ϕ are well-identified with their respective confidence intervals sitting tightly around 0. Looking at equation (4), this suggests limited responsiveness of investment growth to changes in capital utilization and the real interest rate. This implies that, in addition to the weak identification issues discussed above, the mapping from reduced-form to structural parameters is also resulting in unbounded confidence sets for the latter ones, even when the former ones are well-identified. For instance, despite ϕ being well identified when $\rho=0$ or $\rho=0.6$, its reciprocal κ is not, since the confidence interval for ϕ includes 0.[16] Similarly, even though both φ and ϕ are well identified when the investment-specific shock is moderately persistent, the structural parameter ζ is not, since ϕ_k (which is calibrated in our earlier exercises) is very small, therefore resulting in poor identification of ζ.[17]

[16] So that, for example, values of ϕ between 0 and 0.2 imply possible values of κ between five and infinity.

[17] ϕ_k, which depends on β and δ as defined in (CS 10) in Appendix A.1, is 0.0348 given the fairly standard calibration $\beta=0.99$ and $\delta=0.025$.

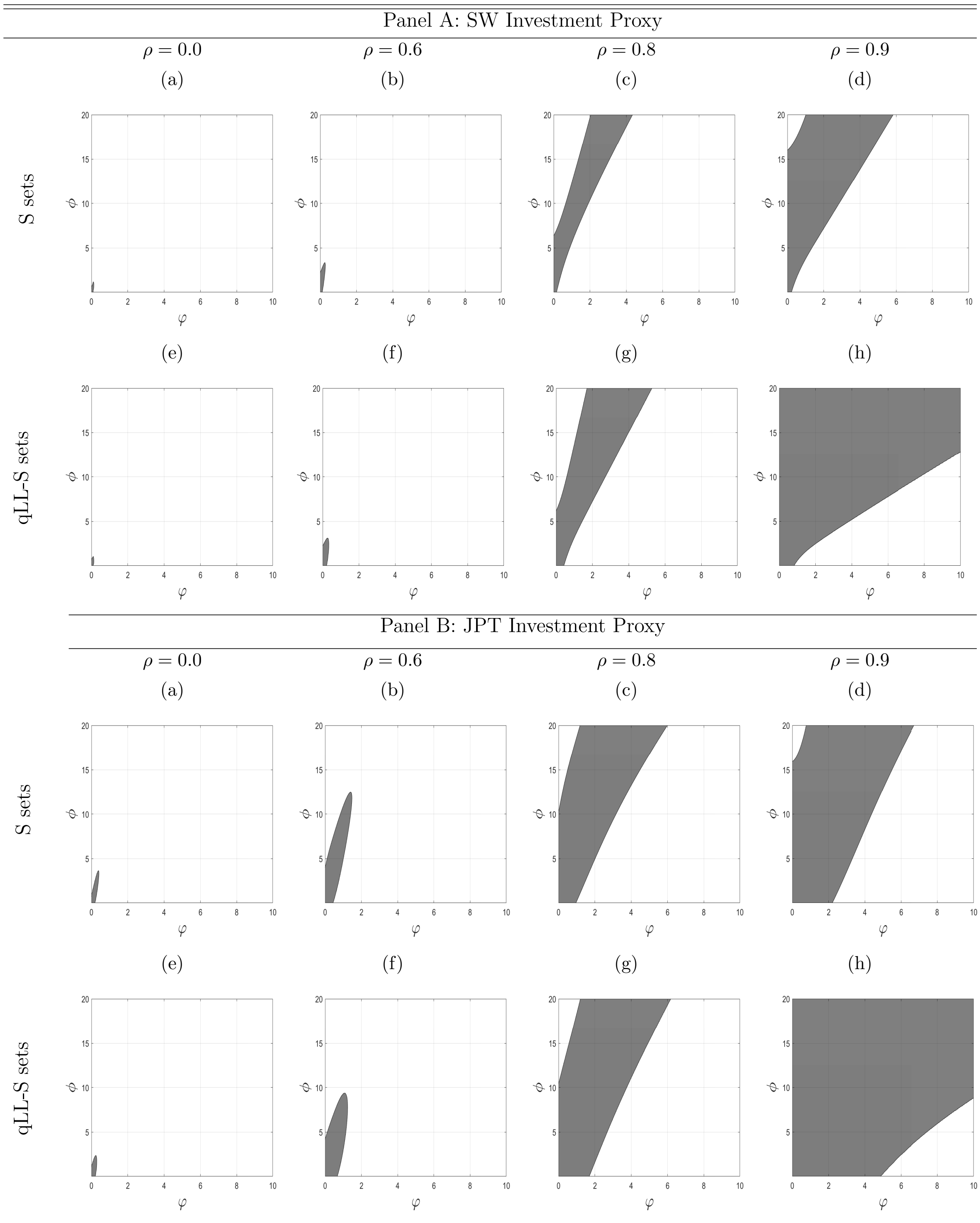

Figure 8: 90% S and qLL-S confidence sets for (φ, ϕ) derived from the investment Euler equation model (4) with Fixed Private Investment and the sum of Gross Private Domestic Investment and Personal Consumption Expenditure on Durable Goods as investment proxies (Panels A and B, respectively). Instruments: a constant, Δi_{t-1}, r^p_{t-2} and u_{t-1}. Newey and West (1987) HAC. Period: 1967Q1-2019Q4.

Table 2: Values of the reduced-form parameters ϕ and φ implied by the estimates of κ and ζ from Figure 1

	ϕ	φ
CEE (2005)	0.40	0.0001
SW (2007)	[0.13-0.25]	[0.003-0.02]
JPT (2010)	[0.26-0.48]	[0.03-0.12]
ACEL (2011)	0.67	0.26
CTW (2011)	[0.05-0.10]	[0.0003-0.002]
CMR (2014)	0.09	0.008
AABC (2020)	[0.18-0.35]	[0.007-0.01]
IKR (2020)	[0.30-0.67]	[0.04-0.15]

Notes: As in Figure 1, the labels refer to the following papers: Christiano et al. - CEE(2005), Smets and Wouters - SW(2007), Justiniano et al. - JPT(2010), Altig et al. - ACEL(2011), Christiano et al. - CTW(2011), Christiano et al. - CMR(2014), Arias et al. - AABC(2020), Inoue et al. - IKR(2020)

As for the sake of comparison, Table 2 shows the estimates of the well-identified reduced-form parameters (φ, ϕ) implied by the estimates of the elasticity of capital utilization cost (ζ) and the investment adjustment cost (κ) in Figure 1. As explained above, because of the mapping from reduced-form to structural parameters, the large differences in the structural parameters does not translate into large differences in the reduced-form ones, whose implied values are consistent with our semi-structural estimation.

To understand why identification of the semi-structural parameters φ and ϕ worsens for higher values of ρ, we look at the autocorrelations of r_t^p and u_{t+1}. The first and second autocorrelations of r_t^p in our baseline sample are 0.90 and 0.83 respectively, while the same autocorrelations for u_{t+1} are 0.96 and 0.87, indicating that both series are highly persistent. Therefore, for small values of ρ, the instruments r_{t-2}^p and u_{t-1} are highly correlated with the endogenous variables of the system, resulting in better identification. In contrast, as the value of ρ increases, $(u_{t+1} - \rho u_t)$ and $(r_t^p - \rho r_{t-1}^p)$ in equation (4) resemble white-noise processes, making lagged endogenous variables weak instruments. This is illustrated in Figure 9 which is a scatter plot of the fitted values of the endogenous regressors $(\phi_k u_{t+1} - \rho \phi_k u_t)$ and $(r_t^p - \rho r_{t-1}^p)$ from their respective first-stage regressions on the baseline set of instruments used to construct Figures 4 and 8. The left column, which plots the results for $\rho = 0$, shows that the instruments are relatively strong as the fitted values mostly lie along the 45-degree line. In contrast, for high values of ρ, as in the right column which plots the results for $\rho = 0.8$, the values of both $(\phi_k u_{t+1} - \rho\phi_k u_t)$ and $(r_t^p - \rho r_{t-1}^p)$ are tightly packed around zero and are uncorrelated with the fitted values from the first-stage regressions, suggesting that instruments are weak.

6 Implications for DSGE Models

In this Section, we discuss the main implications of our analysis. In the Introduction, we argue about the importance of using the GMM methodology to assess the empirical fit in aggregate data of the investment equation commonly used in DSGE models. Our results support the investment equation, in the sense that there is no evidence against it. Moreover, we ask whether the GMM methodology could shed some light on the values of the parameters of this equation given the wide range found in the literature. Unfortunately, the answer is negative, because these parameters are weakly identified. This raises two questions. The first relates to the sensitivity of the DSGE results to changes in the three main parameters of interest: are any of these parameters relatively more important for the dynamic properties of a standard DSGE model, and thus relatively more important to pin down? Second, why have these parameters been estimated more precisely with Bayesian estimation of fully-specified DSGE models such as JPT or SW?

6.1 Sensitivity of DSGE Dynamics to the Parameters of the Investment Equation

Figures 10-11 mimic previous Figure 2. The figures show how the impulse responses of output and investment to the seven structural shocks in JPT's model change when κ and ζ take values in the 90% baseline S confidence set, while the remaining parameters are fixed

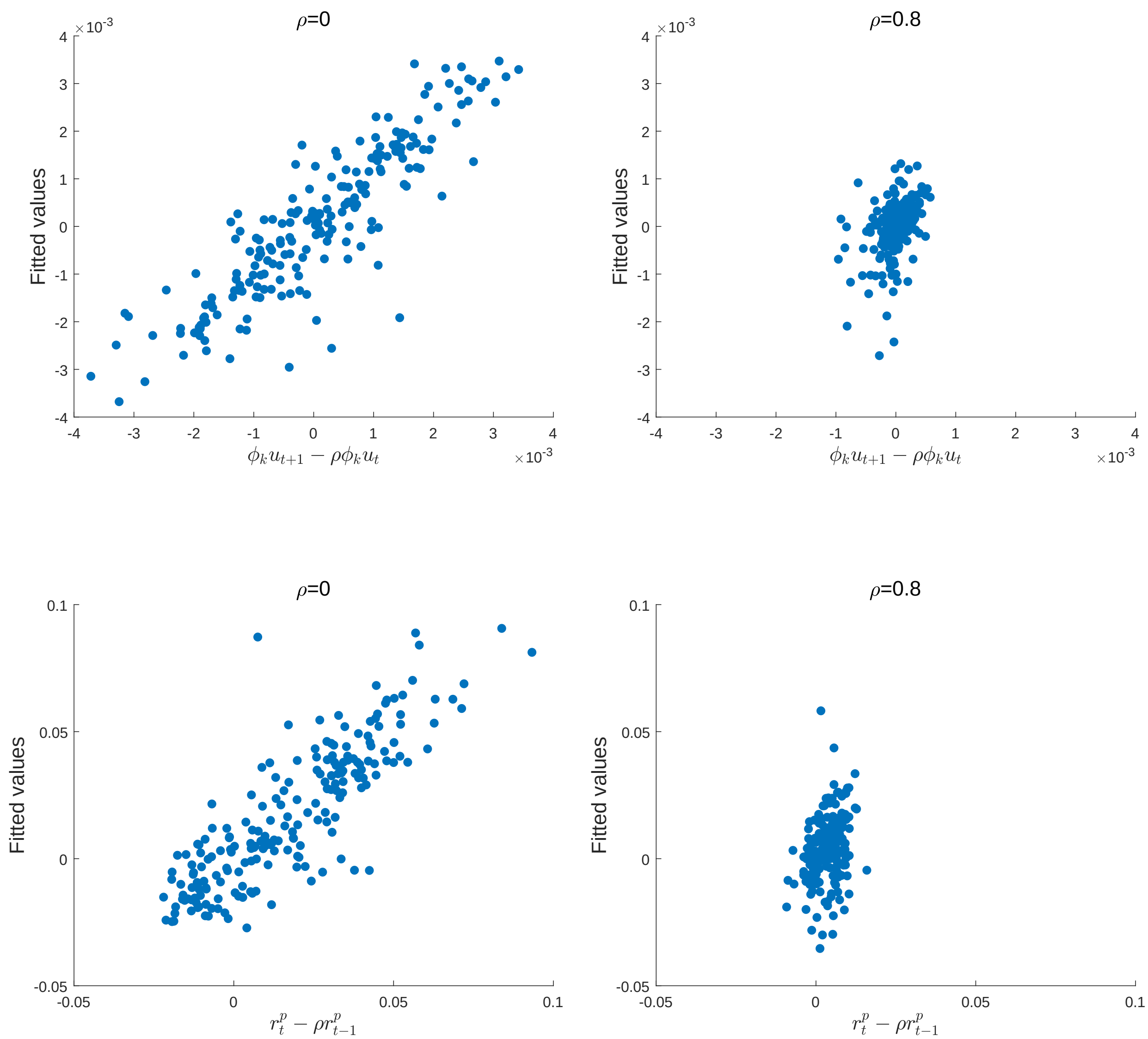

Figure 9: Scatter plot of the fitted values of the endogenous regressors $(\phi_k u_{t+1} - \rho\phi_k u_t)$ and $(r_t^p - \rho r_{t-1}^p)$ from their respective first-stage regressions on the baseline instruments for two different values of ρ.

at the posterior median of JPT, including ρ which is set to 0.72. To highlight the relative sensitivity of the model dynamics to the values of κ and ζ, in both figures the dark shaded areas visualize all the possibilities when both κ and ζ vary in the 90% S confidence set. In Figure 10, the light shaded areas depict all the possibilities when κ ranges in its 90% S confidence set, while ζ is set to the JPT posterior median ($\zeta = 5.30$), while, in Figure 11, the light shaded areas depict all the possibilities when ζ ranges in its 90% S confidence set, while κ is set to the JPT posterior median ($\kappa = 2.85$). Comparing the two figures, it is clear that the IRFs are very sensitive to the value of κ, while they are insensitive to the value of ζ. With the sole exception of the IRFs to a government spending shock (and marginally the wage markup shock), different values of ζ within the 90% S confidence set change the IRFs only slightly, while most of the variation is due to the different values of κ. This is particularly true for the investment-specific shock that directly impacts the investment equation, as well as for the technology and the preference shocks, the other two shocks that in these models are usually found to be the other main drivers of business cycle fluctuations.

Figure 12 uncovers the same message by looking at the forecast error variance decomposition, similarly to Table 1. This figure shows how the variance decomposition of output growth changes when κ and ζ vary in the 90% baseline S confidence set, and, as before, obtained by varying each parameter at a time keeping the other fixed at the JPT posterior median. Even more striking than for the IRFs, almost all the variation in the variance decomposition is due to changes in the values of κ, while the role of ζ is negligible.

Together with Figure 3, these results suggest that κ and ρ are the key parameters that shape the dynamics of investment in a standard medium-scale DSGE model. The former defines the dynamic response of investment (and, consequently, of output) to the different shocks, and, more prominently, to the investment shock. A high value of ρ is fundamental to determine the persistent response of investment to the investment shock. In contrast, ζ does not seem very relevant for determining the dynamic response of investment and output to different structural shocks.

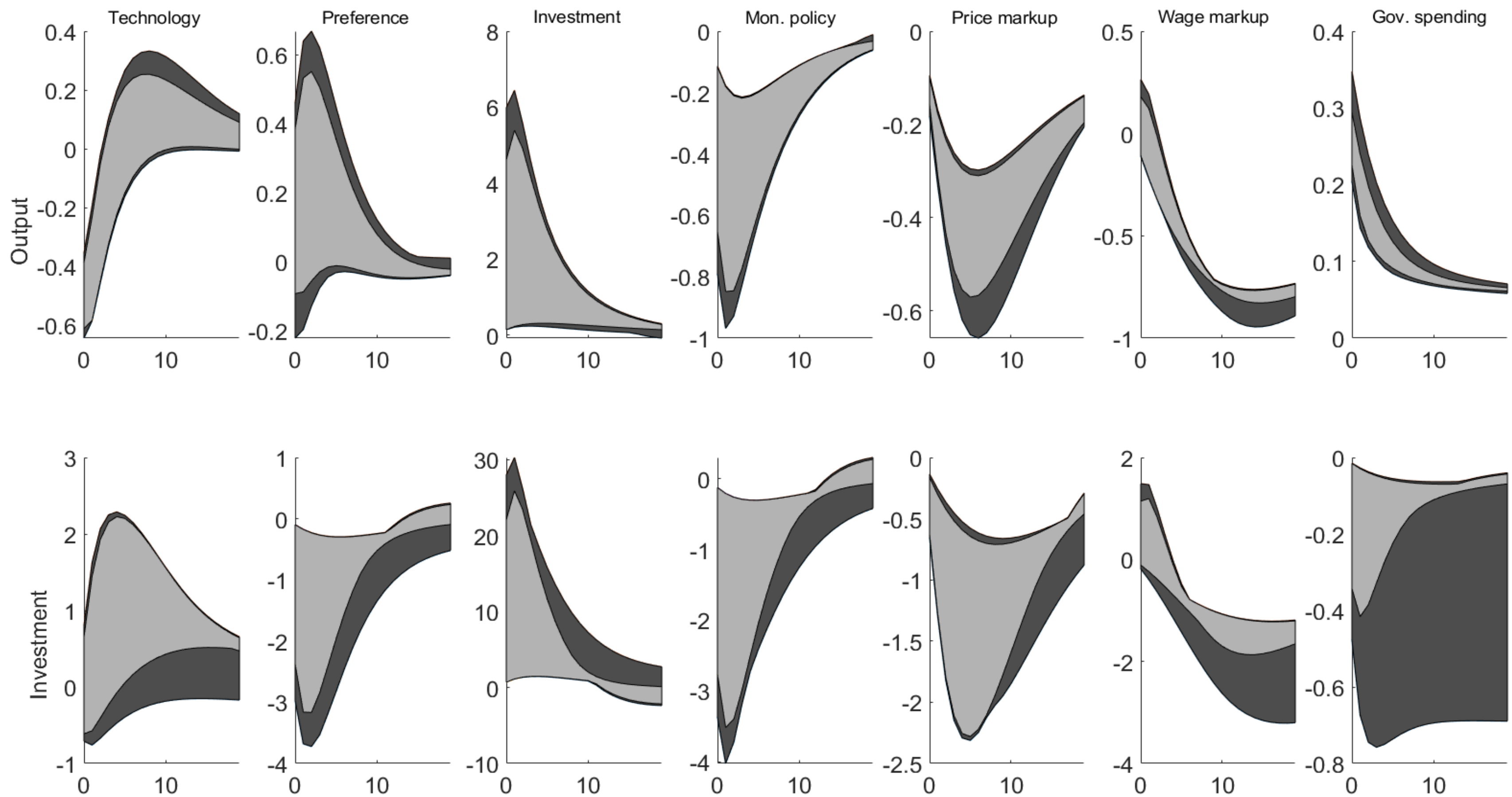

Figure 10: Impulse responses of output and investment to a one-standard deviation structural shock in JPT's model evaluated at the posterior median estimate of all parameters except κ and ζ. Dark shaded areas: both κ and ζ vary in the 90% S confidence set; light shaded areas: only κ vary and $\zeta = 5.30$ (posterior median estimate of JPT).

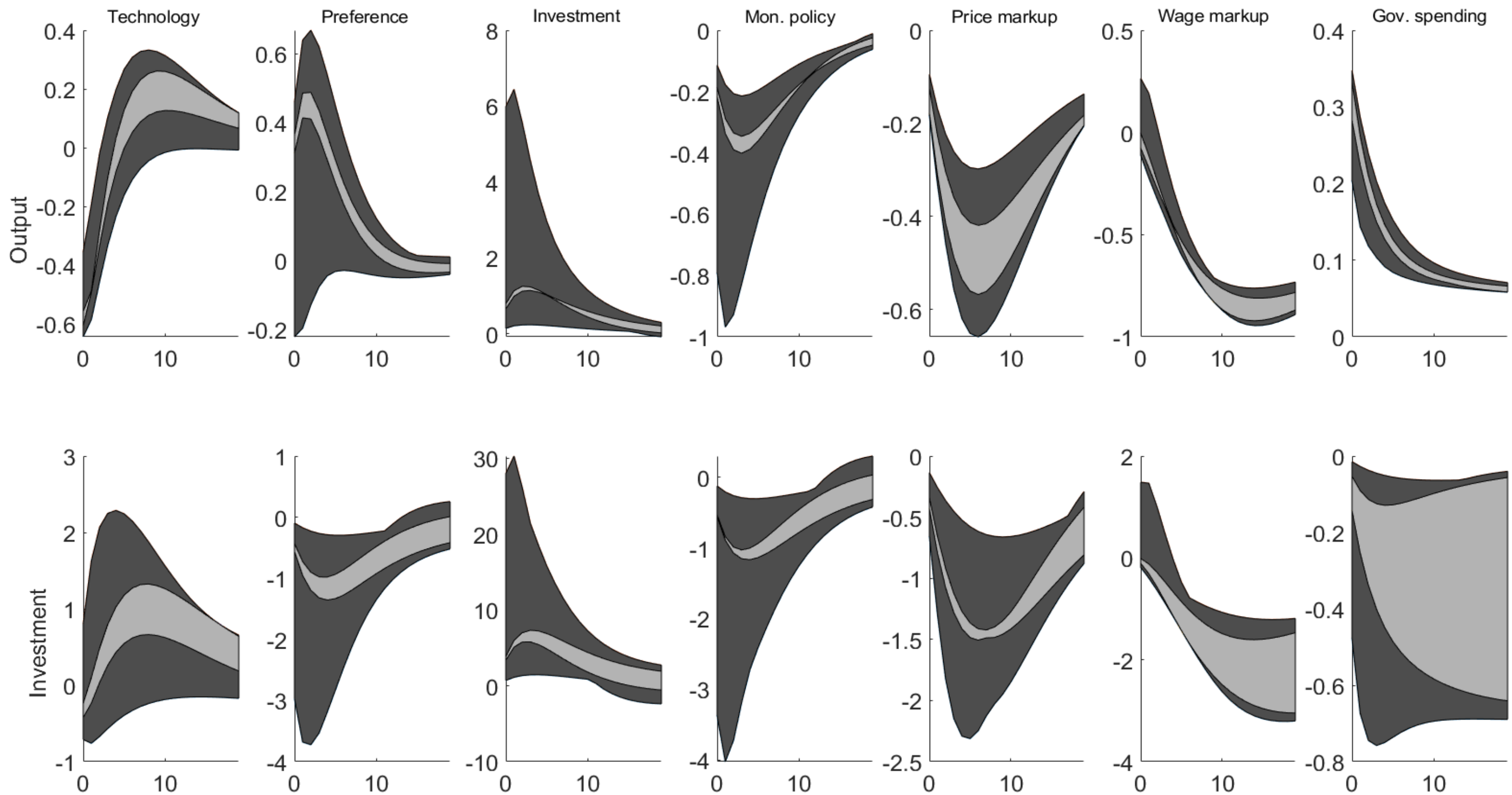

Figure 11: Impulse responses of output and investment to a one-standard deviation structural shock in JPT's model evaluated at the posterior median estimate of all parameters except κ and ζ. Dark shaded areas: both κ and ζ vary in the 90% S confidence set; light shaded areas: only ζ vary and $\kappa = 2.85$ (posterior median estimate of JPT).

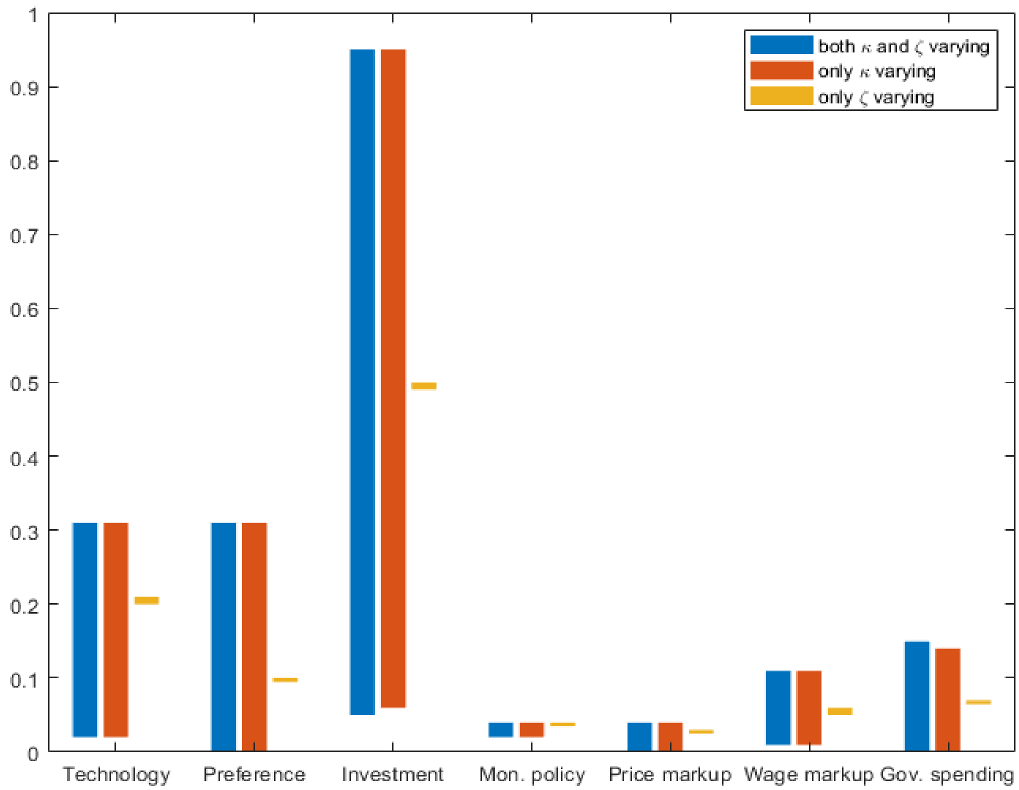

Figure 12: Forecast error variance decomposition of output growth in JPT's model evaluated at the posterior median estimate of all parameters except κ and ζ. Blue bars: both κ and ζ vary in the 90% S confidence set; orange bars: only κ vary and $\zeta = 5.30$ (posterior median estimate of JPT); yellow bars: only ζ vary and $\kappa = 2.85$ (posterior median estimate of JPT).

6.2 Estimation uncertainty in the parameters of the Investment Equation in DSGE Models

In this subsection we study how DSGE models estimated with Bayesian methods obtain more precise estimates of the investment equation parameters than our limited-information estimates previously reported. In comparison with our GMM single-equation approach, the DSGE system-based Bayesian estimation exhibits two main features that can increase estimation precision. The first is the informativeness of the priors. The second is the combination of the joint model dynamics of the variables in the system and the related cross-equation restrictions implied by rational expectations.[18] We would like to distinguish the role of these two features.

Prior informativeness First, we assess the relative importance of the prior for our parameters of interest. Müller (2012) derives a measure of prior informativeness that accounts for the high dimensional interaction between prior and likelihood information. Let Σ_p and Σ_π be the prior and posterior covariance matrices, respectively. Müller (2012) argues that a prior is of "limited overall informativeness" if the maximum eigenvalue of $J := \Sigma_\pi \Sigma_p^{-1}$ is less than unity. To gain some intuition, suppose θ is a scalar. Then the prior is of limited informativeness if the prior variance is higher than the posterior variance. As an illustration, Müller (2012) applies his method to the SW model and finds that the largest eigenvalue of J is 1.25 which implies that the prior is not of limited overall informativeness. Applying the same procedure to JPT, we find that four of the eigenvalues of J are greater than one. Thus, by Müller's measure, the prior in both cases is very informative.

Cross-equation restrictions Given a certain prior, one could investigate the role of cross-equation restrictions in reducing estimation uncertainty by comparing the posteriors of (κ, ζ, ρ) from the baseline DSGE model against the posteriors one would obtain after relaxing all the relevant cross equation restrictions. However, this is difficult for a couple of reasons. First, there are many restrictions in the DSGE model, both on the dynamics (e.g., only lags of inflation appear in the Phillips curve) and on the correlations of the structural shocks. In general, it is not possible to anticipate which of these will be relevant for the parameters of the investment equation. Second, relaxing all of these restrictions requires many additional unknown parameters, and, therefore, the estimation of the unrestricted model will likely be marred with identification problems leading to unreliable results. We are not aware of any method for formally comparing full- versus limited-information Bayesian estimation without

[18] Note that full-information restrictions may reduce estimation uncertainty at the cost of lack of robustness to misspecification of other parts of the system. See Appendix F for an illustration of this point.

running into the above issues. Thus, we will consider two informal approaches.

The first approach involves relaxing only *some* of the cross equation restrictions, in particular, the orthogonality of the structural shocks of the baseline DSGE, to hopefully alleviate some of the concerns regarding identification of the unrestricted model discussed in the previous paragraph. The prior for the correlations of the shocks is Gaussian with zero mean and standard deviation equal to 0.3, and all other priors are the same as in the baseline specification of JPT.[19]

The second approach involves estimating the DSGE-VAR model proposed by Del Negro and Schorfheide (2004). This combines the likelihood and prior of the baseline DSGE model with those of an unrestricted reduced-form VAR using a scalar parameter $\lambda > 0$ that measures the weight placed on the DSGE model. Del Negro and Schorfheide (2004) show that when λ approaches zero the estimated model is close to an unrestricted VAR, while when $\lambda \to \infty$ estimated model coincides with the baseline DSGE model. Unfortunately, this is not exactly what we are after, because we would like to relax only the restrictions in all the DSGE equations *except* the investment equation. Nonetheless, we also report the posteriors (κ, ζ, ρ) from two DSGE-VAR models with $\lambda = 1$ and $\lambda = 5$, as this is a way to indirectly gauge the role of the DSGE restrictions on the dynamics.

The results are reported in Figure 13 separately for each of the parameters κ, ζ and ρ. Each panel in the Figure exhibits the original (informative) prior (grey solid line) and the posterior from the baseline model (black solid line), as well as three additional posteriors: the posterior obtained by allowing the structural shocks of the model to be correlated (dashed line) and two posteriors from DSGE-VAR with $\lambda = 1$ (black dotted) and $\lambda = 5$ (black dash-dotted).[20] Figure 13 shows this exercise using JPT's model and data.[21]

Let us look at the implications of this analysis for the different parameters of interest.

[19]See Table 1 in JPT for a description of the priors.

[20]The VAR model in the DSGE-VAR specification is estimated using four lags and the priors for the structural and shock parameters in the DSGE model are the same as in JPT.

[21]The results obtained when using JPT's model but SW's dataset and SW's model but JPT's prior are similar and are reported in the Appendix, see Figures S.6 and S.7.

Regarding κ (see the first column in Figure 13), the results are mixed. Our first approach suggests the cross-equation restrictions of the model improve estimation accuracy because the dashed line (with correlated shocks) is closer to the prior than the solid line (where no correlations between the shocks are allowed). However, the DSGE-VAR posteriors for $\lambda = 1$ and $\lambda = 5$ are very similar, suggesting that relaxation of the DSGE restrictions does not materially alter the posterior of κ.

Regarding ζ (see the second column in Figure 13), all posteriors coincide with the prior. This is consistent with the view that ζ is not identified with or without the cross-equation restrictions. This is also consistent with JPT, who report a lack of identification of this parameter in their model.[22] Moreover, this is consistent with our analysis in the previous Section 6.1, where we show that changes in the value of ζ are not affecting much the dynamics of the model, thus possibly impairing identification.

Regarding ρ (see the third column in Figure 13), all posteriors differ from each other and from the prior. It appears that cross equation restrictions have an impact on the estimation of ρ. The posterior from the model that relaxes only the orthogonality of the shocks is to the right of the baseline posterior, while both DSGE-VAR posteriors, which relax restrictions on the dynamics as well, are to the left of the baseline posterior. This leads us to conclude that the high baseline estimates of ρ are likely due to the restrictions on the dynamics.

Overall, our results suggest that cross-equation restrictions can partially explain why Bayesian full-information estimates of the investment equation are more precise than our limited-information GMM results in Section 5. However, these cross-equation restrictions could potentially lead to biased estimates whenever some other parts of the system are misspecified, as we show in Section F in the Appendix.

[22] The Fischer information matrix analysis in DYNARE (see Iskrev, 2010) also points to the fact that ζ is not very well identified, contrary to κ and ρ.

Investment adjustment cost (κ)	Elasticity capital utilization cost (ζ)	AR(1) coefficient investment shock (ρ)
(a)	(b)	(c)

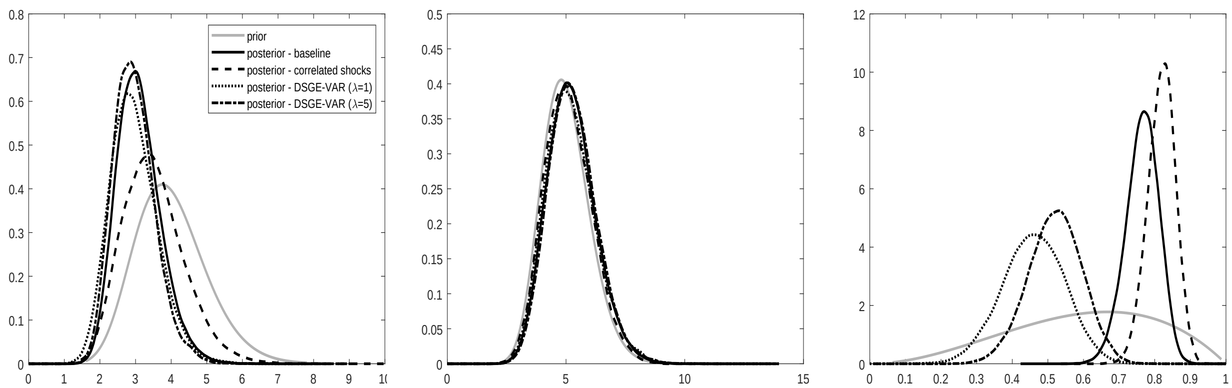

Figure 13: Prior-posterior plots using JPT's model and dataset. All estimations are done using Dynare. The posterior distributions are based on 500,000 draws, with the first 50% draws discarded as burn-in draws. The average acceptance rate is around 25-30%.

7 Conclusions

We assess the empirical performance of the most commonly employed specification of investment behavior in modern operational DSGE models employed for policy analysis. The specification of the investment block of these models is based on the investment adjustment cost specification proposed by Christiano et al. (2005) together with variable capital utilization. We employ the same limited-information methodology that was used in the extant literature on the empirical performance of other key parts of DSGE models, such as the New Keynesian Phillips curve, the monetary policy rule, and the consumption Euler equation.

Our results are mixed. On the one hand, the investment equation is not rejected by the data. On the other hand, there is little information that aggregate data could provide to identify the key structural parameters of the investment block of current medium-scale DSGE models. Hence, this identification has to come from the cross-equation restrictions that other parts of the model imply. However, semi-structural estimation shows that investment is insensitive to changes in capital utilization and the real interest rate. In fact, the semi-structural parameters - the elasticity of investment to the real interest rate and capital utilization - are quite tightly estimated to be near zero when the persistence of the investment-specific shock is assumed to be low. Finally, similar to the results in Ascari et al. (2021), structural change is not as informative for the identification as it was found to be for the NKPC, and there is no evidence of parameter instability.

We also investigate how DSGE models estimated with Bayesian methods obtain more precise estimates of the investment Euler equation parameters than our methodology. Using the JPT model, we find that the investment adjustment cost parameter is identified by the model cross-equation restrictions, while the role of the prior in reducing estimation uncertainty is marginal. The elasticity of capital utilization does not seem to be identified by the data. This is consistent with the evidence in JPT and with our analysis, which shows that changes in the value of ζ are not affecting much the dynamics of the model, thus possibly impairing identification. The data are not informative about the persistence parameter of the investment-specific shock either, which is estimated to be high only if we impose the cross-equation restrictions of the structural model. This is due to the fact that the model wants to match the very persistent dynamics of the observable macroeconomic time series used in the estimation. In conclusion, our results suggest that the identification of the structural parameters in DSGE models could be achieved through cross-equation restrictions implied by the joint dynamics of the full system. However, these assumptions could potentially lead to biased estimates whenever some other parts of the system are misspecified.

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