Fear of Secular Stagnation and the Natural Interest Rate

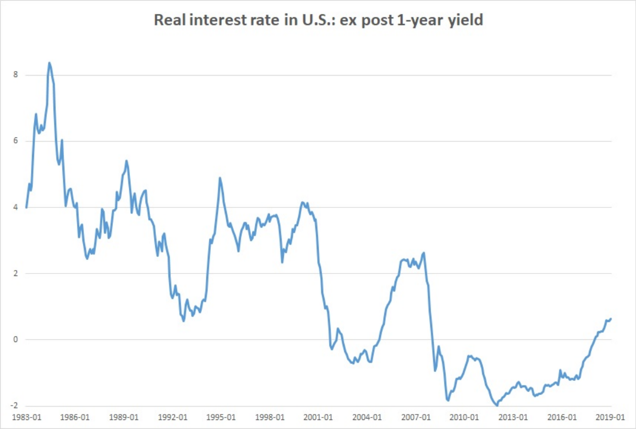
Paolo Bonomolo, *De Nederlandsche Bank*[1]
Valentina Gavazza, *Stockholm University*

22nd Annual DNB Research Conference

Amsterdam, 1 October 2019

[1]The views expressed are those of the authors and do not necessarily reflect official positions of De Nederlandsche Bank

Motivation



Two features of the data:

1. Downward trend
2. Sharp and persistent drop during the great recession

Motivation

Question: *How can we explain the sudden drop in interest rates in the aftermath of the financial crisis?*

- The downward trend has been explained by slow moving forces: demographics, increase in inequality, etc..
- The same factors hardly explain the fast drop in the interest rates

Plausible causes:

- A decrease in productivity that occurred during the crisis
- A change in the **agents' beliefs**

Motivation

The aim of this paper: study the role of agent's beliefs and pessimism in explaining the drop in interest rates during the Great Recession

- Uncertainty about the nature of the shocks that hit the economy: was the decline in GDP persistent but temporary, or permanent?
- Relevant issue in the economic debate: the hypothesis of "Secular Stagnation" (Gordon,2012; Summers, 2014)

Motivation

Conjecture: the attribution of a positive probability to the scenario of secular stagnation acts "per se" as a force that induces a more cautious behavior:

consume less and save more => lower natural interest rate

Income effect due to revision in future conditions (see also Blanchard, Lorenzoni and L'Huillier, 2017)

The higher is the agents' pessimism, the bigger is this effect

This paper:

- Verify if this conjecture is empirically relevant
- Quantify the role of beliefs and pessimism in explaining the decline of the interest rates

Today's presentation

Macro-econometric strategy that serves our purpose:

- Propose a general equilibrium model with growth, where
  - The agents do not observe the determinants of productivity
  - They take into account this uncertainty in their decision making process
  - They can be pessimist
  - Uncertainty over the components of productivity and pessimism can vary over time

Outline

- The environment
  - The technology process and the uncertainty over its components
  - How to model pessimism: Recursive smooth ambiguity preferences
- Taking the model to the data:
  - Perturbation technique
  - Econometric strategy
- The core mechanism through a simple example
- Conclusion

The technology process

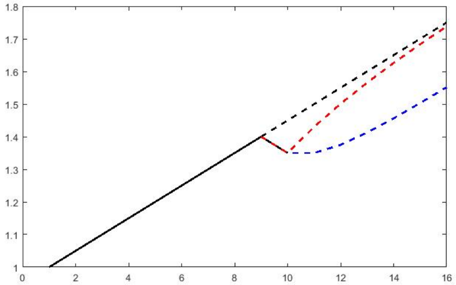
The process for technology is described by the following Dynamic Linear Model(DLM):

$$
\begin{aligned}
\ln(A_t) &= l_t + f_t \\
l_t &= l_{t-1} + \gamma_t \\
\gamma_t &= (1-\rho_\gamma)\,\bar{\gamma} + \rho_\gamma \gamma_{t-1} + \sigma_\gamma \epsilon_{\gamma,t} \\
f_t &= \rho_f f_{t-1} + \sigma_f \epsilon_{f,t}
\end{aligned}
$$

where $(\epsilon_{f,t}, \epsilon_{\gamma,t})' \sim N(0, I)$. The technology process has two components: a trend component (l_t) and a business cycle component (f_t).

The agents observe A_t, but not its components $\theta_t = [\gamma_t \; f_t \; l_t]'$, and do not observe the realization of $\epsilon_{\gamma,t}$ and $\epsilon_{f,t}$. Parameters are known.

The technology process



- This assumption introduces an extra layer of uncertainty
- The distribution of technology tomorrow is not known: its expected value depends on unobserved variables

=> The agents face *Ambiguity*: they consider a set of distributions

A simple example: endowment economy

The preferences

In each period the representative agent decides how much of his endowment A_t to consume and how much to invest in a bond

He has *recursive smooth ambiguity preferences* (Klibanoff, Marinacci and Mukerji, 2005; 2009).

- Under complete information:

$$V_{s^t}\left(B_t\right) = \max_{C_t, B_{t+1}} u\left(C_t\right) + \beta E_{\theta_t} V_{\left(s^t, A_{t+1}\right)}\left(B_{t+1}\right)$$

- Under subjective expected utility (Bayesian):

$$V_{s^t}\left(B_t, \mu_t\right) = \max_{C_t, B_{t+1}} u\left(C_t\right) + \beta\left[E_{\mu_t}\left(E_{\theta_t} V_{\left(s^t, A_{t+1}\right)}\left(B_{t+1}, \mu_{t+1}\right)\right)\right]$$

- Under **smooth ambiguity**:

$$V_{s^t}\left(B_t, \mu_t\right) = \max_{C_t, B_{t+1}} u\left(C_t\right) + \beta\phi^{-1}\left[E_{\mu_t}\phi\left(E_{\theta_t} V_{\left(s^t, A_{t+1}\right)}\left(B_{t+1}, \mu_{t+1}\right)\right)\right]$$

The preferences

$$V_{s^t}\left(B_t,\mu_t\right)=\max_{C_t,B_{t+1}} u\left(C_t\right)+\beta\phi^{-1}\left[E_{\mu_t}\phi\left(E_{\theta_t}V_{\left(s^t,A_{t+1}\right)}\left(B_{t+1},\mu_{t+1}\right)\right)\right]$$

- *Ambiguity*: characterized by the variance of the posterior distribution μ_t.
- *Ambiguity attitude*: characterized by the shape of ϕ
  - concave: ambiguity averse (pessimist)
  - linear: ambiguity neutral (Bayesian)
  - convex: ambiguity loving (optimist)

We assume

$$\phi(y,\alpha)=-\frac{1}{\alpha}\exp\{-\alpha y\}$$

α: coefficient of ambiguity attitude

The equilibrium conditions

The Euler equation:

$$1 = E_{\mu_t}\left[\xi_t\left(\theta_t\right)E_{\theta_t}\left(\beta\frac{A_t}{A_{t+1}}\right)\right]R_{t+1}$$

$$\ln\left(\frac{A_{t+1}}{A_t}\right) = \left(1-\rho_\gamma\right)\bar{\gamma} + \rho_\gamma\gamma_t + \left(\rho_f - 1\right)f_t + \sigma_\gamma\epsilon_{\gamma,t+1} + \sigma_f\epsilon_{f,t+1}$$

where

$$\xi_t\left(\theta_t\right) \equiv \frac{\exp\left\{-\alpha E_{\theta_t}V_{t+1}\right\}}{E_{\mu_t}\left[\exp\left\{-\alpha E_{\theta_t}V_{t+1}\right\}\right]}$$

$\xi_t\left(\theta_t\right)$ creates a wedge between the expectations of a bayesian agent and of an ambiguity-averse agent: *pessimism*

The beliefs distortion

$$\xi_t = \frac{\exp\left[-\alpha E_{\theta_t}\left(V_{t+1}\right)\right]}{E_{\mu_t}\left[\exp\left[-\alpha E_{\theta_t}\left(V_{t+1}\right)\right]\right]}$$

- ξ_t is a Radon-Nikodym derivative with respect to the posterior distribution μ
$$d\mu_t^* = \xi_t d\mu_t$$
- It induces a change of measure from μ_t to the distorted posterior μ_t^*
- Two sources of pessimism:
  - *Ambiguity attitude*: α
  - *Ambiguity*: the variance of the Bayesian posterior distribution

The beliefs distortion: the sources of pessimism

Time variation in the two contributions to pessimism:

- *Ambiguity attitude*: α_t
  We assume, as in Bhandari, Borovicka and Ho (2019):

$$\alpha_t = (1-\rho_\alpha)\bar{\alpha} + \rho_\alpha \alpha_{t-1} + \sigma_\alpha \epsilon_{\alpha t}$$

- *Ambiguity* : the variance of the posterior distribution
  - Under μ_{t-1}, $(\theta_{t-1}|A_{t-1}) \sim N(m_{t-1}, Q_{t-1})$
  - In standard filtering problem this posterior distribution becomes the prior to update beliefs over θ_t
  - We assume time variation in uncertainty through a shock to the variance of the prior distribution:

$$Q^*_{t-1} = Q_{t-1} e^{\sigma_\eta \eta_t}, \;\; \eta_t \sim N(0,1)$$

  - Without the shock η_t, Q_t converges to time invariant variance of the steady state Kalman filter

The perturbation technique

- Estimating the complete non-linear model would require a computational effort above our possibilities:

  => Perturbation technique to approximate the solution of the model under smooth ambiguity

- Risk of the approximation is that we loose the effects we are interested in: they enter non-linearly into the model through the convex function ξ

- We follow Borovicka and Hansen (2014) and Bhandari Borovicka and Ho (2017): joint perturbation of variance of the shocks and coefficient of ambiguity aversion

- We apply this idea to models with smooth ambiguity preferences: additional challenge to keep track of the evolution of beliefs

The perturbation technique

The recursive solution of the model is defined by the following endogenous law of motion

$$x_t = \psi(x_{t-1}, m_t, Q_t, \alpha_t, \theta_t, \omega_t^x)$$

where $\omega_t^x \sim N(0, I)$

We need to keep track of the evolution of beliefs m_t, Q_t and of α_t.

The *approximated solution* (series expansion):

$$x_t = x_0 + q x_{1t} + \frac{q^2}{2} x_{2t} + \ldots$$

where q is the perturbation paramenter.

The approximated beliefs distortion

Pessimism depends on both *ambiguity* and *ambiguity attitude*:

- Under the posterior distribution μ_t

$$\theta_t \sim N\left(m_t, Q_t\right)$$

- Under the distorted distribution μ^*

$$\theta_t \sim N\left(m_t - \alpha_t Q_t B', Q_t\right)$$

- Ambiguity aversion affects only the mean
- Ambiguity affects both the mean and the variance

Back to the endowment economy (first order)

First order approximation: the interest rate is the expected value of the growth rate of technology

- Under complete information:

$$R_{1t} = \beta^{-1} e^{\bar{\gamma}} \left[\begin{array}{ccc} \rho_\gamma & \rho_f - 1 & 0 \end{array} \right] \theta_{1t}$$

- Under subjective expected utility (Bayesian):

$$R_{1t} = \beta^{-1} e^{\bar{\gamma}} \left[\begin{array}{ccc} \rho_\gamma & \rho_f - 1 & 0 \end{array} \right] m_{1t}$$

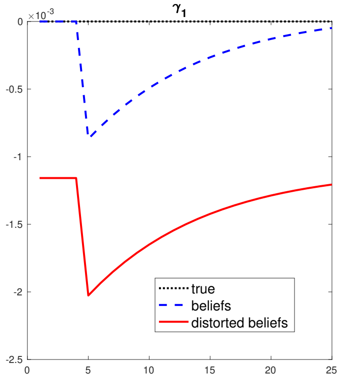
- Under **smooth ambiguty**:

$$R_{1t} = \beta^{-1} e^{\bar{\gamma}} \left[\begin{array}{ccc} \rho_\gamma & \rho_f - 1 & 0 \end{array} \right] \left[m_{1t} - \underbrace{\left(\bar{\alpha} Q_{1t} + Q \alpha_{1t} + \bar{\alpha} Q \right) B'}_{Pessimism} \right]$$

Up to first order we can not distinguish the sources of pessimism
=> Introduce risky assets and use second order approximation

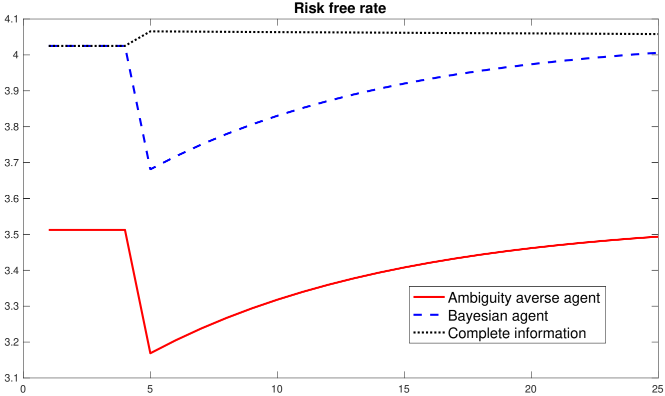
The core mechanism through a simple example

The effect of a negative temporary shock on the agent's beliefs



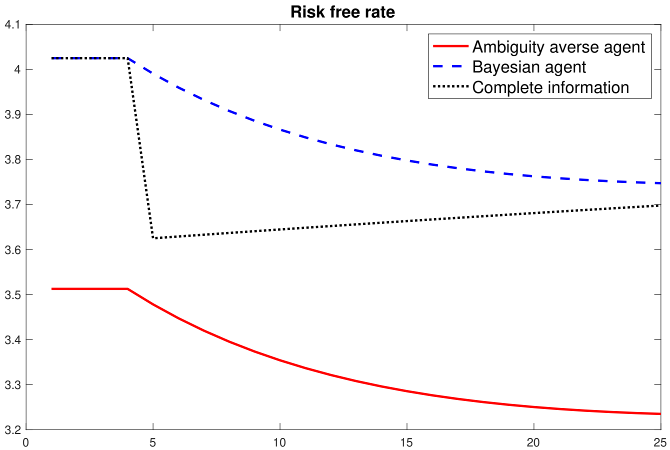
The core mechanism through a simple example

The effect of a negative temporary shock on the interest rate



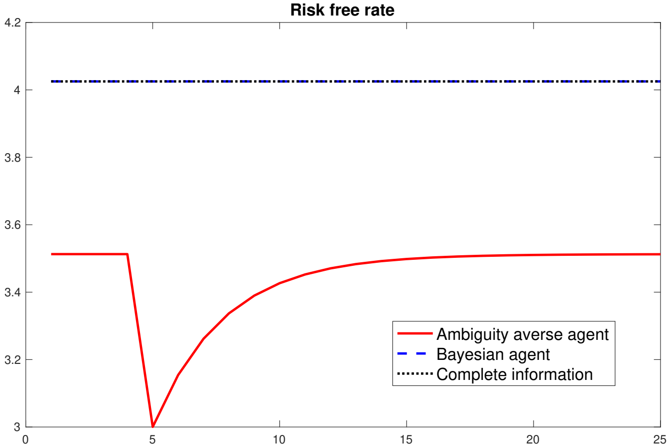
The core mechanism through a simple example

The effect of a negative permanent shock on the interest rate



The core mechanism through a simple example

The effect of an increase in pessimism on the interest rate



Econometric strategy

- Non-linear model (second order approximation)
- Bayesian approach
- Particle filtering strategy
  - Long tradition since Fernández-Villaverde and Rubio-Ramírez (2007)
  - Use an importance distribution that is conditional on data (same spirit as Amisano and Tristani, 2007)

Econometric strategy

The idea of importance sampling:

$$E_p\left[\tilde{x}\right] = \int \tilde{x}p(\tilde{x})d\tilde{x} = \int \tilde{x}\frac{p(\tilde{x})}{g(\tilde{x})}g(\tilde{x})d\tilde{x} = E_g\left[\tilde{x}w(\tilde{x})\right]$$

$g(\tilde{x})$ is called importance distribution: the goal is to have it as close as possible to the posterior

The approximated solution: $x_t \approx x_0 + qx_{1t} + \frac{q^2}{2}x_{2t}$

We are interested in the posterior distribution: $p((x_{1t}, x_{2t})|Data_t)$

- Choose $g(\tilde{x})$ as the posterior of the model approximated to the first order: second order counts less

Conclusions

- Focus: the role of beliefs and pessimism in explaining the interest rates decline after the financial crisis
- Assume uncertainty on the determinants of productivity
- Recursive smooth ambiguity preferences to model pessimism
- Perturbation and estimation strategy

The work ahead:

- The core mechanism in a more realistic model
- Disentangle the sources of pessimism