The Transmission of Monetary Policy through Redistributions and Durable Purchases*

Vincent Sterk
University College London

Silvana Tenreyro
London School of Economics

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Abstract

This paper studies a redistribution channel for the transmission of monetary policy. Using a tractable OLG setting in which the government is a net debtor, we show that standard open market operations (OMO) conducted by Central Banks have significant revaluation effects that alter the level and distribution of wealth in the economy and the real interest rate. Specifically, expansionary OMO generate a negative wealth effect (the private sector as a whole is a net creditor to the government), increasing households' incentives to save for retirement and pushing down the real interest rate. This, in turn, leads to a substitution towards durables, generating a temporary boom in the durable good sector. With search and matching frictions, a form of productive investment is added to the model and the fall in interest rates causes an increase in labour demand, raising aggregate employment. The mechanism can mimic the empirical responses of key macroeconomic variables to monetary policy interventions. The model shows that different monetary interventions (e.g., OMO versus helicopter drops) can have sharply different effects on activity.

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1 Introduction

A central question in monetary economics is how monetary policy interventions transmit to the real economy. This paper contributes to the literature by quantitatively studying a redistribution channel for the transmission of monetary policy that has been unduly glossed over by the literature. Using a tractable quantitative model, the paper shows that this redistribution channel can account for a significant fraction of the empirical response of key macroeconomic aggregates to monetary policy interventions.

An important element for the transmission channel we emphasize is the rather uncontroversial assumption (applicable to the United States and other industrialized countries) that the government is a big net debtor in the economy (while households as a whole are net creditors[1]). Overlapping generations of households consume durable and non-durable goods and work and save for retirement through bond, money holdings, and durable goods.[2] A temporary expansion in monetary policy carried out through open market operations (OMO), whereby the central bank purchases government bonds, pushes down the nominal interest rate and leads to a temporary increase in prices. This price adjustment, needed to close the gap between money supply and demand, causes a downward revaluation of the government debt, generating a negative wealth effect for the household sector.[3] The fall in private wealth induces households to save a larger fraction of their income, as they seek to restore their retirement savings, pushing down the real interest rate. This in turn leads to a substitution towards durable goods, generating a boom in the durable good sector. With search and matching frictions, job vacancies are a form of productive investment, as they create durable employment matches. The decline in the real interest rate thus increases the demand for both durables and productive investment, leading to an increase in aggregate employment and output.

The emphasis on durable goods in the model is motivated by the empirical finding that the response of activity to monetary policy is almost entirely driven by the response of the durable good sector. The introduction of search and matching frictions, while not necessary for the qualitative results, adds realism and generates significant persistence in the responses of economic variables to monetary policy, in line with the empirical evidence. (We study versions

[1] US households tend to hold bank deposits, while banks hold government bonds; we implicitly assume that competitive banks fully pass-through their losses to households and accordingly, in the model, we merge the household and banking sectors.

[2] Bond and money holdings are imperfect substitutes.

[3] Though the intervention redistributes wealth towards the very young and poor, we argue that the dominant effect is the redistribution from the household sector to the government.

of the model with and without search and matching frictions.)

The redistributive channel in our model is motivated by Doepke and Schneider (2006a)'s empirical study, which points out that inflationary episodes can cause significant revaluations of assets and redistributive effects from wealthy, middle age and old households towards the government (the main debtor) and poor, young households. Similar results are found by Adam and Zhu (2014) for European countries and Canada.[4] Despite their stark findings, the literature and the profession at large have not incorporated redistributive elements in standard monetary policy analysis. In this paper, we show that this redistributive effects can have sizeable macroeconomic effects.

We proceed in two steps. First, building on Gertler and Karadi (2015)'s identification strategy, we empirically show that following an unexpected monetary policy expansion, the real value of public debt falls and the price level increases;[5] furthermore, we corroborate that the durable good sector is the key driver of the response of real activity to monetary policy expansions, and show that nondurables and services display virtually no response. Second, we develop a tractable model to quantitatively study the aggregate effects caused by the revaluation of government liabilities due to monetary policy interventions. We show that the model generates responses that are in line with the empirical results: in particular, it leads to a boom driven by the durable good sector and a decline in the real value of public debt. An important element in the model is the presence of a government sector; despite playing a very passive role, its presence is relevant as it leads to a redistribution of wealth away from the private sector—as well as across households—causing a fall in real interest rate and a boom in durables.

An open issue is of course what the government does with its windfalls.[6] Following standard assumptions in the literature, the government in our model is a passive agent; in particular, the model abstracts from government consumption and assumes that the Treasury follows a balanced budget policy, using the increased net income flows to finance a persistent reduction in (non-distortionary) taxes.[7] While these tax cuts help to compensate households for their wealth

[4] Relatedly, Coibion, Gorodnichenko, Kueng and Silvia (2012) find that monetary expansions reduce inequality, as measured by Gini coefficients, suggesting a redistribution away from wealthier individuals.

[5] Our results show a swift and significant response of the aggregate price level (CPI), without the so called "price puzzle" resulting from other identification strategies.

[6] An expansionary OMO improves the financial position of the government via two channels. First, an increase in prices reduces the real value of government debt. Second, the operation increases the Central Bank's bonds holdings and consequently its stream of interest revenues, which are transferred to the Treasury as they are accrued. In the data, these remittances amount to an average of two percent of government expenditures per year, with high variability over time.

[7] An expansionary OMO improves the financial position of the government via two channels. First, an increase

losses, they do not undo the redistributive effects. In particular, old agents emerge as the biggest losers from the operation whereas future (unborn) generations benefit the most. In between these extremes are agents who are in the working phase of their lives when the shock hits. They suffer a negative revaluation of their retirement savings but do not receive full compensation from the Treasury once they retire. So, on net, living agents lose and this breakdown of the Ricardian equivalence (Barro 1974) leads to the non-neutrality of money.

Our model highlights that the real effects of open market operations can be sharply different from the effects of "helicopter drops," that is, tax cuts financed by an increase in the money supply, even though the effects of the two policies on nominal interest rates and prices are similar; an expansionary helicopter drop causes a bust in durables and a decline in output and hours. The difference, as will become clear, is driven by the distributional effects the two policies generate. Our analysis takes Doepke and Schneider (2006a)'s results one step further as it shows that the macroeconomic effects stemming from the revaluation of wealth will ultimately depend on how the policy is implemented.

We conclude by stressing that our model can complement the New Keynesian (NK) model and indeed help in addressing the criticism levied by Barsky, House, and Kimball (2007) against the NK mechanism. These authors integrate durable goods into an otherwise standard sticky-price framework and show that when durables' prices are relatively flexible (as appears to be the case in the data), the model generates a counterfactual *decline* in durables following an expansionary monetary shock.[8] By allowing for redistributions, our model provides a mechanism that counteracts the channel highlighted by Barsky et al. (2007) and can thus generate a boom in durables even when their prices are flexible, helping the standard NK model in mimicking the empirical response.

The paper is organized as follows. Section 2 reviews the literature. Section 3 presents and

in prices reduces the real value of government debt. Second, the operation increases the Central Bank's bonds holdings and consequently its stream of interest revenues, which are transferred to the Treasury as they are accrued. (In the data, these remittances amount to an average of two percent of government expenditures per year, with high variability over time.) The Treasury eventually rebates the money to households, but the beneficiaries are not necessarily the same as those who suffered the negative wealth effect, so when the expansion happens, the negative wealth effect dominates. In standard treatments of helicopter drops, tax rebates are carried out in the period the monetary expansion takes place; as the rebate benefits only the working agents, they enjoy a positive wealth effect, which leads to a reduction in hours work and a bust in activity.

[8]Specifically, when durable goods' prices are relatively flexible, as appears to be the case in the data, these models predict that following a monetary expansion, non-durable purchases increase, while durable purchases, remarkably, decrease. In the case of fully flexible durable prices, the predicted contraction in the durable goods producing sector is so large that the monetary expansion has almost no effect on total aggregate output. See Klenow and Malin (2011) and references therein for a positive link between the durability of the good and the frequency of price adjustment.

discusses the main empirical facts that motivate key features of our model. Section 4 introduces the model. Section 5 performs various numerical exercises and discusses the findings in light of the empirical evidence. Section 6 offers concluding remarks.

2 Relation to the Literature

Our baseline model relies on a frictionless setting with flexible wages and prices. As emphasized by Woodford (2012) in his influential Jackson Hole symposium paper, in standard modern, general-equilibrium, frictionless asset pricing models, open market purchases of securities by Central Banks have no effect on the real economy. This result, which goes back to Wallace (1981)'s seminal article, is at odds with the widely held view that open market operations (OMO) by Central Banks affect interest rates—and at odds indeed with the very practice of Central Banks. The irrelevance result is easiest to see in the context of a representative agent model, as explained by Woodford (2012);[9] however, Wallace (1981)'s widely cited result applies to a more general setting with heterogeneous agents. A key premise for Wallace's irrelevance result, however, is that OMO by the Central Bank are accompanied by fiscal transfers that ensure no change in the income distribution following the monetary policy intervention. In other words, by construction, distributional effects of OMO are muted by fiscal transfers that neutralize distributional changes—and hence preclude any change in individuals' decisions following the intervention.[10]

The goal of this paper is to study the effects of monetary policy interventions when, realistically, OMO are not accompanied by neutralizing fiscal transfers—nor is there a complete set of state-contingent securities that would ensure an unchanged income distribution following the policy intervention. The motivation is necessarily a practical one. When researchers estimate

[9] Suppose the central bank wishes to sell a risky asset (an asset with lower return in a bad state); one would think the private sector would be in principle only willing to buy it at a lower price. However, in the frictionless settings analyzed by Woodford (2012), even if the central bank keeps the risky asset, the risk does not disappear from the economy. The central bank's earning on its portfolio are lower in the bad state and this means lower earnings distributed to the Treasury (and hence higher taxes to be collected from the private sector in the bad state). So the representative household's after-tax income is equally exposed to risk, whether or not the household buys the asset. Thus asset prices are unaffected by the open-market operation.

[10] Wallace (1981) refers to this condition as "unchanged fiscal policy." An unchanged fiscal policy in that context is one in which there is no change in government consumption and no change in the income or wealth distribution. To implement Wallace's OMO without the redistributional effects, a Central Bank needs to ask the Treasury to change transfers and taxes in a particular way to keep the income distribution unchanged. An alternative way of obtaining this result would be to have a complete set of contingent securities that would undo any change in the income distribution.

the causal effects of monetary policy interventions, they do not (cannot) abstract from or control for the distributional effects they cause—and there is no accompanying fiscal policy that undoes them. Hence, to understand the effects of those interventions on activity, researchers need to take into account the potential impact of the redistribution caused by the policy intervention.

To better understand the importance of agents' life cycle savings considerations, we also study a limit case of our baseline model with an infinitely-lived representative agent. In this limit case, monetary neutrality is obtained, as in Sidrauski (1967). This is because agents suffering a revaluation effect on their financial assets are compensated in equal amounts by current and future transfers from a fiscal authority rebating lump-sum transfers, thus precluding wealth effects and any change in behavior. In the absence of nominal rigidities, real wages and relative prices are thus entirely determined by real factors. Nominal wage income and durable good prices therefore increase in tandem in the presence of inflation, and the increase in nominal wage income exactly offsets the desire to bring forward durable good purchases. This is true even though inflation does reduce the real value of financial wealth.[11] Money neutrality in our model obtains under the same conditions in which Ricardian Equivalence holds (Barro 1974). By (realistically) precluding risk sharing of aggregate monetary policy shocks across generations, the model yields money non-neutrality even with flexible prices.[12]

The methodological strategy followed in this paper has much in common with the NK literature (e.g., Christiano, Eichenbaum, and Evans, 2005): we construct a dynamic equilibrium model and compare the responses to monetary policy shocks to those obtained from a structural Vector Auto-Regressive (VAR) model.[13] We argue that our model can match many of the patterns in the data without relying on nominal rigidities; the latter can, however, be easily added to the analysis.

[11] Recall the assumption that the government makes lump-sum transfers from seigniorage revenues to agents. Following Weil (1991)'s arguments, based on an endowment economy with helicopter drops, we show that also in an economy with production and durable goods, the reduction in wealth caused by OMO is exactly offset by future increases in government transfers, which renders money neutral. This intuition is perhaps not too easy to see in Weil (1991) because of a key mistake in the derivation of the formula for money holdings, going from equation 3.12 to 3.18, which blurs the interpretation.

[12] Allowing for fiscal transfers to exactly offset the heterogenous effects of monetary policy across different agents would restore the money neutrality in our model. Realistically, however, monetary policy shocks are not accompanied by targeted fiscal transfers aimed at undoing the monetary effects. Hence, to interpret the data and in particular the empirical evidence on the effect of monetary policy interventions, one cannot assume away the redistributional effects of monetary policy.

[13] Models in the New-Keynesian literature rarely allow for household heterogeneity due to computational challenges. An exception is Gornemann, Kuester and Nakajima (2012) who study monetary policy shocks in a model with sticky prices and uninsurable labor market risk. Other exceptions are McKay and Reis (2013) and Ravn and Sterk (2013).

The paper connects with a small but growing literature which seeks to study other channels for the transmission of monetary policy that can complement the standard channel based on nominal rigidities. Examples in this literature are Grossman and Weiss (1983), Rotemberg (1984), and Alvarez and Lippi (2012), who study the role of segmentation in financial markets and the redistributive effects caused by monetary policy.[14] Lippi, Ragni, and Trachter (2013) provide a general characterization of optimal monetary policy in a setting with heterogeneous agents and incomplete markets. More quantitative analyses can be found in Doepke and Schneider (2006b), Meh, Ríos-Rull, and Terajima (2010), Algan, Allais, Challe and Ragot (2012) and Gottlieb (2012). Like us, they numerically analyze the effects of monetary policy and/or inflation in a flexible price economy with aggregate dynamics and heterogeneous-agents. However, none of these papers models open market operations or consumer durables, both key elements of the transmission mechanism we highlight. More crucially, they do not consider the critical role played by the government as net debtor, which leads to the negative wealth effect in the private sector. The qualitative effects are also different: Doepke and Schneider (2006b) and Meh et al. (2010) generate a contraction in activity following a monetary policy expansion, whereas our model generates a boom in activity driven by the durable good sector. Finally, the heterogeneity in these models typically requires computationally heavy methods, which makes it difficult to incorporate shock processes that are realistic enough for a comparison to responses from a structural VAR.[15] By contrast, our model is solved quickly using standard linearization methods, allowing for a straightforward comparison to VARs as well as New-Keynesian DSGE models. To achieve this, we follow a simple stochastic ageing structure introduced in Gertler (1999), but work out a computational strategy that allows for standard preferences.[16]

Last by not least, our paper relates to recent work by Auclert (2015), who focuses on the redistribution of wealth across agents with different marginal propensities to consume and different exposure to interest rate changes. This is of course an important redistribution channel, but distinct from the revaluation effect that we study. Our focus is on the distribution away from the private sector (the negative wealth effect), which is what drives most of our results—the redistribution across households plays a more subdued role in our model. In terms of aggregate

[14] In our model, there is no financial segmentations: all agents can in principle participate in financial markets, though naturally some may endogenously choose not to hold any positions.

[15] Doekpe and Schneider (2006) and Meh et al. (2010) model a one-time, unanticipated inflationary episode rather than recurring monetary policy shocks. Algan et al. (2012) and Gottlieb (2012) discretize the monetary shock process, giving rise to relatively stylized dynamics.

[16] Gertler's approach requires the utility function to be in a class of nonexpected utility preferences, excluding for example standard CRRA utility functions, whereas our model is instead compatible with the latter.

effects from the redistribution, we find that the revaluation effect can have a large impact in the US economy, whereas Auclert (2015) finds relatively small effects for the United States (the effects are larger for countries with a prevalence of adjustable interest rates, like the United Kingdom). Needless to say, the two mechanisms complement each other and should be taken into account when evaluating redistributive effects of monetary policy.

3 Empirical Evidence

In this Section we first revisit the empirical evidence on the effects of monetary policy shocks on the macroeconomy, highlighting the role of durables and the government debt. We do so by estimating a structural VAR model using Gertler and Karadi (GK, 2015)'s identification strategy. The model, identification, and results are described in turn. Next, we discuss the empirical evidence on redistributive effects from monetary policy.

3.1 Monetary expansions and the response of durables and prices

Policy and academic discussions on the economic effects of monetary policy interventions often rely on the relatively high sensitivity of the durables sector to interest rate changes. We corroborate this premise by studying U.S. evidence using a VAR approach. We find that monetary expansions lead to a boom in consumer durables, with little increase in non-durables. This motivates the introduction of consumer durables in our model, as a key variable in the monetary transmission mechanism. Further, we find that monetary expansions trigger a substantial and swift increase in prices, another important element of the transmission mechanism which underlies a redistribution of wealth.[17]

The empirical analysis for measuring the effects of monetary policy shocks relies on a general linear dynamic model of the macroeconomy whose structure is given by the following system of equations:[18]

$$Y_t = \sum_{s=1}^{S} \mathbf{A}_s Y_{t-s} + \sum_{s=1}^{S} \mathbf{B}_s P_{t-s} + \varepsilon_t^y, \tag{1}$$

[17] A typical finding in earlier empirical studies is a puzzling gradual *decrease* of prices following an expansionary monetary shock. However, many of these studies exploit a recursive identification strategy which restricts prices to respond only with a one-period lag to the shock. Our results highlight the limitations of such restrictions.

[18] See for example, Olivei and Tenreyro (2007) and the references therein.

$$P_t = \sum_{s=1}^{S} \mathbf{C}_s Y_{t-s} + \sum_{s=1}^{S} \mathbf{D}_s P_{t-s} + \varepsilon_t^p. \tag{2}$$

Here, Y_t is a vector of non-policy variables, P_t is a policy indicator, $\mathbf{A}_s$, $\mathbf{B}_s$, $\mathbf{C}_s$ and $\mathbf{D}_s$ are coefficient matrices, ε_t^y is a vector of reduced-form residuals associated with the non-policy block of the VAR, and ε_t^p is the residual of the policy equation. The reduced-form residuals are linear combinations of structural shocks:

$$\begin{bmatrix} \varepsilon_t^y \\ \varepsilon_t^p \end{bmatrix} = \mathbf{\Psi} \begin{bmatrix} v_t^y \\ v_t^p \end{bmatrix}, \tag{3}$$

where v_t^p is a monetary policy shock, v_t^y is a vector of other structural shocks, and $\mathbf{\Psi}$ is an unknown matrix which governs the contemporaneous impact of structural shocks on the variables in the VAR.[19] Together, Equations (1), (2) and (3) state that the variables in the VAR depend on lagged values of Y and P, as well as on the structural shocks.

The coefficient matrices $\mathbf{A}_s$, $\mathbf{B}_s$, $\mathbf{C}_s$ and $\mathbf{D}_s$ are estimated by Ordinary Least Squares (OLS) and do not depend on how the contemporaneous effects of structural shocks are identified. Following GK, we use monthly data starting in July 1979, when Paul Volcker took office as chairman of the Federal Reserve System, and end the sample in July 2012. Also following GK, we include twelve lags of data and use the one-year rate on government bonds as the policy indicator. The non-policy variables in the system include the seasonally adjusted Consumer Price Index (CPI), as well as expenditures on durables and non-durables, both seasonally adjusted and deflated with the CPI. Further, we control for the Gilchrist and Zakrajšek (2012) excess bond premium, following GK. Finally, we include total public debt, deflated by the CPI, which is relevant for the monetary transmission mechanism that we study.[20] This data series has been retrieved manually from the Monthly Statements of Public Debt of the United States, available online via www.treasurydirect.gov.

A standard assumption to identify the effects of a monetary policy shock is that non-policy variables respond to the shock only with a lag, which amounts to the assumption that the right

[19] Shocks are assumed to have zero mean and to be uncorrelated among each other and over time. Independence from contemporaneous economic conditions is considered part of the definition of an exogenous policy shock. The standard interpretation of v^p is a combination of various random factors that might affect policy decisions, including data errors and revisions, preferences of participants at the FOMC meetings, politics, etc. (See Bernanke and Mihov 1998).

[20] We have also estimated specifications extended with Industrial Production and the Civilian Unemployment Rate, and obtained very similar results.

column of $\boldsymbol{\Psi}$ consists of zeros except for its bottom element. While this "recursive identification" assumption is debatable in general, it is especially ill-suited for our purposes, since the redistribution channel that we study relies on a change in prices when a monetary shock hits, which is ruled out by assumption under the recursive identification scheme. We therefore resort to an alternative approach proposed by Gertler and Karadi (2015), which we describe below.

3.2 High-frequency identification of monetary policy shocks

Our approach to identifying monetary policy shocks follows GK, who use the methodology of Mertens and Ravn (2013). A key element of the approach is the use of an instrumental variable which is correlated with the monetary policy shock, v_t^p, but not with the other macroeconomic shocks, contained in v_t^y. The instrument used is the change in the three-month ahead futures rate during a 30 minute window around announcements by the Federal Open Market Committee (FOMC).[21,22].

The instrumental variables estimator is implemented following a simple two-stage procedure. The first stage is to regress the reduced-form policy residual ε_t^p on the instrument. The fitted value of this regression, denoted $\widehat{\varepsilon}_t^p$, captures variations in the one-year interest rate that are purely due to monetary policy surprises around FOMC meetings. The second stage is to estimate the linear model $\varepsilon_t^y = \psi^p \widehat{\varepsilon}_t^p + \xi_t$, where ξ_t is a vector of i.i.d. residuals and ψ^p is a vector of coefficients, which captures the impact on the non-policy variables of a monetary surprise associated with a unit increase in the policy instrument. Up to a scaling's factor, the right column of $\boldsymbol{\Psi}$ is thus estimated as $[\widehat{\psi^p}; 1]$, where $\widehat{\psi^p}$ is the OLS estimate of ψ^p. Given this vector and the estimates of $\mathbf{A}_s$, $\mathbf{B}_s$, $\mathbf{C}_s$ and $\mathbf{D}_s$, Impulse Response Functions (IRFs) can be computed by iterating on the VAR. We scale the IRFs such that the one-year rate declines by 75 basis points on impact.

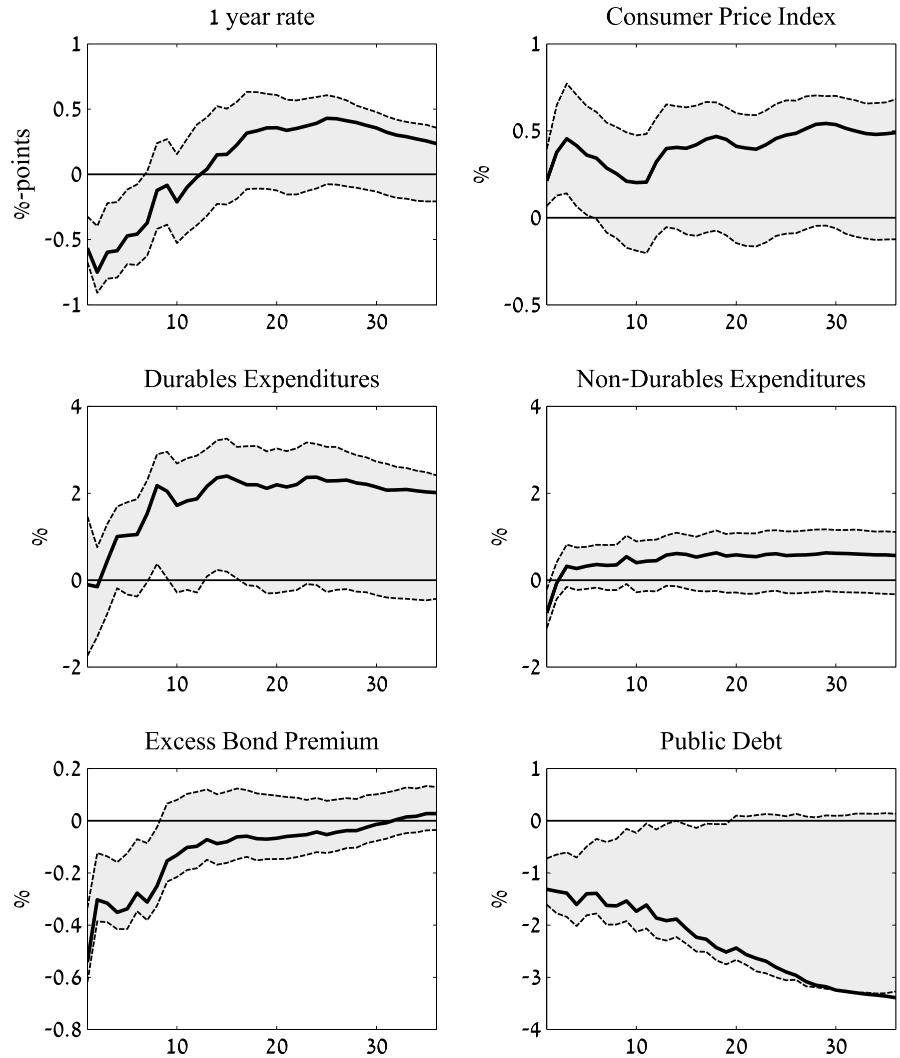
The estimated IRFs are depicted in Figure 1, together with 90 percent confidence bands. Following the monetary expansion, prices increase quickly by about 50 basis points, a substantial increase. Thus, our results do not exhibit a "price puzzle".[23] Further, there is a large, somewhat

[21] The data series for the instrumental variable is taken from GK, who convert the surprises to a monthly frequency using a weighting procedure which accounts for the precise timing of each FOMC within the month. The instruments are available over the period 1990-2012.

[22] See Gürkaynak, Sack and Swanson (2005) for an early analysis of the effects of monetary surprises using the high-frequency approach.

[23] For other VAR approaches that avoid the price puzzle, see e.g. Bernanke, Boivin and Eliasz (2005) and Castelnuovo and Surico (2010).

Figure 1: Responses to an Expansionary Monetary Policy Shock in the VAR.



Note: horizontal axes denote months after the shock.

gradual increase in durables expenditures, up to about 2 percent. By contrast, the increase in non-durables expenditures is small and insignificant. On impact, non-durables even decline significantly. Furthermore, public debt shows a large and significant decline.[24]

3.3 Redistributive Effects of Monetary Policy

A main goal of our paper is to study the redistributive effects of monetary policy and their impact on aggregate variables in a quantitative model. A number of recent empirical papers substantiate our motivation. In particular, Doepke and Schneider (2006a) document significant

[24]There is also a decline in the excess bond premium which is in line with the results of GK (given the size and the sign of the shock).

wealth redistributions in the US economy following (unexpected) inflationary episodes. Their analysis is based on detailed data on assets and liabilities held by different segments of the population, from which they calculate the revaluation effects caused by inflation. The authors find that the main winners from a monetary expansion are the government as well as poor, young households, whereas the losers tend to be richer, middle age and older households (in their forties or above). Note that households as a whole are net creditors and the government is a net debtor in the US economy. Adam and Zhu (2014) document similar patterns for Euro area countries and Canada, and update the results for the United States. As for the US economy, in most euro-area countries, the household sector is a net creditor and the government is a net debtor.[25]

Our model embeds these redistributive revaluation effects and brings two additional considerations to the analysis. The first consideration is how these redistributive effects alter the various demographic groups' incentives to work, consume, and save in different types of assets, the hiring decision of firms, and finally, how these changes affect the macroeconomy. The second consideration is how the Treasury redistributes the higher revenues stemming from an expansionary monetary policy intervention. These higher revenues consist of i) higher value of remittances received from the Central Bank as a result of the interest on bonds earned by the Central Bank; and ii) gains from the revaluation of government debt—assuming the government is a net debtor. The revaluation gains by the government can be large, as Doepke and Schneider (2006a)'s calculations illustrate. The remittances are also considerable, amounting to an average of two percent of total government revenues during our period of analysis, with significant volatility. In the baseline model, we assume that these remittances are rebated to the young (working agents), as in practice the taxation burden tends to fall on the working population. However, the framework can be adjusted to allow for different tax-transfer configurations.

An additional empirical paper motivating our analysis is Coibion et al. (2012), who find that unexpected monetary contractions as well as permanent decreases in the inflation target lead to an increase in inequality in earnings, expenditures, and consumption. Their results rely on the CEX survey, and thus exclude top income earners. The authors however argue that their estimates provide lower bounds for the increase in inequality following monetary policy contractions. This is because individuals in the top one-percent of the income distribution receive a third of their income from financial assets—a much larger share than any other segment of the

[25] Looking at the disaggregated data for households, the age at which households become net creditors differs across countries, with the turning point being around 40-45.

population; hence, the income of the top one-percent likely rises even more than for most other households following a monetary contraction.

Consistent with these findings, in our model, monetary policy expansions cause a redistribution of income from old agents, who rely more heavily on wealth, to young agents and future tax payers. The consumption of goods by the young increases relative to that of old agents following a monetary expansion. These results are more directly examined by Wong (2014), who finds that total expenditures by the young increases relatively to those of older people following a monetary policy expansion, the latter identified through a recursive VAR assumption. In the Appendix, using a different identification strategy, we study the responses by different demographic groups and furthermore explore the differences in the responses of durables and nondurables by the various groups. We find that indeed young households see an increase in expenditures relative to old households and that this response is almost entirely driven by the purchases of durable goods. These results lend support to the mechanism in our model, which generates a relative increase in durable consumption by young (working) agents vis-à-vis old (retired) agents following a monetary expansion.

4 Monetary policy shocks in a tractable heterogeneous agents model

We study the dynamic effects of monetary policy shocks in a general equilibrium model which embeds overlapping generations and a parsimonious life cycle structure with two stages: working life and retirement. Transitions from working life to retirement and from retirement to death are stochastic but obey fixed probabilities, following Gertler (1999). Financial markets are incomplete in the sense that there exists no insurance against risks associated with retirement and longevity. As a result, agents accumulate savings during their working lives, which they gradually deplete once retired. These savings can take the form of money, bonds, and durable consumption goods.

The money supply is controlled by a Central Bank, who implements monetary policy using open market operations, that is, by selling or buying bonds. Realistically, we assume that the Central Bank transfers its profits to the Treasury. The Treasury in turn balances its budget by setting lump-sum transfers to households. In this environment we study the dynamic effects of persistent monetary policy shocks. We contrast our benchmark model with an alternative economy in which the Central Bank uses “helicopter drops” of money rather than OMO to implement monetary policy.

We solve the model using a standard numerical method.[26] This may seem challenging given the presence of heterogeneous households and incomplete markets. In particular, the presence of aggregate fluctuations implies that a time-varying wealth distribution is part of the state of the macroeconomy. To render the model tractable, we introduce a government transfer towards newborn agents which eliminates inequality among young agents.[27] We show that aggregation then becomes straightforward and only the distribution of wealth between the group of young and old agents is relevant for aggregate outcomes. At the same time, our setup preserves the most basic life-cycle savings pattern: young agents save for old age and retired agents gradually consume their wealth.

Another advantage of our model with limited heterogeneity is that it straightforwardly nests a model with an infinitely-lived representative agent. One can show analytically that monetary policy shocks do not affect real activity under the representative agent assumption, provided that money and consumption enter the utility function separably.[28] This result is closely related to the fact that by construction redistributive effects are absent in an economy without heterogeneity.

We consider two versions of the model. The baseline version does not incorporate any form of product or labor market friction. Hence, the monetary transmission in the model is very different to the transmission in New Keynesian models, which typically abstract from demographics and household heterogeneity in wealth. We first discuss the baseline model. In Section 4.8, we analyze a modified version of the model which incorporates search and matching frictions in the labour market. Section 4.9 discusses a special case of the model with a representative agent, in which the transmission mechanism falls apart due to a lack of redistributional effects.

4.1 Agents and demographics

We model a closed economy which consists of a continuum of households, a continuum of perfectly competitive firms and a government, which is comprised of a Treasury and a Central Bank. In

[26] Specifically, we use first-order perturbation, exploiting its certainty-equivalence property. See the appendix for details.

[27] Wealth inequality among retired agents, as well as between young and old, is preserved in our framework.

[28] This result by itself is not surprising, as (super)neutrality results for representative agent models with productive durables, have been known since the seminal work of Sidrauski (1967) and Fischer (1979). Sidrauski (1967) shows that when money enters the utility function separably, the rate of inflation does not affect real outcomes in the steady state. Fischer (1979) shows that under logarithmic utility this is also true along transition paths. Under alternative utility functions this is generally not true, but in quantitative exercises deviations from neutrality are often found to be quantitatively small, see for example Danthine, Donaldson and Smith (1987). In our benchmark model we will assume logarithmic utility and thus focus on a different source of non-neutrality.

every period a measure of new young agents is born. Young agents retire and turn into old agents with a time-invariant probability $\rho_o \in [0,1)$ in each period. Upon retirement, agents face a time invariant death probability $\rho_x \in (0,1]$ in each period, including the initial period of retirement. The population size and distribution over the age groups remains constant over time and the total population size is normalized to one. The fraction of young agents in the economy, denoted ν, can be solved for by exploiting the implication that the number of agents retiring equals the number of deaths in the population, i.e.

$$\rho_o \nu = \rho_x \left(1 - \nu + \rho_o \nu\right). \tag{4}$$

The age status of an agent is denoted by a superscript $\mathbf{s} \in \{\mathbf{n}, \mathbf{y}, \mathbf{o}\}$, with $\mathbf{n}$ denoting a newborn young agent, $\mathbf{y}$ a pre-existing young agent, and $\mathbf{o}$ an old agent.

Households derive utility from non-durables, denoted $c \in \mathbb{R}^+$, a stock of durables, $d \in \mathbb{R}^+$, and real money balances, denoted $m \in \mathbb{R}^+$. They can also invest in nominal bonds, the real value of which we label $b \in \mathbb{R}$. Bonds pay a net nominal interest rate $r \in \mathbb{R}^+$.

Young agents, including the newborns, supply labor to firms on a competitive labor market whereas old agents are not productive. Durables depreciate at a rate $\delta \in (0,1)$ per period and are produced using the same technology as non-durables. Because of the latter, durables and non-durables have the same market price. All agents take laws of motion of prices, interest rates, government transfers and idiosyncratic life-cycle shocks as given. We describe the decision problems of the agents in turn.

4.2 Old agents

Agents maximize expected lifetime utility subject to their budgets, taking the law of motion of the aggregate state, denoted by Γ, as given. Letting primes denote next period's variables, we can express the decision problem for old agents ($\mathbf{s} = \mathbf{o}$) recursively and in real terms as:

$$\begin{aligned}
&V^{\mathbf{o}}(a, \Gamma) = \max_{c,d,m,b} U(c,d,m) + \beta \left(1 - \rho_x\right) \mathbb{E} V^{\mathbf{o}}(a', \Gamma') \\
&s.t. \\
&c + d + m + b = a + \tau^{\mathbf{o}}, \\
&a' \equiv (1 - \delta)\, d + \frac{m}{1 + \pi'} + \frac{(1 + r)\, b}{1 + \pi'}, \\
&c, d, m \geq 0,
\end{aligned} \tag{5}$$

where $V^{\mathbf{o}}(a,\Gamma)$ is the value function of an old agent which depends on the aggregate state and the real value of wealth, denoted by a, $\mathbb{E}$ is the expectation operator conditional on information available in the current period, $\beta \in (0,1)$ is the agent's subjective discount factor, and $\pi \in \mathbb{R}$ is the net rate of inflation. $U(c,d,m)$ is a utility function and we assume that $U_j(c,d,m) > 0$, $U_{jj}(c,d,m) < 0$ and $\lim_{j\to 0} U_j(c,d,m) = \infty$ for $j = c,d,m$. Finally, $\tau^{\mathbf{s}} \in \mathbb{R}$ is a transfer from the government to an agent with age status $\mathbf{s}$, so $\tau^{\mathbf{o}}$ is the transfer to any old agent.

The budget constraint implies that old agents have no source of income other than from wealth accumulated previously. Implicit in the recursive formulation of the agent's decision problem is a transversality condition $\lim_{t\to\infty} \mathbb{E}_t \beta^t \left(1-\rho_x\right)^t U_{c,t} x_t = 0$, where $x = d, m, b$ and where $U_{c,t}$ denotes the marginal utility of non-durable consumption. Finally, we assume that agents derive no utility from bequests and that the wealth of the deceased agents is equally distributed among the currently young agents.

4.3 Young agents

Young agents supply labor in exchange for a real wage $w \in \mathbb{R}^+$ per hour worked. The optimization problem for newborn agents ($\mathbf{s} = \mathbf{n}$) and pre-existing young agents ($\mathbf{s} = \mathbf{y}$) can be written as:

$$
\begin{aligned}
& V^{\mathbf{s}}(a,\Gamma) = \max_{c,d,m,b,h} U(c,d,m) - \zeta \frac{h^{1+\kappa}}{1+\kappa} + \beta\left(1-\rho_o\right)\mathbb{E}V^{\mathbf{y}}(a',\Gamma') + \beta\rho_o\left(1-\rho_x\right)\mathbb{E}V^{\mathbf{o}}(a',\Gamma') \\
& \mathbf{s} = \mathbf{n}, \mathbf{y} \qquad\qquad (6) \\
& s.t. \\
& c + d + m + b = a + wh + \tau^{bq} + \tau^{\mathbf{s}}, \\
& a' \equiv (1-\delta)\, d + \frac{m}{1+\pi'} + \frac{(1+r)\, b}{1+\pi'}, \\
& c, d, m \geq 0,
\end{aligned}
$$

where young agents too obey transversality conditions. The term $\zeta \frac{h^{1+\kappa}}{1+\kappa}$ captures the disutility obtained from hours worked, denoted h, with $\zeta > 0$ being a scaling's parameter and $\kappa > 0$ being the Frisch elasticity of labor supply. Bequests from deceased agents are denoted τ^{bq}; as before, $\tau^{\mathbf{s}}$ is a lump-sum transfer from the government. When making their optimal decisions, young agents take into account that in the next period they may be retired, which occurs with probability $\rho_o\left(1-\rho_x\right)$, or be deceased which happens with probability $\rho_o \rho_x$. We thus assume that upon retirement, young agents may be immediately hit by a death shock.

4.4 Firms

Goods are produced by a continuum of perfectly competitive and identical goods firms. These firms operate a linear production technology:

$$y_t = h_t. \tag{7}$$

Profit maximization implies that $w_t = 1$, that is, the real wage equals one.

4.5 Central bank

Although we do not model any frictions within the government, we make a conceptual distinction between a Central Bank conducting monetary policy and a Treasury conducting fiscal policy. We make this distinction for clarity and in order to relate the model to real-world practice.

The Central Bank controls the nominal money supply, $M_t \in \mathbb{R}^+$, by conducting open market operations. In particular, the Central Bank can sell or buy government bonds. We denote the nominal value of the bonds held by the Central Bank by $B_t^{\mathbf{cb}} \in \mathbb{R}$. The use of these open market operations implies that in every given period the change in bonds held by the Central Bank equals the change in money in circulation, that is,

$$B_t^{\mathbf{cb}} - B_{t-1}^{\mathbf{cb}} = M_t - M_{t-1}. \tag{8}$$

The Central Bank transfers its accounting profit—typically called seigniorage- to the Treasury.[29] The real value of the seigniorage transfer, labeled $\tau_t^{\mathbf{cb}} \in \mathbb{R}$, is given by:

$$\tau_t^{\mathbf{cb}} = \frac{r_{t-1} b_{t-1}^{\mathbf{cb}}}{1 + \pi_t}. \tag{9}$$

The above description is in line with how Central Banks conduct monetary policy, as well as with the typical arrangement between a Central Bank and the Treasury. By contrast, many models of monetary policy assume monetary policy is implemented using "helicopter drops," that is expansions of the money supply that are not accompanied by a purchase of assets but instead by a fiscal transfer that is equal to the change in the money supply. Modern monetary models are often silent on how monetary policy is implemented and directly specify an interest rate rule. In our framework, however, the specific instruments used to implement monetary

[29] We abstract from operational costs incurred by the central bank.

policy are critical, since the associated monetary-fiscal arrangements pin down redistributive effects and hence the impact of changes in monetary policy on the real economy.

When we implement the model quantitatively, we simulate exogenous shocks to monetary policy. We do so by specifying a stochastic process that affects the growth rate of the money supply M_t. The change in M_t is engineered using open market operations.

4.6 Treasury

The Treasury conducts fiscal policy. For simplicity, we abstract from government purchases of goods and assume that the Treasury follows a balanced budget policy. The government has an initial level of bonds $B^{\mathbf{g}}_{t-1}$ which gives rise to interest income (or expenditure if the government has debt) on top of the seigniorage transfer from the Central Bank. To balance its budget, the government makes lump-sum transfers to the households, which can be either positive or negative. The government's budget policy satisfies:

$$\nu \rho_o \tau^{\mathbf{n}}_t + \nu \left(1 - \rho_o\right) \tau^{\mathbf{y}}_t + \left(1 - \nu\right) \tau^{\mathbf{o}}_t = \frac{r_{t-1} b^{\mathbf{g}}_{t-1}}{1 + \pi_t} + \tau^{\mathbf{cb}}_t. \tag{10}$$

Here, the left-hand size denotes the total transfer. In particular, $\nu \rho_o \tau^{\mathbf{n}}_t$ is the total transfer to the newborns, $\nu \left(1 - \rho_o\right) \tau^{\mathbf{y}}_t$ is the transfer to pre-existing young agents and $b^{\mathbf{g}}_t$ is the real value of government bonds. The right-hand side denotes total government income.

For tractability we also assume that the government provides newborn agents with an initial transfer that equalizes the wealth levels with the average after-tax wealth among pre-existing agents, that is,

$$\tau^{\mathbf{n}}_t = a^{\mathbf{y}}_t + \tau^{\mathbf{y}}_t, \tag{11}$$

where $a^{\mathbf{y}}_t \equiv \int_{i:\mathbf{s}=\mathbf{y}} a_{i,t} di$ is the *average* wealth among pre-existing young agents (before transfers). Since before-tax wealth is the only source of heterogeneity among young agents, all young agents make the same decisions and what arises is a representative young agent. This implication makes the model tractable. Note that although we eliminate heterogeneity among young agents by assumption, we do preserve heterogeneity between young and old agents, as well as heterogeneity among old agents.

Finally, we assume that only productive agents are affected by transfers/taxes, i.e. we set $\tau^{\mathbf{o}}_t = 0$. This assumption is motivated by the reality that the majority of the tax burden falls on people in their working life, due to the progressivity of tax systems.[30]

4.7 Market clearing and equilibrium

Aggregate non-durables and durables are given by:

$$c_t = \nu c_t^{\mathbf{y}} + (1-\nu)\, c_t^{\mathbf{o}}, \tag{12}$$

$$d_t = \nu d_t^{\mathbf{y}} + (1-\nu)\, d_t^{\mathbf{o}}, \tag{13}$$

where superscripts $\mathbf{y}$ and $\mathbf{o}$ denote the averages among young and old agents, defined analogously to the definition of $a_t^{\boldsymbol{y}}$.[31] Clearing in the markets for goods, money and bonds requires:

$$c_t + d_t = \nu h_t^{\mathbf{y}} + (1-\delta)\, d_{t-1}, \tag{14}$$

$$m_t = \nu m_t^{\mathbf{y}} + (1-\nu)\, m_t^{\mathbf{o}}, \tag{15}$$

$$0 = b_t^{\mathbf{g}} + b_t^{\mathbf{cb}} + \nu b_t^{\mathbf{y}} + (1-\nu)\, b_t^{\mathbf{o}}. \tag{16}$$

Finally, the size of the bequest received per young agent is given by:

$$\tau_t^{bq} = \frac{\rho_x a_t^{\mathbf{o}} + \rho_o \rho_x a_t^{\mathbf{y}}}{\nu}. \tag{17}$$

We are now ready to define a recursive competitive equilibrium:

> **Definition.** *A recursive competitive equilibrium is defined by policy rules for non-durable consumption, $c^{\mathbf{s}}(a,\Gamma)$, durable consumption, $d^{\mathbf{s}}(a,\Gamma)$, money holdings, $m^{\mathbf{s}}(a,\Gamma)$, bond holdings, $b^{\mathbf{s}}(a,\Gamma)$, labor supply, $h^{\mathbf{s}}(a,\Gamma)$, with $\mathbf{s} = \mathbf{n}, \mathbf{y}, \mathbf{o}, \mathbf{cb}, \mathbf{g}$, as well as laws of motion for inflation, the nominal interest rate and the real wage, such that households optimize their expected life-time utility subject to their constraints and the law of motion for the aggregate state, the Treasury and Central Banks follow their specified policies, and the markets for bonds, money, goods and labor clear in every period. The aggregate state Γ includes the value of the monetary policy shock, the distribu-*

[30] We have solved a version of our model in which instead taxes are proportional to wealth levels, and obtained results similar to the ones obtained from our benchmark model. An alternative, behavioural assumption, suggested by David Laibson, would be to realistically assume that agents are not aware of these future transfers. We do not follow this avenue here, but we highlight that this would cause even bigger perceived wealth effects and intensify the aggregate responses we document.

[31] Due to the transfer to newborns $c_t^{\mathbf{y}} = c_t^{\mathbf{n}}$, $d_t^{\mathbf{y}} = d_t^{\mathbf{n}}$, $b_t^{\mathbf{y}} = b_t^{\mathbf{n}}$ and $m_t^{\mathbf{y}} = m_t^{\mathbf{n}}$.

tion of wealth among agents, as well as the initial holdings of assets by households, the Treasury and the Central Bank.

4.8 Adding search and matching frictions

In the baseline model described above, fluctuations in aggregate output due to monetary policy shocks arise from labour supply effects. To appreciate this point, recall that labour is the only input in production and note that the young households' first-order condition for labour can be written as:

$$w_t \lambda_t = \zeta h_t^{\kappa},$$

where λ_t is the Lagrange multiplier on the young households' budget constraint which measures the marginal utility of wealth. After a negative shock to wealth, λ_t increases, which pushes up aggregate labour supply and therefore aggregate output. Vice versa, any increase in aggregate output following a monetary expansion derives from an increase in labour supply.[32] Various empirical studies indicate that reductions in wealth can depress labour supply, see e.g. Imbens, Rubin and Sacerdote (2001) and Holtz-Eakin, Joulfaian and Rosen (1993). However, at high frequency and for small shocks, the labour supply response may not be strong.

We verify robustness of our transmission mechanism in an environment in which the labour supply channel is suppressed completely. The new assumptions we introduce are arguably more realistic and in line with the macro-labour literature. Specifically, we introduce search and matching frictions in the labour market. Workers inelastically supply labour if they have a job and firms hire workers by posting costly vacancies. Operational firms make positive profits and hence firm equity is a valuable asset, which is a form of savings to households alongside money, bonds and consumer durables.

We introduce matching frictions following the approach of Diamond, Mortensen and Pissarides. Young agents can be either unemployed or matched with a firm.[33] A separation between a worker and a firm takes place if the worker retires at the end of the period. If the worker does not retire, the match dissolves with an exogenous probability ρ_s. The overall separation rate, denoted $\widetilde{\rho}_s$, is therefore given by $\widetilde{\rho}_s = \rho_o + (1 - \rho_o)\ \rho_s$. Newborn agents enter the workforce as unemployed. It follows that the number of job searchers in the economy, which we denote s_t,

[32] Recall that $w_t = 1$, so any increase in h_t must be accompanied by an increase in λ_t.

[33] We set $\zeta = 0$ in this model version, i.e. there is no disutility from work. We do not model unemployment benefits.

is given by $s_t = \rho_o \nu + (1-\rho_o)\, \rho_s n_{t-1}$. Hiring takes place at the beginning of the period, after aggregate and individual shocks have realized, but before production takes place. The evolution of the employment rate among young agents, denoted n_t, is given by:

$$n_t = (1-\widetilde{\rho}_s)\, n_{t-1} + g_t,$$

where g_t denotes the number of new hires in period t. We assume that there is full income sharing among workers, following Merz (1995) and many others. Hence, we preserve our setup without heterogeneity among young agents.

Firms are either matched with a worker or are inactive. The equity value of an active firm is given by:

$$V_t = \theta - w_t + (1-\widetilde{\rho}_s)\, \mathbb{E}_t \Lambda_{t,t+1} V_{t+1}, \tag{18}$$

where w_t is the real wage, θ is worker productivity, and $\Lambda_{t,t+1}$ is the stochastic discount factor of the owner of the firms. Inactive firms may search on the labor market for a worker after posting a vacancy, which comes at a flow cost χ_0 per period. If the firm is successful in finding a worker, the firm pays a fixed cost χ_1 to hire the worker. The latter cost represents all hiring costs that are not proportional to the duration of the vacancy, such as training costs, see Pissarides (2009). Creating an inactive firm is costless which gives rise to the following free-entry condition:

$$\frac{\chi_0}{\lambda_t} + \chi_1 \leq V_t,$$

where $\lambda_t \in [0,1]$ is the probability of filling a vacancy. The free-entry condition states that the total (expected) cost of activating a firm cannot exceed the equity value. We calibrate the model such that the condition holds with equality at all times. Given a number of vacancies and a number of searchers, the total number of new matches follows from an aggregate matching function given by $g_t = \nu s_t^{\alpha} v_t^{1-\alpha}$,where v_t is the aggregate number of vacancies, ν is a scaling's parameter and α is the elasticity of the number of new matches with respect to the number of searchers. The probability of filling vacancy is given by $\lambda_t = \frac{g_t}{v_t}$. We assume the real wage is fixed, i.e. $w_t = w < \theta$.[34] Further, we assume that firms use the young agents' stochastic discount

[34] We normalize θ to obtain a steady-state wage of one, as in the baseline model.

factor.[35,36]

4.9 Absence of wealth effects in a representative agent model

A special case of our model is obtained when we set the death probability to one, i.e. $\rho_x = 1$. In this case, agents immediately die upon retirement and old agents are effectively removed from the model. Given the absence of heterogeneity among young agents, the model becomes observationally equivalent to one with an infinitely-lived representative household with a subjective discount factor equal to $\widetilde{\beta} = \beta(1-\rho_o)$. Without heterogeneity among households, shocks to monetary policy do not create net wealth effects and do not impact on real economic activity. This subsection explains why this is the case, using arguments that closely follow Sidrauski (1967), Barro (1978) and Weil (1991).

Consider the baseline model with $\rho_x = 1$.[37] We assume that non-durables, durables and real money balances enter the utility function separably. In particular, we assume the following logarithmic preferences: $U(c,d,m) = \ln c + \eta \ln d + \mu \ln m_t$, where $\eta, \mu > 0$ are preference parameters. This special case is useful to understand the role of household heterogeneity in the transmission of monetary policy, as several analytical results can be derived. The first result is:

Result 1. *Monetary policy is neutral with respect to real activity in the representative agent model.*

The arguments for the monetary neutrality follow Sidrauski (1967). The representative agents' first-order conditions for durables and labor supply, and the aggregate resource constraint are, respectively:

$$U_{c,t} = U_{d,t} + \widetilde{\beta}(1-\delta)\,\mathbb{E}_t U_{c,t+1}, \tag{19}$$

$$U_{c,t} = h_t^{\kappa}, \tag{20}$$

$$c_t + d_t = h_t + (1-\delta)\, d_{t-1}, \tag{21}$$

[35] Thus, the firms' discount factor is given by $\Lambda_{t,t+1} = \beta(1-\rho_o)\frac{U^{\mathbf{y}}_{c,t+1}}{U^{\mathbf{y}}_{c,t}} + \beta\rho_o(1-\rho_x)\frac{U^{\mathbf{yo}}_{c,t+1}}{U^{\mathbf{y}}_{c,t}}$ This assumption simplifies the analysis but is not very restrictive since it can be shown that the stochastic discount factor of all households is the same to a first-order approximation.

[36] Consistent with this assumption we assume that agents sell off all firm their equity upon retirement. The budget constraint of a young agent becomes: $c_t + d_t + m_t + b_t + V_t\left(x_t - (1-\widetilde{\rho}_s)\, x_{t-1}\right) = a_t + (\theta - w_t)\, x_t + w n_t + \tau^{bq} + \tau^{\mathbf{s}}$,where x_t is the amount of firm equity held by the household. The aggregate supply of firm equity is equal to n_t.

[37] We focus on the baseline model for simplicity. It is straightforward to show that the same results are obtained in a representative agent version of the model with search and matching frictions.

where $U_{c,t}=\frac{1}{c_t}$, $U_{d,t}=\frac{\eta}{d_t}$ and for $t=0,1,...$ Given an initial level of durables and given that the utility function is separable in its arguments, these three equations pin down the equilibrium solution paths for c_t, d_t, and h_t in any period t without any reference to variables related to monetary policy. Given this solution it is straightforward to pin down output and the real interest rate as well.

Next, we consider the wealth effects of monetary policy shocks and derive the following key result:

Result 2. *Changes in monetary policy do not create net wealth effects in the representative agent model.*

To arrive at this result, consider the government's consolidated (expected) present value budget constraint. The Appendix demonstrates that this constraint can be written as:

$$\mathbb{E}_t \sum_{s=t}^{\infty} D_s \frac{r_s}{1+r_s} m_s = \frac{m_{t-1}-(1+r_{t-1})\left(b^{\mathbf{g}}_{t-1}+b^{\mathbf{cb}}_{t-1}\right)}{1+\pi_t} + \mathbb{E}_t \sum_{s=t}^{\infty} D_s \tau^{\mathbf{g}}_s, \tag{22}$$

where $D_s \equiv \prod_{k=t}^{s-1} \frac{1+\pi_{k+1}}{1+r_k}$ is the agent's valuation of one unit of nominal wealth received in period $s>t$, $D_t \equiv 1$, and $\tau^{\mathbf{g}}_t \equiv \nu\rho_o\tau^{\mathbf{n}}_t + \nu\left(1-\rho_o\right)\tau^{\mathbf{y}}_t$ is the total transfer to the household sector in period t. The left-hand side of Equation (22) represents the expected present value of government income, in real terms. Here, $\frac{r_t}{1+r_t}m_t$ is the opportunity cost that households pay for holding money. This cost to the households represents a source of income to the government, which enjoys an interest-free liability. The left-hand side of the equation represents the present value of this income to the government. The right hand side captures the present value of government liabilities. The first term represents the real value of the outstanding stock of money and government debt, whereas the second term is the present value of transfers to households, another liability to the government.

Importantly, both components of government liabilities are a source of wealth to the household. Equation (22) makes clear that a monetary shock can affect household wealth via two channels. First, it can trigger a change in current inflation, π_t, affecting the real value of wealth held in nominal assets by the households. Second, monetary policy shocks can affect the present value of transfers to households, via Central Bank remittances to the Treasury.

The Appendix demonstrates that the left hand side of Equation (22) does not respond to monetary policy shocks. In particular, from Result 1 it follows that both D_s and $\frac{r_s}{1+r_s}m_s$, with $s\geq 1$, remain constant. It follows that the right-hand side of the equation remains constant as well. Thus, monetary policy shocks have no *net* wealth effects on households: any downward (upward) revaluation of nominal wealth due to a change in the price level is *exactly* offset by an increase (decline) in the present value of transfers. This insight is closely related to the seminal work of Barro (1978) and was spelled out by Weil (1991) in the context of a monetary model.

5 Quantitative simulations

In this Section we analyze the effects of open market operations in our model using numerical simulations. Before doing so we specify the details of household preferences and the monetary policy rule. We assume that the utility function is a CES basket of non-durables, durables and money, nested in a CRRA function:

$$\begin{aligned} U(c_{i,t}, d_{i,t}, m_{i,t}) &= \frac{x_{i,t}^{1-\sigma} - 1}{1-\sigma}, \\ x_{i,t} &\equiv \left[c_{i,t}^{\frac{\epsilon-1}{\epsilon}} + \eta d_{i,t}^{\frac{\epsilon-1}{\epsilon}} + \mu m_{i,t}^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}}, \end{aligned} \tag{23}$$

where $\epsilon, \sigma, \eta, \mu > 0$. Here, ϵ is the elasticity of substitution between non-durables, durables and money, σ is the coefficient of relative risk aversion, and η and μ are parameters giving utility weights to durables and money, respectively. Computation of the dynamic equilibrium path seems complicated due to the high dimensionality of the aggregate state Γ_t. In the Appendix we show that solving the model using a standard first-order perturbation (linearization) method is nonetheless straightforward under the above preference specification.[38]

The Central Bank is assumed to set the money supply according to the following process:

$$\frac{M_t}{M_{t-1}} = 1 + z_t \tag{24}$$

where z_t is an exogenous shock process to the rate of nominal money growth, assumed to be of the following form:

$$z_t = \xi \left(\overline{m} - m_{t-1} \right) + \varepsilon_t, \;\; \xi \in (0,1), \tag{25}$$

where ε_t is an i.i.d. shock innovation and $\overline{m}$ is the steady-state value of real money balances. A

[38] In particular, we exploit the properties of first-order perturbation and show that the implied certainty equivalence with respect to the aggregate state allows us to express the decision rules of the old agents as linear functions of their wealth levels. This in turn implies that aggregation is straightforward and that only the distribution of wealth between *between* old agents and young agents is relevant for aggregate outcomes.

positive shock increases the money supply on impact. The above feedback rule implies that this increase is gradually reversed in subsequent periods when $\xi \in (0, 1)$.[39]

5.1 Parameter values

The model period is set to one quarter. The parameter values for the baseline model and the version with search and matching frictions are presented in Table 1.

Baseline model. The subjective discount factor, β, is set to 0.9732 which implies an annual real interest rate of 4 percent in the deterministic steady state. The steady state real interest rate is lower than the subjective discount rate, $1/\beta - 1$, due to the retirement savings motive arising in the presence of incomplete insurance markets. The durable preference parameter η is chosen to target a steady-state consumption spending ratio of 20 percent on durables. To set the money preference parameter, we target a quarterly money velocity, defined as $\frac{y}{m}$, of 1.8. The intratemporal elasticity of substitution between non-durables, durables and money, ϵ, is set equal to one, as is the coefficient of relative risk aversion, σ. These two parameter settings imply that money and consumption enter the utility function additively in logs. Hence, our benchmark results are not driven by non-separability of money and consumption in the utility function. In the baseline model, we set the Frisch elasticity of labor supply κ equal to one following many macro studies. (We shut down the labour supply response in the extension.) The parameter scaling the disutility of labor, ζ, is set so as to normalize aggregate quarterly output to one.

Life-cycle transition parameters are set to imply a life expectancy of 60 years, with an expected 40 years of working life and expected 20 years of retirement. Accordingly, we set $\rho_o = 0.0063$ and $\rho_x = 0.0125$ which imply $\nu = 0.6677$. The depreciation rate of durables, δ, is set to 0.04 following Baxter (1996). The initial level of government debt is set to sixty percent of annual output. For simplicity we assume that the Central Bank starts off without any bond holdings or debt. The shock process parameter ξ is set to 0.2 which implies that the half life of the response for the nominal interest rate is about 2.5 years.

Model with search and matching frictions. The calibration of the model with search and matching friction targets the same steady-state values for the interest rate, the durables spending ratio, and money velocity as the baseline model. Accordingly, β, η and μ are set to,

[39] In equilibrium, both real an nominal money balances increase following the shock. Also, the rule implies that the net rate of inflation is zero in the steady state.

respectively, 0.0049, 0.31 and 0.0049. The labour utility parameters κ and ζ are irrelevant in the search and matching version. Instead, four parameter pertaining to the labour market frictions are calibrated: α, χ_0, χ_1 and ζ. The matching function elasticity, α, is set to 0.5, a conventional value in the search and matching literature. The other three parameters are set to hit three steady-state targets. The first target is a steady-state unemployment rate of 5 percent. Second, we target the ratio of the vacancy cost to the fixed cost of hiring, χ_1/χ_0, equal to 10, which is the mid point of the range considered by Pissarides (2009). Finally, set we set ζ to 0.7, which delivers a vacancy filling probability of 0.74, in line with Den Haan, Ramey and Watson (2000). Finally, the persistence parameter, ξ, is set to 0.4, in order to obtain a degree of persistence in the nominal interest rate that is similar to the baseline model. All other parameter values are the same as in the baseline model.

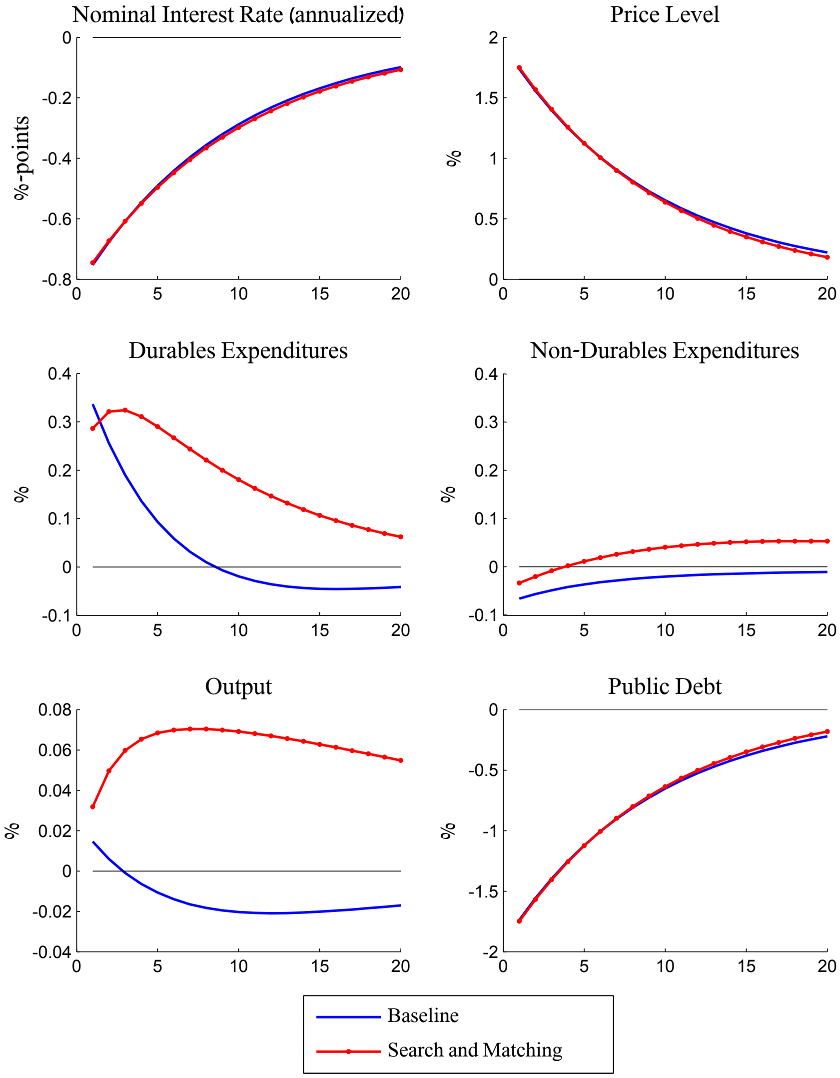
Table 1. Parameter values for the baseline model and the Search and Matching (SaM) model.

	baseline	SaM	description	motivation
β	0.9732	0.9755	subjective discount factor	4% s.s. annual interest rate
η	0.31	0.31	durables preference param.	20% s.s. spending on durables (NIPA)
μ	0.0068	0.0049	money preference param.	1.8 s.s. M2 velocity ($\frac{y}{m}$) (FRB/NIPA)
σ	1	1	coef. rel. risk aversion	convention literature
ϵ	1	1	intratemp. elast. of subst.	convention literature
κ	1	–	inv. elasticity labour supply	convention literature
ζ	0.5795	–	disutility of labor	normalize agg. output to one
ρ_o	0.0063	0.0063	ageing probability	avg duration working life 40 years
ρ_x	0.0125	0.0125	death probability	avg duration retirement 20 years
δ	0.04	0.04	depreciation rate durables	Baxter (1996)
b_0^g	-2.4	-2.4	initial bonds Treasury	government debt 60% of annual output
b_0^{cb}	0	0	initial bondsCentral Bank	no initial central bank debt/bonds
ξ	0.2	0.4	coefficient monetary rule	half life nominal interest rate 2.5 years
χ_0	–	0.0044	variable hiring cost	$\chi_1/\chi_0 = 10$ (Pissarides (2009))
χ_1	–	0.0004	fixed hiring	$\chi_1/\chi_0 = 10$ (Pissarides (2009))
α	–	0.5	matching function elasticity	convention search literature
ν	–	0.7	scaling matching function	vacancy filling probability 0.74

5.2 The dynamic effects of open market operations

Figure 2 presents the responses to an expansionary monetary policy shock, implemented using open market operations. The blue lines show the responses in the baseline model whereas the red lines show the responses when labour market frictions are added. The magnitude of the shock is scaled to imply a reduction in the nominal interest rate of about 75 basis points.

Figure 2: Responses to an Expansionary Monetary Policy Shock in the Baseline Model and the Model with Search and Matching Frictions.



Note: horizontal axes denote quarters after the shock.

First consider the baseline model. Following the monetary expansion, the price level increases.[40] In the periods after the initial shock, the nominal interest rate and the price level

[40] The intuition for the price increase is standard. As the central bank buys government bonds, it increases

gradually revert back to their initial levels, which happens as a result of the reversion in the monetary policy rule. The monetary expansion increases aggregate output on impact. The responses of durables and non-durables make clear that this increase in output is entirely driven by an increase in expenditures on durables. Non-durables decline on impact, although the magnitude of the response is much smaller than the response of durables. Finally, there is a decline in the real value of public debt (i.e. debt issued by the Treasury), which mirrors the increase in prices and which reflects a financial gain for the government at the expense of the public due to a revaluation of its debt.[41]

Introducing search and matching frictions significantly increases the persistence of the response of durables expenditures. Further, the response of non-durable expenditures turns positive soon after the initial shock. Thus, adding search and matching frictions renders the responses of consumption expenditures more in line with the empirical responses. Further, the output response is more persistent and displays a hump shape. The responses of the nominal interest rate, prices and public debt, by contrast, are very similar to the baseline model.

Figure 3 plots several variables that provide insight into the impact of monetary policy shocks as well as their endogenous propagation over time. The real interest rate, plotted in the upper left panel, declines in both models, reflecting an increased desire to save. The top right panel plots the transfer to the young households as a fraction of output, which on impact increases by about 0.6 - 0.7 percent, after which it gradually reverts back to the steady state. Thus, the government gradually remits its financial gains from the monetary expansion back to the households.

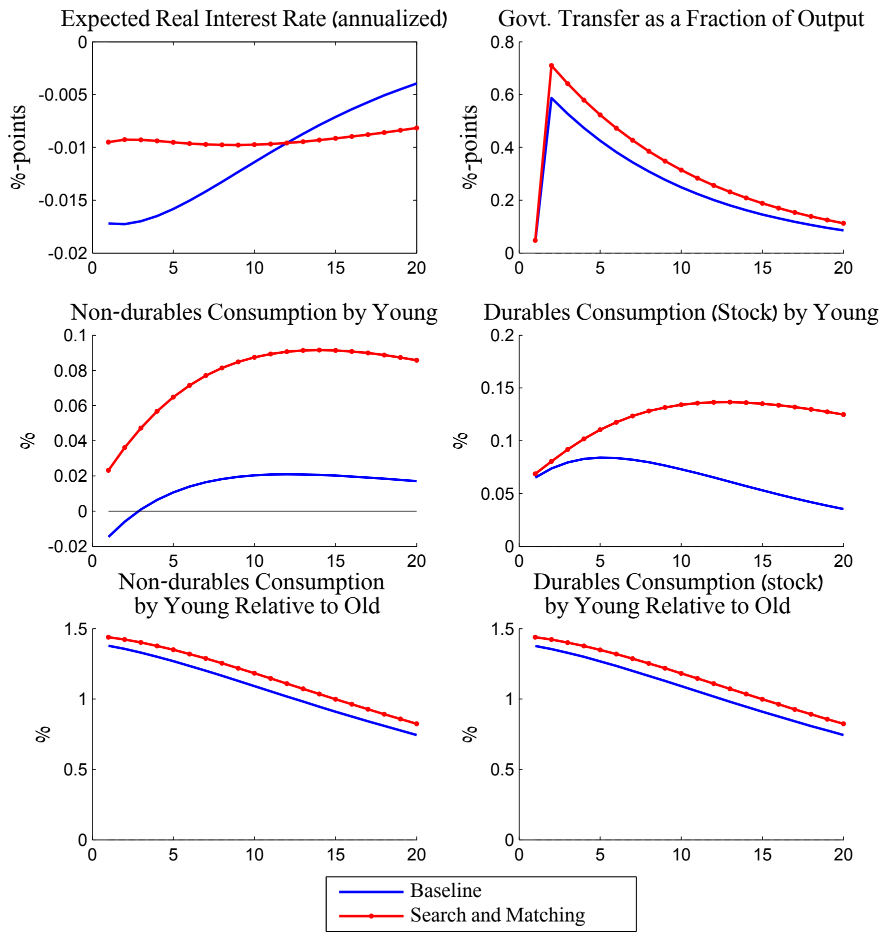
The middle two panels show the responses of consumption by the young, whereas the bottom panels show the consumption responses of the young vis-à-vis the old agents. Relative to the old, consumption of durables and non-durables by the young increases in both models. All households face a reduction in their real wealth due to the increase in prices, but the old are not compensated by an increase in transfers; hence, they lose relative to the young.[42] In absolute terms, consumption of durables by the young increases as well. The response of non-durables

the amount of money in circulation. Since agents' utility is concave in real money holdings, they are induced to substitute some of the extra cash for consumption goods. The increased demand for goods in turn drives up prices, which dampens the demand increase as it reduces the real value of money holdings.

[41] A second financial gain for the government stems from a downward revaluation of the outstanding stock of money, which is a liability to the government alongside debt.

[42] Additionally, for old agents wealth is the only source of income, whereas the young agents also receive wage income, which in real terms is not directly affected by inflation. This is another reason why the young agents are less vulnerable to inflation.

Figure 3: Responses to an Expansionary Monetary Policy Shock in the Baseline Model and the Model with Search and Matching Frictions.



Note: horizontal axes denote quarters after the shock.

expenditures by the young is slightly negative in the baseline model, but positive in the model with search and matching frictions.

To understand effects of monetary policy on real activity more deeply, first consider the baseline model. The increase in prices creates a negative wealth effect to the households as it reduces the real value of their money and bond holdings, who are only partly compensated via an increase in (expected) government transfers. Thus, the policy shock reduces the households' permanent income levels. Further, households become less well insured against idiosyncratic shocks after a decline in the value of their assets. These effects induce the households to consume less and enjoy less leisure, that is, to work more, in order to re-build their savings. However, the aggregate resource constraint, Equation (14), makes clear that in equilibrium it is not possible for the household sector as a whole to reduce both consumption expenditures and work more, since the additional labour effort generates more output. Thus, while the household sector desires to save a larger fraction of the real income that it generates through production, it is not possible to increase its aggregate holdings of bonds since the economy is closed and the government's financial position is determined by its policies. However, it is possible for households to save more by accumulating more durables, which are partly consumption goods and partly assets. This implies a substitution from non-durables expenditures towards durables expenditures. Thus, the negative wealth effect triggered by a monetary expansion induces households to work more and save more for retirement, which leads to an expansion in output and a substitution of consumption towards durables.

In the model with search and matching frictions, the labour supply channel is absent and aggregate output is determined by firms' hiring decisions. In this economy, the household sector can increase real savings not only through consumer durables, but also via investment in firm equity. An increased desire to save among households pushes up the market value of the firms, which encourages vacancy posting and boosts employment.[43] Thus, in this version of the model aggregate output increases because of an increase in labor demand rather than in labor supply. Further, aggregate output dynamics are governed by the employment rate, which is a slow-moving state variable which adds to the degree of endogenous persistence in the model.

5.3 Helicopter drops

We now contrast the effects of open market operations to the effects of shocks in an alternative economy in which monetary policy is implemented using "helicopter drops" of money. In the interest of space, we present the results for the modified version of the model with labour market frictions (results for the baseline model paint a similar picture). By a helicopter drop we mean an expansion in the money supply that is not accompanied by an increase in Central Bank bond holdings, but rather by an outright transfer to the Treasury.[44] It then follows that the total transfer from the Treasury to the households is given by its interest earnings on bond holdings (which can be negative) plus the change in the money supply. In real terms, the transfer to the households becomes:

[43] From Equation (18) it can be seen that an increase in the discount factor, $\Lambda_{t,t+1}$, leads to an increase in the firm value, V_t. The free-entry condition dictates that an increase in V_t must be offset by a decline in λ_t,the rate at which vacancies are filled. From the matching function it the follows that hiring increases.

[44] Consequently, $b_t^{\mathbf{cb}}$ remains zero at all times.

$$m_t - \frac{m_{t-1}}{1+\pi_t} + \frac{r_{t-1} b^{\mathbf{g}}_{t-1}}{1+\pi_t} = \nu \rho_o \tau^{\mathbf{n}}_t + \nu \left(1-\rho_o\right) \tau^{\mathbf{y}}_t + \left(1-\nu\right) \tau^{\mathbf{o}}_t \tag{26}$$

We assume again that helicopter drops are gradually reversed after the initial shock, following the same feedback rule as used in the economy with market operations.[45]

Figures 4 plots the responses for the economy with helicopter drops, together with those for the economy with open market operations. Note first that the response of the nominal interest rate is virtually the same as it was before in the case of OMO. The figures show that although response of prices to the helicopter drop is comparable to the one in our economy with OMO, the effects on real economic outcomes are drastically different. In particular, with helicopter drops output and durable expenditures *decline* following an expansion of the money supply, whereas the real interest rate *increases* several periods after the shock. Thus, the transmission of monetary policy depends importantly on the operating procedures of the Central Bank and the associated monetary-fiscal arrangements.

The response of government transfers, plotted in the lower right panel, reveals why the effects of a monetary expansion are so different when helicopter drops are used. Upon impact, there is a large one-time positive transfer with a magnitude of about 1.5 percent of annual GDP. This transfer more than offsets negative revaluation of households' assets, which can be seen from Equation 26.[46] Thus, the household sector now gains following the expansion, at the expense of the government. After the initial shock, the increase in the money supply is gradually reversed by a series of small interventions in the opposite direction, which lower the transfers to households relative to the steady state. Thus, the households who are alive when the shock hits enjoy on aggregate a favorable net redistribution: they receive the entire initial transfer while part of the costs are borne by future generations. Note that under OMO the exact opposite is true. As a result, the transmission mechanism is reversed when helicopter drops are used.

6 Concluding remarks

We study the redistributive and aggregate effects of monetary policy in an economy in which the government is a large net debtor. An expansionary open market operation causes a downward

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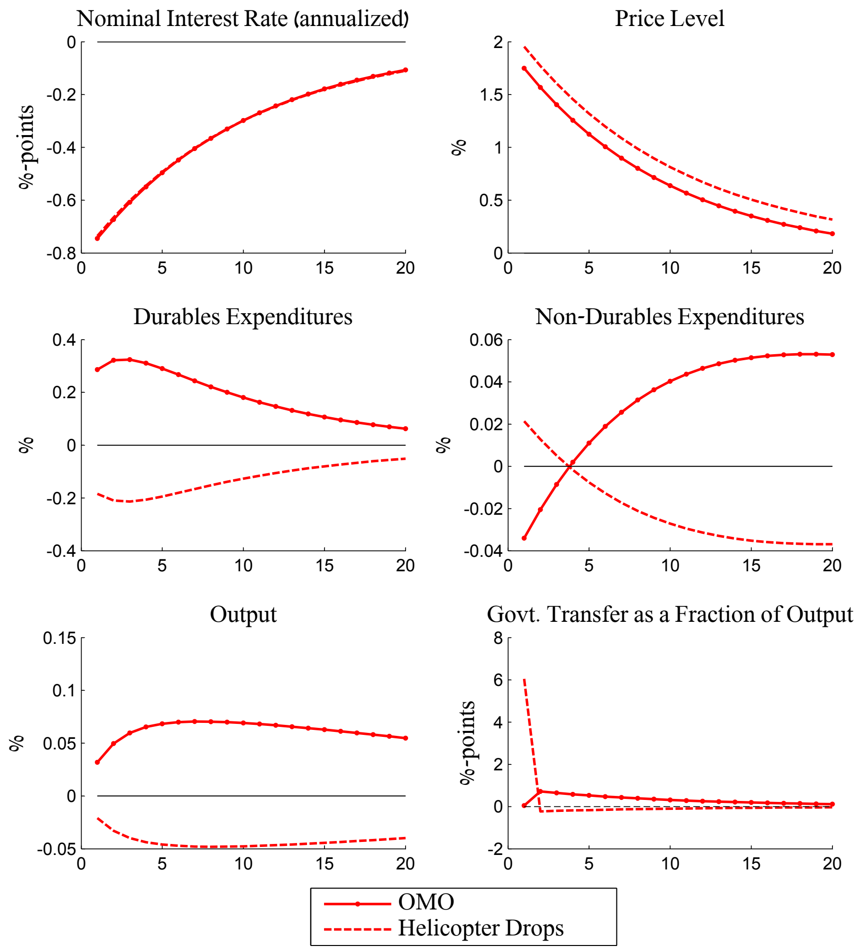
[45] For comparability, we do not re-scale the magnitude of the shock relative to the benchmark model.

[46] To see this explicitly, note that the the change in $\nu \rho_o \tau^{\mathbf{n}}_t + \nu \left(1-\rho_o\right) \tau^{\mathbf{y}}_t + \left(1-\nu\right) \tau^{\mathbf{o}}_t - \frac{m_{t-1}}{1+\pi_t} - \frac{r_{t-1} b^{\mathbf{g}}_{t-1}}{1+\pi_t} = m_t$ captures the effect of the transfer on households' wealth, net of the negative revaluation of households' nominal assets. Given that real money balances, m_t, increases following the intervention, this net effect is positive.

Figure 4: Responses to an Expansionary Monetary Policy Shock in the Model with Search and Matching Frictions: OMO versus Helicopter Drops.



Note: horizontal axes denote quarters after the shock.

revaluation of public debt and a negative wealth effect in the private sector, as households' revaluation losses are not fully compensated by fiscal rebates. Households respond to the fall in wealth by increasing their saving rate, which pushes down the real interest rate. Lower interest rates generate a substitution towards durable goods, causing a boom in the durable good sector. In the baseline model, aggregate hours worked increase due to a labour supply effect. With search and matching frictions, aggregate hours increase as firms post more vacancies. In all, the expansionary OMO causes an increase in output driven by the durable good sector. This response, together with the redistributive effects embedded in the model are consistent with the empirical evidence on the effects of monetary interventions in the US economy.

Our model thus offers a setting consistent with i) the way in which Central Banks affects the policy rate; ii) empirical estimates on how such changes affects the macroeconomy and more specifically, the durable good sector and the real value of public debt; and iii) empirical evidence on the distributional effects of monetary policy. Our results address the challenge posed by Barsky, House and Kimball (2007), who pointed out to a counterfactual prediction of the standard New Keynesian representative-agent model with durable goods. The mechanism emphasized in our model can thus be used to complement the workhorse New Keynesian model in monetary policy analyses.

The model also shows that implementation matters: specifically, expansionary OMO can have sharply different effects from helicopter drops. We stress that in economies with a largely indebted government sector, monetary policy can have significant fiscal repercussions and it is hence important to take them into account to fully understand the effect of monetary interventions. Understanding how the government redistributes its losses or windfalls through spending, investment and taxes is important and we plan to study this second round of redistributions in future work.

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Appendix

In this Appendix we present additional evidence supplementing the empirical results, provide full derivation of the model equations and study extensions of the model that allow for search and matching frictions in the labour market as well as wage rigidity.

A1. The response of different demographic groups

Wong (2014) explores de response of expenditures to monetary policy shocks by different demographic groups. In this Section, we replicate her results and decompose expenditures in durables and nondurables to check the soundness of our model. We find that nondurable expenditures respond very little to monetary policy expansions, as in the aggregate results—and consistent with our model. This is true for all demographic groups. Durable good expenditures are the key variable responding to monetary interventions. Consistent with our model, we find that the increase in expenditures during a monetary expansion is almost entirely driven by the response of young people; hence as in the model we present, the relative response of durables by the young vis-à-vis the old, increases significantly following an expansion. In what follows, we describe the data and approach.

A1.1. Data

The longitudinal data is based on the microdata of the consumer expenditure survey obtained from the ICPSR at the University of Michigan for years 1980-2007[47]. Each household is surveyed for 4 subsequent quarters, where they report monthly expenditures at a disaggregated level. Information about the household demographics and finances are also available. Following Aguiar and Hurst (2012), only households that respond for all quarters (that is, with at least 12 months of data) are kept. Moreover, we keep only urban households, as rural households were not surveyed in the first covered years. This leaves a total of about 80,000 households.

Our measure of durables includes residential investments and other long-term expenditures such as vehicle purchases (new cars, parts) or recreational equipment. Nondurables include

[47] The CEX data is available up to 2012, but the analysis here is restricted to take place before 2007.

services as well as food, alcohol, tobacco, clothing, fossils consumption, and other miscellaneous categories.

The identified monetary policy shocks were obtained through the methodology of Romer and Romer (2004), extended up to 2007.

A1.2. Method

We run the following regression:

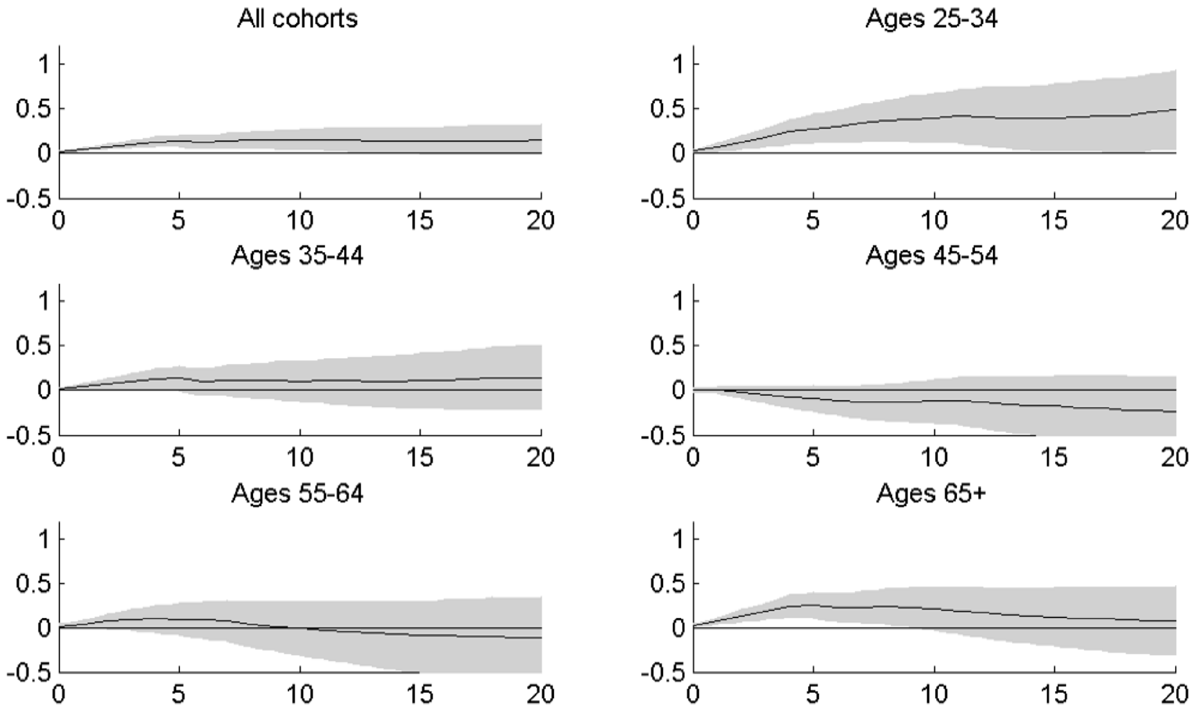
$$\Delta \ln y_{it} = \sum_{s=0}^{20} \beta_s mps_{t-s} + \text{Dummies} + e_{it}$$

Where the left hand-side is the log-change in consumption for household i at time t, mps_{t-s} is the monetary policy shock at t with s lags, and dummies include household and cohort (the year of birth) fixed-effects, as well as family size, the only demographic variable whose coefficient is significant. As we work in log-changes, we compute the cumulative IRF, that is, the cumulative sum of the beta coefficients. For the 95 percent confidence interval band, we follow Romer & Romer (2004)'s Monte Carlo approach in that we draw 10,000 coefficients from a multivariate normal distribution with mean vector and variance-covariance matrix from the OLS regression. For each of these draws, the cumulative IRF is computed, and the 2.5 and 97.5 quantiles are kept to produce the bands of the confidence interval. Then, this regression is run for the entire cohort, and for different age groups.

A1.3. Results

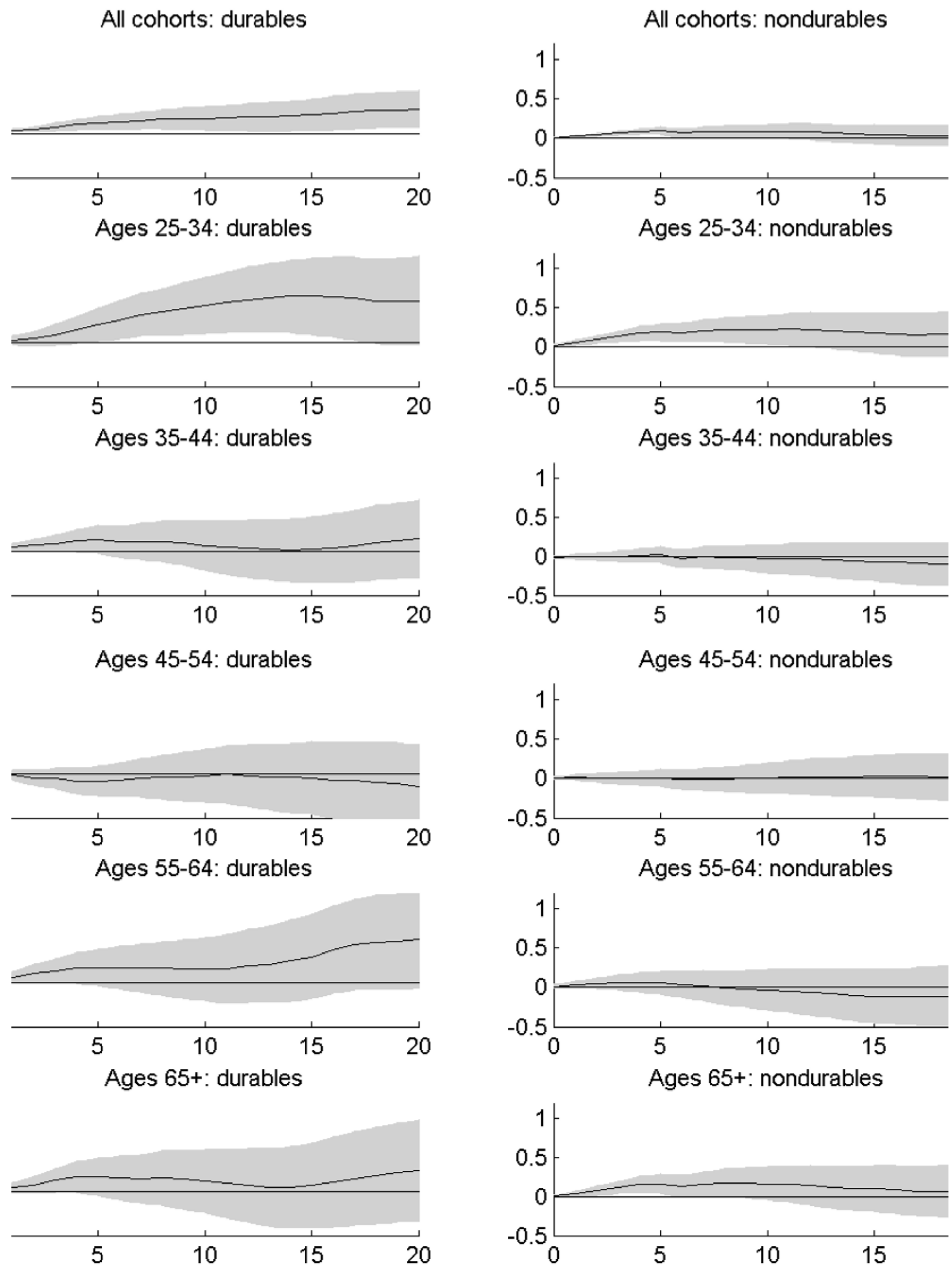
The responses of household expenditures to expansionary monetary policy shocks suggest that the increase in consumption is triggered mainly by young households (25-34), as is shown in Figure A4. More specifically, the increase is preliminary due to the durable good response, as Figure A5 indicates.

Figure A4. Response of Total Expenditures by Age group



Note: The plots show the response of total expenditures to an identified monetary policy shock using Romer&Romer dates. Shaded areas show 90 percent confidence bands.

Figure A5. Response of Durable and Non-durable Good Expenditures by Age group



Note: The plots show the response of durable and non-durable good expenditures to an identified monetary policy shock using Romer&Romer dates. Shaded areas show 90 percent confidence bands.

A2. Model derivations

This Section derives the present-value budget constraint of the government and provides details on the model and the solution strategy.

A2.1 The government's budget constraint

The consolidated government budget constraint in real terms can be written as:

$$b_t^{\mathbf{g}} + b_t^{\mathbf{cb}} - m_t = \frac{1+r_{t-1}}{1+\pi_t}\left(b_{t-1}^{\mathbf{g}} + b_{t-1}^{\mathbf{cb}}\right) - \frac{m_{t-1}}{1+\pi_t} - \tau_t^{\mathbf{g}}$$

where $\tau^{\mathbf{g}} \equiv \nu\rho_o\tau_t^{\mathbf{n}} + \nu\left(1-\rho_o\right)\tau_t^{\mathbf{y}} + \left(1-\nu\right)\tau_t^{\mathbf{o}}$ is the total transfer to the households. We now derive the present-value government budget constraint, see also Ireland (2005). Define:

$$\varpi_{t+1} \equiv \frac{1+r_t}{1+\pi_{t+1}}\left(b_t^{\mathbf{g}} + b_t^{\mathbf{cb}}\right) - \frac{m_t}{1+\pi_{t+1}}$$

and use this definition to express the period-t budget constraint as:

$$\begin{aligned}
\varpi_{t+1} &= \frac{1+r_t}{1+\pi_{t+1}}\left(\frac{1+r_{t-1}}{1+\pi_t}\left(b_{t-1}^{\mathbf{g}} + b_{t-1}^{\mathbf{cb}}\right) - \frac{m_{t-1}}{1+\pi_t} - \tau_t^{\mathbf{g}} + \frac{r_t}{1+r_t}m_t\right), \\
&= \frac{1+r_t}{1+\pi_{t+1}}\left(\varpi_t - \tau_t^{\mathbf{g}} + \frac{r_t}{1+r_t}m_t\right).
\end{aligned}$$

Also, define D_s as in the main text note that $\frac{1+r_s}{1+\pi_{s+1}}D_{s+1} = D_s$. Consider budget constraint for period s and multiply both sides by D_{s+1}:

$$D_{s+1}\varpi_{s+1} = D_s\left(\varpi_s - \tau_s^{\mathbf{g}} + \frac{r_s}{1+r_s}m_s\right).$$

Sum all constraints from period t to infinity:

$$\sum_{s=t}^{\infty} D_{s+1}\varpi_{s+1} = \sum_{s=t}^{\infty} D_s\left(\varpi_s - \tau_s^{\mathbf{g}} + \frac{r_s}{1+r_s}m_s\right),$$

where we impose the limit condition $\sum_{s\to\infty}^{\infty} D_s\varpi_s = 0$. Finally, rearrange to obtain:

$$\sum_{s=t}^{\infty} D_s\left(\frac{r_s}{1+r_s}m_s - \tau_s^{\mathbf{g}}\right) = \frac{m_{t-1} - \left(1+r_{t-1}\right)\left(b_{t-1}^{\mathbf{g}} + b_{t-1}^{\mathbf{cb}}\right)}{1+\pi_t}$$

Furthermore, in the representative agent version of the model we can express the household's first-order condition for money and bonds, respectively, as:

$$\begin{aligned} U_{c,t} &= U_{m,t} + \widetilde{\beta}\mathbb{E}_t \frac{1}{1+\pi_{t+1}} U_{c,t+1} \\ U_{c,t} &= \widetilde{\beta}\mathbb{E}_t \frac{1+r_t}{1+\pi_{t+1}} U_{c,t+1} \end{aligned}$$

which can be combined as:

$$U_{c,t} = \frac{1+r_t}{r_t} U_{m,t}$$

Under the logarithmic preferences assumed in Section 3.1.8 this equation becomes $\mu c_t = \frac{r_t}{1+r_t} m_t$. Given that non-durable consumption is not affected by monetary policy in the representative agent version, it follows that $\frac{r_t}{1+r_t} m_t$ is not affected either.

A2.2 Solving the model

The model is solved using first-order perturbation (linearization). This part of the Appendix describes the first-order conditions for the optimization problems of the individuals and discusses aggregation of the individuals' choices.

Old agents and aggregation. Although the model features a representative young agent, there is wealth heterogeneity among the old agents. Typically, dynamic models with a large number of heterogeneous agents are challenging to solve. For our model, however, it turns out that the decision rules of the old are linear in wealth, which implies that aggregation is straightforward. Hence we can solve for aggregates without reference to the distribution of wealth among old agents. Wealth heterogeneity between young and old agents, however, is a key factor driving aggregate dynamics.

We exploit that the use of first-order perturbation implies certainty equivalence (see Schmitt-Grohé and Uribe (2004)). As a consequence, first-order approximations to the equilibrium laws of motion of the model coincide with those obtained for a version without aggregate uncertainty.[48] In what follows, we therefore omit expectations operators.[49]

The first-order conditions for the choices of durables, money and bonds by an old household

[48] Both versions preserve *idiosyncratic* uncertainty.

[49] Alternatively, one could first linearize the model equations and then perform the steps described below.

i can be written, respectively, as:

$$
\begin{aligned}
U_{c,i,t} &= U_{d,i,t} + \beta\left(1-\rho_x\right)\left(1-\delta\right)U_{c,i,t+1}, \\
U_{c,i,t} &= U_{m,i,t} + \frac{\beta\left(1-\rho_x\right)}{1+\pi_{t+1}}U_{c,i,t+1}, \\
U_{c,i,t} &= \frac{\beta\left(1-\rho_x\right)\left(1+r_t\right)}{1+\pi_{t+1}}U_{c,i,t+1}.
\end{aligned}
$$

Now introduce four auxiliary variables $\gamma_{c,i,t} \equiv \frac{c_{i,t}}{a_{i,t}}$, $\gamma_{d,i,t} \equiv \frac{d_{i,t}}{a_{i,t}}$, $\gamma_{m,i,t} \equiv \frac{m_{i,t}}{a_{i,t}}$ and $\gamma_{b,i,t} \equiv \frac{b_{i,t}}{a_{i,t}}$. The crucial step is to show that there are four restrictions that pin down $\gamma_{c,i,t}$, $\gamma_{d,i,t}$, $\gamma_{m,i,t}$ and $\gamma_{b,i,t}$ as functions of *only* aggregate variables. To find these coefficients, first combine the first-order conditions to obtain:

$$
\begin{aligned}
U_{c,i,t} &= U_{d,i,t} + \left(1-\delta\right)\left(1+\pi_{t+1}\right)\left(U_{c,i,t}-U_{m,i,t}\right) \\
U_{c,i,t} &= \left(1+r_t\right)\left(U_{c,i,t}-U_{m,i,t}\right)
\end{aligned}
$$

Under the assumed nested CES preferences we obtain:

$$
\begin{aligned}
U_{c,i,t} &= x_{i,t}^{-\sigma}\frac{\epsilon}{\epsilon-1}\left[c_{i,t}^{\frac{\epsilon-1}{\epsilon}} + \eta d_{i,t}^{\frac{\epsilon-1}{\epsilon}} + \mu m_{i,t}^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}-1}\frac{\epsilon-1}{\epsilon}c_{i,t}^{\frac{\epsilon-1}{\epsilon}-1}, \\
&= x_{i,t}^{\frac{-\sigma\epsilon+1}{\epsilon}}c_{i,t}^{\frac{-1}{\epsilon}}, \\
U_{d,i,t} &= x_{i,t}^{\frac{-\sigma\epsilon+1}{\epsilon}}\eta d_{i,t}^{\frac{-1}{\epsilon}}, \\
U_{m,i,t} &= x_{i,t}^{\frac{-\sigma\epsilon+1}{\epsilon}}\mu m_{i,t}^{\frac{-1}{\epsilon}}.
\end{aligned}
$$

The combined first-order conditions become:

$$
\gamma_{c,i,t}^{\frac{-1}{\epsilon}} = \eta\gamma_{d,i,t}^{\frac{-1}{\epsilon}} + \left(1-\delta\right)\left(1+\pi_{t+1}\right)\left(\gamma_{c,i,t}^{\frac{-1}{\epsilon}} - \mu\gamma_{m,i,t}^{\frac{-1}{\epsilon}}\right) \tag{27}
$$

$$
\gamma_{c,i,t}^{\frac{-1}{\epsilon}} = \left(1+r_t\right)\left(\gamma_{c,i,t}^{\frac{-1}{\epsilon}} - \mu\gamma_{m,i,t}^{\frac{-1}{\epsilon}}\right) \tag{28}
$$

To get the third restriction, consider the Euler equation for bonds, which can be written as:

$$
\left(\frac{x_{i,t}}{x_{i,t+1}}\right)^{\frac{-\sigma\epsilon+1}{\epsilon}}\left(\frac{c_{i,t}}{c_{i,t+1}}\right)^{\frac{-1}{\epsilon}} = \frac{\beta\left(1-\rho_x\right)\left(1+r_t\right)}{\left(1+\pi_{t+1}\right)} \tag{29}
$$

and use the fact that $a_{i,t+1} = \left((1-\delta)\,\gamma_{d,i,t} + \frac{\gamma_{m,i,t}}{1+\pi_{t+1}} + \frac{(1+r_t)\gamma_{b,i,t}}{1+\pi_{t+1}} \right) a_{i,t}$ to write:

$$
\begin{aligned}
\frac{c_{i,t}}{c_{i,t+1}} &= \frac{\gamma_{c,i,t}}{\gamma_{c,i,t+1}\left((1-\delta)\,\gamma_{d,i,t} + \frac{\gamma_{m,i,t}}{1+\pi_{t+1}} + \frac{(1+r_t)\gamma_{b,i,t}}{1+\pi_{t+1}} \right)} \\
\frac{x_{i,t}}{x_{i,t+1}} &= \left[\left(\frac{\gamma_{c,i,t}^{\frac{\epsilon-1}{\epsilon}} + \eta\gamma_{d,i,t}^{\frac{\epsilon-1}{\epsilon}} + \mu\gamma_{m,i,t}^{\frac{\epsilon-1}{\epsilon}}}{\gamma_{c,i,t+1}^{\frac{\epsilon-1}{\epsilon}} + \eta\gamma_{d,i,t+1}^{\frac{\epsilon-1}{\epsilon}} + \mu\gamma_{m,i,t+1}^{\frac{\epsilon-1}{\epsilon}}} \right) \right]^{\frac{\epsilon}{\epsilon-1}} \frac{1}{(1-\delta)\,\gamma_{d,i,t} + \frac{\gamma_{m,i,t}}{1+\pi_{t+1}} + \frac{(1+r_t)\gamma_{b,i,t}}{1+\pi_{t+1}}}
\end{aligned}
$$

The budget constraint gives the fourth restriction since it can be written as:

$$
\gamma_{c,i,t}a_{i,t} + \gamma_{d,i,t}a_{i,t} + \gamma_{m,i,t}a_{i,t} + \gamma_{b,i,t}a_{i,t} = a_{i,t}
$$

or:

$$
\gamma_{c,i,t} + \gamma_{d,i,t} + \gamma_{m,i,t} + \gamma_{b,i,t} = 1 \tag{30}
$$

Equations (27)-(30) pin down $\gamma_{c,i,t}$, $\gamma_{d,i,t}$, $\gamma_{m,i,t}$ and $\gamma_{b,i,t}$ as functions of only aggregate variables, as we have substituted out individual wealth from all the equations. Hence we can omit individual i-subscripts for these variables. Given the average wealth level among old agents, $a_t^{\mathbf{o}}$, we can now compute averages for the old agents' decision variables as $c_t^{\mathbf{o}} = \gamma_{c,t}a_t^{\mathbf{o}}$, $d_t^{\mathbf{o}} = \gamma_{d,t}a_t^{\mathbf{o}}$, $m_t^{\mathbf{o}} = \gamma_{m,t}a_t^{\mathbf{o}}$ and $b_t^{\mathbf{o}} = \gamma_{b,t}a_t^{\mathbf{o}}$. Note that these objects do not depend on the distribution of wealth among old agents. Finally, we can express aggregate wealth owned by the old agents as:

$$
\begin{aligned}
a_t^{\mathbf{o}} &= (1-\rho_x)\left((1-\delta)\,d_{t-1}^{\mathbf{o}} + \frac{m_{t-1}^{\mathbf{o}} + (1+r_{t-1})b_{t-1}^{\mathbf{o}}}{1+\pi_t} \right) \\
&\quad + \rho_o\,(1-\rho_x)\,\frac{\nu}{1-\nu}\left[(1-\delta)\,d_{t-1}^{\mathbf{y}} + \frac{m_{t-1}^{\mathbf{y}} + (1+r_{t-1})b_{t-1}^{\mathbf{y}}}{1+\pi_t} \right].
\end{aligned}
$$

Young agents. As discussed in the main text there is effectively a representative young agent. Its first-order conditions for the choices of labour, durables, money and bonds can be written, respectively, as:

$$
\begin{aligned}
U_{c,t}^{\mathbf{y}} &= \zeta h_t^{\kappa} \\
U_{c,t}^{\mathbf{y}} &= U_{d,t}^{\mathbf{y}} + \beta\,(1-\rho_o)\,(1-\delta)\,U_{c,t+1}^{\mathbf{y}} + \beta\rho_o\,(1-\rho_x)\,(1-\delta)\,U_{c,t+1}^{\mathbf{yo}}, \\
U_{c,t}^{\mathbf{y}} &= U_{m,t}^{\mathbf{y}} + \beta\left(\frac{1-\rho_o}{1+\pi_{t+1}} \right) U_{c,t+1}^{\mathbf{y}} + \beta\frac{\rho_o\,(1-\rho_x)}{1+\pi_{t+1}}U_{c,t+1}^{\mathbf{yo}}, \\
\frac{U_{c,t}^{\mathbf{y}}}{(1+r_t)} &= \beta\frac{1-\rho_o}{1+\pi_{t+1}}U_{c,t+1}^{\mathbf{y}} + \beta\frac{\rho_o\,(1-\rho_x)}{1+\pi_{t+1}}U_{c,t+1}^{\mathbf{yo}}.
\end{aligned}
$$

Here, $U^{\mathbf{y}}_{c,t}$ and $U^{\mathbf{yo}}_{c,t}$ are the marginal utility of non-durables of the young and newly retired agents, respectively, which satisfy:

$$\begin{aligned} U^{\mathbf{y}}_{c,t} &= \left(x^{\mathbf{y}}_t\right)^{\frac{-\sigma\epsilon+1}{\epsilon}} \left(c^{\mathbf{y}}_t\right)^{\frac{-1}{\epsilon}} \\ U^{\mathbf{y}}_{d,t} &= \left(x^{\mathbf{y}}_t\right)^{\frac{-\sigma\epsilon+1}{\epsilon}} \eta \left(d^{\mathbf{y}}_t\right)^{\frac{-1}{\epsilon}} \\ U^{\mathbf{y}}_{m,t} &= \left(x^{\mathbf{y}}_t\right)^{\frac{-\sigma\epsilon+1}{\epsilon}} \mu \left(m^{\mathbf{y}}_t\right)^{\frac{-1}{\epsilon}} \\ U^{\mathbf{yo}}_{c,t} &= \left(x^{\mathbf{yo}}_t\right)^{\frac{-\sigma\epsilon+1}{\epsilon}} \left(c^{\mathbf{yo}}_t\right)^{\frac{-1}{\epsilon}} \end{aligned}$$

where $x^{\mathbf{yo}}_t = \left[\left(c^{\mathbf{yo}}_t\right)^{\frac{\epsilon-1}{\epsilon}} + \eta\left(d^{\mathbf{yo}}_t\right)^{\frac{\epsilon-1}{\epsilon}} + \mu\left(m^{\mathbf{yo}}_t\right)^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}}$. Note that for the newly retired agents it holds that $c^{\mathbf{yo}}_t = \gamma_{c,t} a^{\mathbf{y}}_t$. Finally, the wealth of a young agent can be expressed as:

$$\begin{aligned} a^{\mathbf{y}}_t &= \left(1-\rho_o+\rho_o\rho_x\right)\left(\left(1-\delta\right)d^{\mathbf{y}}_{t-1} + \frac{m^{\mathbf{y}}_{t-1}+(1+r_{t-1})b^{\mathbf{y}}_{t-1}}{1+\pi_t}\right) \\ &\quad + \frac{1-\nu}{\nu}\rho_x\left(\left(1-\delta\right)d^{\mathbf{o}}_{t-1} + \frac{m^{\mathbf{o}}_{t-1}+(1+r_{t-1})b^{\mathbf{o}}_{t-1}}{1+\pi_t}\right). \end{aligned}$$

The full system. We are now ready to collect the equations and summarize the entire model. Old agents:

$$\gamma_{c,t}^{\frac{-1}{\epsilon}} = \eta\gamma_{d,t}^{\frac{-1}{\epsilon}} + \left(1-\delta\right)\left(1+\pi_{t+1}\right)\left(\gamma_{c,t}^{\frac{-1}{\epsilon}} - \mu\gamma_{m,t}^{\frac{-1}{\epsilon}}\right) \tag{31}$$

$$\gamma_{c,t}^{\frac{-1}{\epsilon}} = \left(1+r_t\right)\left(\gamma_{c,t}^{\frac{-1}{\epsilon}} - \mu\gamma_{m,t}^{\frac{-1}{\epsilon}}\right) \tag{32}$$

$$\frac{\beta\left(1-\rho_x\right)\left(1+r_t\right)}{\left(1+\pi_{t+1}\right)} = \left(\Phi_t\right)^{\frac{-\sigma\epsilon+1}{\epsilon}}\left(\frac{\gamma_{c,t+1}}{\gamma_{c,t}}\left(\left(1-\delta\right)\gamma_{d,t} + \frac{\gamma_{m,t}}{1+\pi_{t+1}} + \frac{\left(1+r_t\right)\gamma_{b,t}}{1+\pi_{t+1}}\right)\right)^{\frac{1}{\epsilon}} \tag{33}$$

$$\Phi_t = \left[\left(\frac{\gamma_{c,t}^{\frac{\epsilon-1}{\epsilon}} + \eta\gamma_{d,t}^{\frac{\epsilon-1}{\epsilon}} + \mu\gamma_{m,t}^{\frac{\epsilon-1}{\epsilon}}}{\gamma_{c,t+1}^{\frac{\epsilon-1}{\epsilon}} + \eta\gamma_{d,i,t+1}^{\frac{\epsilon-1}{\epsilon}} + \mu\gamma_{m,t+1}^{\frac{\epsilon-1}{\epsilon}}}\right)\right]^{\frac{\epsilon}{\epsilon-1}} \frac{1}{\left(1-\delta\right)\gamma_{d,t} + \frac{\gamma_{m,t}}{1+\pi_{t+1}} + \frac{(1+r_t)\gamma_{b,t}}{1+\pi_{t+1}}} \tag{34}$$

$$c^{\mathbf{o}}_t = \gamma_{c,t} a^{\mathbf{o}}_t \tag{35}$$

$$d^{\mathbf{o}}_t = \gamma_{d,t} a^{\mathbf{o}}_t \tag{36}$$

$$m^{\mathbf{o}}_t = \gamma_{m,t} a^{\mathbf{o}}_t \tag{37}$$

$$b^{\mathbf{o}}_t = \gamma_{b,t} a^{\mathbf{o}}_t \tag{38}$$

$$a^{\mathbf{o}}_t = \left(1-\rho_x\right)\left(\left(1-\delta\right)d^{\mathbf{o}}_{t-1} + \frac{m^{\mathbf{o}}_{t-1}+(1+r_{t-1})b^{\mathbf{o}}_{t-1}}{1+\pi_t}\right) \tag{39}$$

$$+\rho_o\left(1-\rho_x\right)\frac{\nu}{1-\nu}\left[\left(1-\delta\right)d^{\mathbf{y}}_{t-1} + \frac{m^{\mathbf{y}}_{t-1}+(1+r_{t-1})b^{\mathbf{y}}_{t-1}}{1+\pi_t}\right] \tag{40}$$

$$a^{\mathbf{o}}_t = c^{\mathbf{o}}_t + d^{\mathbf{o}}_t + m^{\mathbf{o}}_t + b^{\mathbf{o}}_t \tag{41}$$

Young agents:

$$\left(x_t^{\mathbf{y}}\right)^{\frac{-\sigma\epsilon+1}{\epsilon}}\left(c_t^{\mathbf{y}}\right)^{\frac{-1}{\epsilon}} = \zeta h_t^{\kappa} \tag{42}$$

$$\begin{aligned}\left(c_t^{\mathbf{y}}\right)^{\frac{-1}{\epsilon}} &= \eta\left(d_t^{\mathbf{y}}\right)^{\frac{-1}{\epsilon}} + \beta\left(1-\rho_o\right)\left(1-\delta\right)\left(\frac{x_{t+1}^{\mathbf{y}}}{x_t^{\mathbf{y}}}\right)^{\frac{-\sigma\epsilon+1}{\epsilon}}\left(c_{t+1}^{\mathbf{y}}\right)^{\frac{-1}{\epsilon}} \\ &\quad +\beta\rho_o\left(1-\rho_x\right)\left(1-\delta\right)\left(\frac{x_{t+1}^{\mathbf{yo}}}{x_t^{\mathbf{y}}}\right)^{\frac{-\sigma\epsilon+1}{\epsilon}}\left(c_{t+1}^{\mathbf{yo}}\right)^{\frac{-1}{\epsilon}},\end{aligned} \tag{43}$$

$$\begin{aligned}\left(c_t^{\mathbf{y}}\right)^{\frac{-1}{\epsilon}} &= \mu\left(m_t^{\mathbf{y}}\right)^{\frac{-1}{\epsilon}} + \beta\left(\frac{1-\rho_o}{1+\pi_{t+1}}\right)\left(\frac{x_{t+1}^{\mathbf{y}}}{x_t^{\mathbf{y}}}\right)^{\frac{-\sigma\epsilon+1}{\epsilon}}\left(c_{t+1}^{\mathbf{y}}\right)^{\frac{-1}{\epsilon}} \\ &\quad +\beta\frac{\rho_o\left(1-\rho_x\right)}{1+\pi_{t+1}}\left(\frac{x_{t+1}^{\mathbf{yo}}}{x_t^{\mathbf{y}}}\right)^{\frac{-\sigma\epsilon+1}{\epsilon}}\left(c_{t+1}^{\mathbf{yo}}\right)^{\frac{-1}{\epsilon}},\end{aligned} \tag{44}$$

$$\begin{aligned}\left(c_t^{\mathbf{y}}\right)^{\frac{-1}{\epsilon}} &= \beta\frac{\left(1-\rho_o\right)\left(1+r_t\right)}{1+\pi_{t+1}}\left(\frac{x_{t+1}^{\mathbf{y}}}{x_t^{\mathbf{y}}}\right)^{\frac{-\sigma\epsilon+1}{\epsilon}}\left(c_{t+1}^{\mathbf{y}}\right)^{\frac{-1}{\epsilon}} \\ &\quad +\beta\frac{\rho_o\left(1-\rho_x\right)\left(1+r_t\right)}{1+\pi_{t+1}}\left(\frac{x_{t+1}^{\mathbf{yo}}}{x_t^{\mathbf{y}}}\right)^{\frac{-\sigma\epsilon+1}{\epsilon}}\left(c_{t+1}^{\mathbf{yo}}\right)^{\frac{-1}{\epsilon}}.\end{aligned} \tag{45}$$

$$\begin{aligned}a_t^{\mathbf{y}} &= \left(1-\rho_o+\rho_o\rho_x\right)\left(\left(1-\delta\right)d_{t-1}^{\mathbf{y}} + \frac{m_{t-1}^{\mathbf{y}}+(1+r_{t-1})b_{t-1}^{\mathbf{y}}}{1+\pi_t}\right) \\ &\quad +\frac{1-\nu}{\nu}\rho_x\left(\left(1-\delta\right)d_{t-1}^{\mathbf{o}} + \frac{m_{t-1}^{\mathbf{o}}+(1+r_{t-1})b_{t-1}^{\mathbf{o}}}{1+\pi_t}\right)\end{aligned} \tag{46}$$

$$c_t^{\mathbf{y}} + d_t^{\mathbf{y}} + m_t^{\mathbf{y}} + b_t^{\mathbf{y}} = a_t^{\mathbf{y}} + h_t^{\mathbf{y}} + \tau_t^{\mathbf{s}} \tag{47}$$

$$c_t^{\mathbf{yo}} = \gamma_{c,t}a_t^{\mathbf{y}} \tag{48}$$

$$x_t^{\mathbf{yo}} = \left[\left(\gamma_{c,t}a_t^{\mathbf{y}}\right)^{\frac{\epsilon-1}{\epsilon}} + \eta\left(\gamma_{d,t}a_t^{\mathbf{y}}\right)^{\frac{\epsilon-1}{\epsilon}} + \mu\left(\gamma_{m,t}a_t^{\mathbf{y}}\right)^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}} \tag{49}$$

$$x_t^{\mathbf{y}} = \left[\left(c_t^{\mathbf{y}}\right) + \eta\left(d_t^{\mathbf{y}}\right)^{\frac{\epsilon-1}{\epsilon}} + \mu\left(m_t^{\mathbf{y}}\right)^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}} \tag{50}$$

Government policy:

$$\frac{r_{t-1}\left(b_{t-1}^{\mathbf{g}}+b_{t-1}^{\mathbf{cb}}\right)}{1+\pi_t} = \nu\left(1-\rho_o\right)\tau_t^{\mathbf{s}} \tag{51}$$

$$\frac{m_t}{m_{t-1}}\left(1+\pi_t\right) = 1+z_t \tag{52}$$

$$z_t = \xi\left(\overline{m}-m_{t-1}\right)+\varepsilon_t \tag{53}$$

Market clearing:

$$
\begin{aligned}
c_t + d_t &= \nu h_t^{\mathbf{y}} + (1-\delta)\, d_{t-1} && (54)\\
c_t &= \nu c_t^{\mathbf{y}} + (1-\nu)\, c_t^{\mathbf{o}} && (55)\\
d_t &= \nu d_t^{\mathbf{y}} + (1-\nu)\, d_t^{\mathbf{o}} && (56)\\
m_t &= \nu m_t^{\mathbf{y}} + (1-\nu)\, m_t^{\mathbf{o}} && (57)\\
0 &= b_t^{\mathbf{g}} + b_t^{\mathbf{cb}} + \nu b_t^{\mathbf{y}} + (1-\nu)\, b_t^{\mathbf{o}} && (58)
\end{aligned}
$$

These are 28 equations in 28 variables, being c_t, $c_t^{\mathbf{o}}$, $c_t^{\mathbf{yo}}$, $c_t^{\mathbf{y}}$, d_t, $d_t^{\mathbf{o}}$, $d_t^{\mathbf{y}}$, m_t, $m_t^{\mathbf{o}}$, $m_t^{\mathbf{y}}$, $b_t^{\mathbf{o}}$, $b_t^{\mathbf{y}}$, $b_t^{\mathbf{g}}$, $b_t^{\mathbf{cb}}$, $x_t^{\mathbf{y}}$, $x_t^{\mathbf{yo}}$,Φ_t, $\gamma_{c,t}$, $\gamma_{d,t}$, $\gamma_{m,t}$, $\gamma_{b,t}$, $h_t^{\mathbf{y}}$, r_t, π_t, $\tau_t^{\mathbf{s}}$, z_t, $a_t^{\mathbf{o}}$, and $a_t^{\mathbf{yo}}$. We leave out the government's budget constraint, which is redundant by Walras' law.

Special cases We present two simplifying special cases of the model.

Special case 1 ($\epsilon = 1$). When the utility elasticity ϵ equals one, the utility function becomes a Cobb-Douglas basket nested in a CRRA function:

$$
U(c_{i,t}, d_{i,t}, m_{i,t}) = \frac{\left(c_{i,t} d_{i,t}^{\eta} m_{i,t}^{\mu}\right)^{1-\sigma} - 1}{1-\sigma}
$$

and the marginal utilities become $U_{c,i,t} = \frac{x_{i,t}^{1-\sigma}}{c_{i,t}}$ $U_{d,i,t} = \eta \frac{x_{i,t}^{1-\sigma}}{d_{i,t}}$ and $U_{m,i,t} = \mu \frac{x_{i,t}^{1-\sigma}}{m_{i,t}}$. In the system to be solved, we correspondingly set:

$$
\begin{aligned}
x_t^{\mathbf{y}} &= \left(c_t^{\mathbf{y}}\right)\left(d_t^{\mathbf{y}}\right)^{\eta}\left(m_t^{\mathbf{y}}\right)^{\mu}\\
x_t^{\mathbf{yo}} &= \left(c_t^{\mathbf{yo}}\right)\left(d_t^{\mathbf{yo}}\right)^{\eta}\left(m_t^{\mathbf{yo}}\right)^{\mu}\\
\Phi_t &= \left(\frac{\gamma_{c,t}}{\gamma_{c,t+1}}\right)\left(\frac{\gamma_{d,t}}{\gamma_{d,t+1}}\right)^{\eta}\left(\frac{\gamma_{m,t}}{\gamma_{m,t+1}}\right)^{\mu}\left((1-\delta)\,\gamma_{d,t} + \frac{\gamma_{m,t}}{1+\pi_{t+1}} + \frac{(1+r_t)\,\gamma_{b,t}}{1+\pi_{t+1}}\right)^{-(1+\eta+\mu)}
\end{aligned}
$$

Special case 2 ($\sigma = \epsilon = 1$). When both the risk aversion coefficient σ and the utility elasticity ϵ are unity, the utility function further simplifies to:

$$
U(c_{i,t}, d_{i,t}, m_{i,t}) = \ln c_{i,t} + \eta \ln d_{i,t} + \mu \ln m_{i,t}
$$

and the marginal utilities become $U_{c,i,t} = \frac{1}{c_{i,t}}$, $U_{d,i,t} = \frac{\eta}{d_{i,t}}$ and $U_{m,i,t} = \frac{\mu}{m_{i,t}}$. We can therefore set $\left(x_t^{\mathbf{y}}\right)^{\frac{-\sigma\epsilon+1}{\epsilon}} = \left(x_t^{\mathbf{yo}}\right)^{\frac{-\sigma\epsilon+1}{\epsilon}} = \left(\Phi_t\right)^{\frac{-\sigma\epsilon+1}{\epsilon}} = 1$.

Additional References

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