Non-Neutrality of Open-Market Operations*

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Abstract

We analyze the effects on inflation and output of unconventional open-market operations due to the possible income losses on the central bank's balance sheet. We first state a general Neutrality Property, and characterize the theoretical conditions supporting it. We then discuss three non-neutrality results. First, when treasury's support is absent, sizeable balance-sheet losses can undermine central bank's solvency and should be resolved through a substantial increase in inflation. Second, a financially independent central bank – i.e. averse to income losses – commits to a more inflationary stance and delayed exit strategy from a liquidity trap. Third, if the treasury is unable or unwilling to tax households to cover central bank's losses, the wealth transfer to the private sector also leads to higher inflation. Finally, we argue that non-neutral open-market operations can be used to escape suboptimal policies during a liquidity trap.

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1 Introduction

The recent financial crisis has shown an unprecedented intervention of central banks around the world in an attempt to mitigate the adverse effects on the economy through the purchases of long-term risky securities. The Bank of England, the Bank of Japan, the European Central Bank, the Federal Reserve System and the Swedish Central Bank have all enlarged at various stages, and with different speed and composition, their asset holdings to include long-term private securities and government debt of different maturities and credit worthiness.

All these policies have raised worries about the possible stress that the central bank's balance sheet could suffer in terms of income losses and declining net worth.[1] In this paper, we take a general-equilibrium perspective in order to understand under which conditions equilibrium prices and output respond to unconventional open-market operations because of the possible income losses that they imply on the central bank's balance sheet.

To this end, we have to challenge an important property, discussed first by Wallace (1981), affirming the irrelevance of standard open-market operations for equilibrium prices and quantities. We extend Wallace's result to any unconventional composition of central bank's assets in a model in which, among other features, central bank and treasury have separate budget constraints and where the central bank can issue both money and reserves. Even in this more general form, the intuition behind neutrality is simple. If the central bank bears some risk that was before in the hands of the private sector, the equilibrium allocation of prices and quantities does not change simply because the possible realization of that risk is ultimately borne by the private sector, through appropriate lump-sum taxes, which are collected by the treasury and transferred back to the central bank to cover income losses.[2]

The key observation to break neutrality is that it hinges on particular specifications of the transfer policies.[3] Two channels are at work identified by the possibility that symmetric and appropriate state-contingent transfers can occur 1) between the treasury and the central

[1]A recent literature has evaluated these risks for the U.S. economy based on some projection analysis and concluded that they can be in general of minor importance (see Carpenter et al., 2015, Christensen et al., 2015, Greenlaw et al., 2013).

[2]This neutrality result goes straight to the heart of a long-lasting debate on how central banks should control the value of money in connection with the assets that they hold in their balance sheet. Indeed, under unconventional asset holdings, it is not gold, nor reserves, nor "real bills" that help to back the value of money. Taxpayers do. Before unconventional monetary policy took place, what seemed to be the prevailing view shares common traits with the "real bills" doctrine (Smith, 1809), according to which central banks should issue money backed by short-term securities free of risk. In a system of this kind, it is understood that the central bank can control the value of money by setting the interest rate on the safe assets held in its portfolio. Sargent (2011) discusses Wallace's irrelevance result in light of the "real bills" doctrine.

[3]Sargent and Smith (1987, p.91), who provide a neutrality result in a model where money is dominated in return, also argue that "irrelevance requires that fiscal policy be held constant in a precise sense". In our work we give a thorough understanding on what "held constant" means. See also the discussion of Sargent (2011).

bank, 2) between the private sector and the treasury. To question neutrality some risk should stay in the hands of the central bank or the treasury.

We provide three non-neutrality results.

In the first, we break channel 1) by assuming lack of treasury support, i.e. a commitment to never recapitalize the central bank. Sizeable balance-sheet losses can undermine central bank's solvency and therefore require a change in the monetary policy stance that tilts equilibrium prices and output. The value of money should change – i.e. inflation rises – up to the point in which private agents are forced to hold more currency, so that the seigniorage earnings of the central bank can increase and profitability be restored.[4]

The second non-neutrality result is one leg of what we call the *impossible trinity* in central banking, that is to reconcile any balance-sheet policy with any generic interest-rate (or money-supply) rule while preserving at the same time financial independence. The latter captures the central bank's commitment to limit the size or duration of balance sheet's losses. Non-neutrality emerges when a central bank engages in purchases of risky securities and wants to maintain financial independence since both conditions may only accord through a change in the conventional monetary policy stance (interest-rate or money-supply rule).[5]

The last non-neutrality result is derived by breaking channel 2) and assuming that the treasury is unable or unwilling to tax the private sector to cover losses made by the central bank. When the central bank purchases risky securities, the materialization of risk remains in the hands of the whole government and represents a positive transfer of wealth to the private sector. Whoever unloaded the risky securities to the central bank experiences a positive wealth gain. Demand will surge and so will inflation. The value of money will fall.[6]

We apply this theoretical framework to economies in which the long-term assets held by the central bank are subject to either credit or interest-rate risk, in the latter case as a consequence of exit strategies from a liquidity trap. Our numerical examples show three results: 1) in the absence of treasury's support, large losses, mainly due to credit events, should be resolved by a substantial increase in inflation in order to restore the long-run profitability of the central bank; 2) a central bank which is averse to income losses commits to a more inflationary monetary policy stance and delayed exit strategies from a liquidity trap; 3) if the treasury is unable or unwilling to tax households to cover central bank's losses,

[4]Conversely, if losses are relatively small in size, neutrality can emerge if the central bank can cover losses through retained future profits at the expenses of lower remittances to the treasury. At the end, for neutrality to hold, any loss and associated lower remittances should be entirely paid by higher taxes levied on the private sector.

[5]In early 2002, the market questioned the willingness of the Bank of Japan to pursue quantitative easing policies on the ground of the possible losses that could result after an eventual rise in interest rates (see Stella, 2005, for an insightful discussion).

[6]This monetary/fiscal policy regime represents one of the alternative ways to implement the so-called "helicopter money".

the wealth transfer to the private sector also leads to higher inflation.

Asset purchases can be inflationary not because the central bank "prints money" or increases the size of its balance sheet, but because a higher inflation is either the desirable response to sizeable income losses in order to regain central bank's profitability or the outcome of a policy geared toward avoiding losses altogether or, in general, a consequence of an indirect (or direct) wealth transfer from the government to the private sector.

We also argue that non-neutral open-market operations during a liquidity trap can be a way to escape suboptimal monetary policies by possibly making them inconsistent with the existence of a rational expectations equilibrium, and credibly signal a shift to a different monetary policy stance.

Our work first contributes to the tradition of the irrelevance results of Wallace (1981), Chamley and Polemarchakis (1984), Sargent and Smith (1987), Sargent (1987) and Eggertsson and Woodford (2003).[7] In this direction, we state a general Neutrality Property and characterize the theoretical conditions supporting it, in an economy where treasury and central bank have separate budget constraints and the central bank issues both non-interest and interest bearing liabilities (money and reserves, respectively). In particular, distinguishing the budget constraint of the central bank from that of the treasury requires to detail the transfer policy that each institution should follow in order to obtain neutrality. Moreover, allowing the central bank to issue reserves enlarges the set of irrelevance results.

In addition, we provide a meaningful departure from that tradition by identifying and discussing several non-neutrality cases of practical interest, focusing in particular on transfer policies between central bank and treasury. In this direction, our paper is inspired by the seminal works of Sims (2000, 2005) who has first emphasized in theoretical models the importance for policy analysis of separating the budget constraint of the treasury from that of the central bank and by the more recent Bassetto and Messer (2013), Del Negro and Sims (2015), Hall and Reis (2015).[8] Unlike our analysis, all this literature is not concerned about stating a general Neutrality Property or about characterizing the theoretical conditions for non-neutrality. However, Sims (2000, 2005) and Del Negro and Sims (2015) underline that when there is lack of treasury's support the central bank may no longer be able to maintain control of inflation when committing to a certain Taylor's rule because this would lead to

[7]The framework of Wallace (1981) and Chamley and Polemarchakis (1984) is extended to economies where money is dominated in return by Sargent and Smith (1987) and Sargent (1987). Eggertsson and Woodford (2003) generalize the neutrality result to a context in which the central bank's balance sheet includes unconventional asset purchases and the only liability is money. However, they consolidate the budget constraints of treasury and central bank when specifying the transfer policy that delivers neutrality.

[8]Drawing from the experience of several central banks, Stella (1997, 2005) has also provided evidence for the relationship between central-bank financial strength and monetary policy. See also the recent work of Adler et al. (2012).

insolvency. This is consistent with one of the non-neutrality results that we discuss. Bassetto and Messer (2015), instead, mainly focus on the fiscal consequences of alternative compositions of central bank's assets emphasizing the different accounting procedures and remittance policies between treasury and central bank.[9] Reis (2013, 2015) and Hall and Reis (2015) take instead equilibrium inflation as given and are interested in analyzing the consequences of central bank's insolvency for financial stability, i.e. a non-exploding path of central bank's reserves.

Our case of financially independent central bank, which implies non-neutrality of balance-sheet policies, shares some similarities with Bhattarai et al. (2015), with two main, important differences. First, in our model a financially independent central bank penalizes only negative remittances to the treasury, while they assume a quadratic penalty for non-zero remittances. Accordingly, their environment features non-neutral effects of any balance-sheet policy, while in our case only those that imply potential losses on risky securities. Second, we analyze the optimal allocation under full commitment, while they only consider the case of discretion.

Berriel and Bhattarai (2009) and Park (2015) also consider a separate budget constraints of treasury and central bank, but they analyze the case in which the central bank holds only short-term assets and therefore losses are not possible, unlike our model. In particular, Berriel and Bhattarai (2009) show how optimal policy changes considering different central-bank remittances' rule. Park (2015) investigates the determinacy of the equilibrium under alternative remittances' and fiscal policies. Our study, instead, focuses on the equilibrium consequences of alternative balance-sheet policies holding "constant" the specification of remittances' and fiscal policy, which is the relevant comparison to make in order to evaluate neutrality.[10]

Our work is also related to a more extensive literature which has studied the monetary policy consequences of alternative assumptions on fiscal policy (see among others Sargent and Wallace, 1981, Sargent, 1982, Leeper, 1991, Sims, 1994, 2013, and Woodford, 1994, 2001) but which, on the contrary, has disregarded the distinction between the balance sheets of treasury and central bank. This separation is key in our analysis.

The plan of this work is the following. Section 2 presents a simple monetary model while Section 3 states the Neutrality Property and studies neutrality and non-neutrality results.

[9] There is a substantial literature which has analyzed the different central-bank accounting procedures and remittance policies as, among others, Stella (1997, 2005) and Archer and Moser-Boehm (2013).

[10] Zhu (2004) also distinguishes between the budget constraint of the central bank and that of the treasury but he focuses on how the properties of equilibrium determinacy change when the interest-rate rule followed by the central bank reacts also to variations in its net worth with respect to a target. Jeanne and Svensson (2007) discuss the importance of balance-sheet considerations as a credible device to exit from a liquidity trap; however, their focus is on the balance-sheet losses possibly arising because of the effect of exchange-rate movements on the value of reserves.

In Section 4, a numerical analysis evaluates the results of Section 3. Section 5 concludes.

2 Model

We present our analysis in a simple infinite-horizon monetary economy, along the lines of Bassetto and Messer (2013), featuring three sets of agents: households, the treasury and the central bank. A key assumption of our analysis, as mentioned in the introduction, is the separation between the balance sheets of treasury and central bank. There is a financial friction in the model since money is the only asset that can be used to buy goods. This friction is not important at all for our results. It is only useful to capture features of current economies in which money has a relevant role for transactions and for partly financing central bank's assets.[11] Our model economy is perturbed by three stochastic disturbances. We allow for a credit shock and a preference shock, to capture credit and interest-rate risk, respectively. These are the two most relevant risks in thinking about the consequences of recent asset purchases by central banks in advanced economies. Finally, the third stochastic disturbance is the endowment of the only traded good in the economy. In Section 4, we generalize this model along several dimensions, including endogenous production, nominal rigidities and additional shocks perturbing the economy.

2.1 Households

Households have an intertemporal utility of the form:

$$E_{t_0}\left\{\sum_{t=t_0}^{\infty}\beta^{t-t_0}\xi_t U(C_t)\right\} \tag{1}$$

where E_t denotes the standard conditional expectation operator, β is the intertemporal discount factor with $0 < \beta < 1$, ξ is a stochastic disturbance, which affects the intertemporal preferences of the consumer and is assumed to follow a Markov process, with transition density $\pi_\xi(\xi_{t+1}|\xi_t)$ and initial distribution f_ξ. We assume that (π_ξ, f_ξ) is such that $\xi \in [\xi_{\min}, \xi_{\max}]$.[12] C is a consumption good and $U(\cdot)$ is a concave function.

The timing of markets' opening follows that of Lucas and Stokey (1987). In a generic period t, the asset market opens first, followed by the goods market. There is a financial friction since only money can be used to purchase goods and only in the goods market. In

[11] See Lucas (1984) for further discussion of the usefulness of this class of models for monetary theory and more recently Sargent (2014).

[12] We restrict our analysis to exogenous stochastic processes with finite state space.

the asset market, households can adjust their portfolio according to

$$M_t+\frac{B_t+X_t}{1+i_t}+Q_tD_t \leq B_{t-1}+X_{t-1}+(1-\varkappa_t)(1+\delta Q_t)D_{t-1}+P_{t-1}Y_{t-1}-T_t^F+(M_{t-1}-P_{t-1}C_{t-1}). \tag{2}$$

Households invest their financial wealth in money, M_t – a non-interest-bearing asset issued by the central bank which provides liquidity services – in central bank's reserves, X_t, which carry a risk-free nominal return i_t. Finally they can lend or borrow using short-term, B_t, and long-term, D_t, securities at a price $1/(1+i_t)$ and Q_t, respectively. In the case of long-term debt, the security available has decaying coupons: by lending Q_t units of currency at time t, geometrically decaying coupons are delivered equal to $1, \delta,\ \delta^2, \delta^3...$ in the following periods and in the case of no default.[13]

In the case of long-term lending or borrowing, the stochastic disturbance $\varkappa_t$ on the right-hand side of (2) captures the possibility that long-term securities can be partially seized by exogenous default; in particular $\varkappa_t$ follows a Markov process with transition density $\pi_\varkappa(\varkappa_{t+1}|\varkappa_t)$ and initial distribution $f_\varkappa$. We assume that $(\pi_\varkappa, f_\varkappa)$ is such that $\varkappa \in [0,1)$. To shorten the writing of (2), we are including only those securities, among the ones traded, that households can exchange externally with the treasury and central bank. In addition, in each period, households can trade with each other in a set of state-contingent nominal securities spanning all states of nature which they face in the next period. It is assumed that the payoffs of these securities are enough to "complete" the financial markets.

In the budget constraint (2), Y_{t-1} is the time $t-1$ endowment of the only good traded which is the third stochastic disturbance that also follows a Markov process with transition density $\pi_y(Y_{t+1}|Y_t)$ and initial distribution f_y. We assume that (π_y, f_y) is such that $Y \in [Y_{\min}, Y_{\max}]$. In the budget constraint (2), $P_{t-1}Y_{t-1}$ are the revenues the household obtains by selling the endowment in $t-1$ which are deposited in the financial account only in period t; T_t^F are lump-sum taxes levied by the treasury. Unspent money in the previous-period goods market is deposited in the financial account.

When asset market closes, goods market opens and households can use money to purchase goods according to

$$M_t \geq P_tC_t. \tag{3}$$

The households' problem is subject to initial conditions $B_{t_0-1}, X_{t_0-1}, D_{t_0-1}, M_{t_0-1}$ and a

[13]The stock of long-term asset (or debt) follows the law of motion $D_t = Z_t + (1-\delta)D_{t-1}$, where Z_t is the amount of new long-term lending, if positive, or borrowing, if negative, supplied at time t. See among others Woodford (2001).

borrowing limit of the form[14]

$$\lim_{T\to\infty} E_t\left[R^n_{t,T}\mathcal{W}_T\right] \geq 0, \tag{4}$$

looking forward from each time t where $R^n_{t,T}$ is the nominal stochastic discount factor that is used to evaluate nominal wealth $\mathcal{W}_T$ in a generic contingency at time T with respect to nominal wealth at time t, with $T > t$. Nominal wealth $\mathcal{W}_t$ is given by

$$\mathcal{W}_t = \tilde{\mathcal{W}}_t + M_t + \frac{B_t + X_t}{1+i_t} + Q_t D_t$$

which includes the nominal values of the portfolio of state-contingent securities $\tilde{\mathcal{W}}_t$. It is also required for the existence of an intertemporal budget constraint that

$$E_t\left\{\sum_{T=t}^{\infty} R^n_{t,T}\left[P_{T-1}C_{T-1} + \frac{i_T}{1+i_T}M_T\right]\right\} < \infty \tag{5}$$

looking forward from any date t, since there is no limit to the ability of households to borrow against future income.[15]

Households choose consumption, and asset allocations to maximize utility (1) under the constraints (2), (3), (4) and (5), given the initial conditions. The optimal choice with respect to consumption, assuming an interior solution, requires that

$$\xi_t U_c(C_t) = (\varphi_t + \beta E_t \lambda_{t+1})P_t, \tag{6}$$

where λ_t and φ_t are the non-negative Lagrange multipliers associated with constraints (2) and (3), respectively. The first-order condition with respect to money holdings

$$\lambda_t - \varphi_t = \beta E_t \lambda_{t+1}, \tag{7}$$

implies in (6) that the marginal utility of nominal wealth is simply given by $\lambda_t = \xi_t U_c(C_t)/P_t$, which is positive.

The optimality conditions with respect to the holdings of short-term treasury bills or central bank's reserves determine the nominal interest rate according to

$$\frac{1}{(1+i_t)} = E_t R^n_{t,t+1}, \tag{8}$$

[14] There are also initial conditions on $P_{t_0-1}Y_{t_0-1}$ and $P_{t_0-1}C_{t_0-1}$ but we assume that they sum to zero as in equilibrium.

[15] It is important to note that the expression in the curly bracket of (5) is never negative since consumption, money holdings, prices and interest rates are all non-negative.

where the equilibrium nominal stochastic discount factor $R^n_{t,t+1}$ is implied by the optimality conditions with respect to the state-contingent securities and given by

$$R^n_{t,t+1} = \beta \frac{\lambda_{t+1}}{\lambda_t}. \tag{9}$$

Combining (6)–(9) it follows

$$\varphi_t = \frac{i_t}{1+i_t}\lambda_t \tag{10}$$

from which $i_t \geq 0$, since $\varphi_t \geq 0$ and $\lambda_t > 0$. The complementary slackness condition on the constraint (3) can be written as

$$\varphi_t(M_t - P_t C_t) = 0.$$

The first-order condition with respect to lending or borrowing using long-term fixed-rate securities implies that the price Q_t follows

$$Q_t = E_t[R^n_{t,t+1}(1-\varkappa_{t+1})(1+\delta Q_{t+1})]. \tag{11}$$

To conclude the characterization of the household's problem, a transversality condition applies and therefore (4) holds with equality, given the equilibrium nominal stochastic discount factor $R^n_{t,T} = \beta^{T-t}\lambda_T/\lambda_t$.

2.2 Treasury

The treasury raises lump-sum taxes T^F_t (net of transfers) from the private sector and receives remittances T^C (when T^C is positive) or makes transfers to the central bank (when T^C is negative). The treasury can finance its deficit through short-term (B^F) and long-term (D^F) debt, at the prices $1/(1+i_t)$ and Q_t respectively, facing the following flow budget constraint

$$Q_t D^F_t + \frac{B^F_t}{1+i_t} = (1-\varkappa_t)(1+\delta Q_t)D^F_{t-1} + B^F_{t-1} - T^F_t - T^C_t$$

given initial conditions $D^F_{t_0-1}$, $B^F_{t_0-1}$.[16]

2.3 Central bank

The central bank issues non-interest-bearing-liabilities, money M^C_t, and interest-bearing liability, reserves X^C_t, to finance a portfolio of assets including short-term and long-term

[16]It is worth reminding that to simplify the analysis we have assumed that there is only one long-term security, which is issued either by the private sector or by the treasury. In particular, without losing generality, we assume $D^F_t = 0$ if $D_t < 0$ and $D_t \geq 0$ if $D^F_t > 0$.

fixed-rate securities, B_t^C and D_t^C respectively. Central bank's net worth, N_t^C – the difference between the market value of assets and liabilities – is given by

$$N_t^C \equiv Q_t D_t^C + \frac{B_t^C}{1+i_t} - M_t^C - \frac{X_t^C}{1+i_t}, \tag{12}$$

while its law of motion depends on the profits that are not distributed to the treasury:

$$N_t^C = N_{t-1}^C + \Psi_t^C - T_t^C \tag{13}$$

where Ψ_t^C are central bank's profits, which depend on the composition of its balance sheet:[17]

$$\Psi_t^C = \frac{i_{t-1}}{1+i_{t-1}}(B_{t-1}^C - X_{t-1}^C) + \left[(1-\varkappa_t)(1+\delta Q_t) - Q_{t-1}\right] D_{t-1}^C. \tag{14}$$

They can also be written as

$$\Psi_t^C = i_{t-1}(N_{t-1}^C + M_{t-1}^C) + (r_t - i_{t-1}) Q_{t-1} D_{t-1}^C \tag{15}$$

having used the definition $(1+r_t) \equiv (1+\delta Q_t)(1-\varkappa_t)/Q_{t-1}$.[18] Central bank's profits depend on two components: the first captures the revenues obtained by issuing non-interest bearing liabilities – net worth is indeed part of the non-interest bearing liabilities; the second component, instead, represents the excess gains or losses of holding long-term securities with respect to a riskless portfolio. Since the realized excess return on these securities can be negative, the latter component may as well be negative – the more so the larger are the holdings of long-term securities – producing income losses for the central bank. Combining (12) and (13), we can write the central bank's flow budget constraint as follows:

$$Q_t D_t^C + \frac{B_t^C}{1+i_t} - M_t^C - \frac{X_t^C}{1+i_t} = (1-\varkappa_t)(1+\delta Q_t) D_{t-1}^C + B_{t-1}^C - X_{t-1}^C - M_{t-1}^C - T_t^C,$$

given initial conditions $D_{t_0-1}^C, B_{t_0-1}^C, X_{t_0-1}^C, M_{t_0-1}^C$.

2.4 Equilibrium

Here, we describe in a compact way the equations that characterize the equilibrium allocation.

[17]Profits of central bank are defined as the net income derived from the portfolio of assets and liabilities which once fully distributed are such to keep the central bank's nominal net worth constant. This is in line with similar definitions given by Bassetto and Messer (2013), Del Negro and Sims (2015) and Hall and Reis (2015). We abstract from the dividends that the central bank gives to the member banks. In the US, this amounts to 6% of capital (see Carpenter et al., 2015).

[18]The definition is only valid for positive asset prices.

The Euler equations for short-term and long-term securities, (8) and (11), imply that

$$\frac{1}{1+i_t} = E_t\left\{\beta\frac{\xi_{t+1}U_c(Y_{t+1})}{\xi_t U_c(Y_t)}\frac{P_t}{P_{t+1}}\right\}, \tag{16}$$

and

$$Q_t = E_t\left\{\beta\frac{\xi_{t+1}U_c(Y_{t+1})}{\xi_t U_c(Y_t)}\frac{P_t}{P_{t+1}}(1-\varkappa_{t+1})(1+\delta Q_{t+1})\right\}, \tag{17}$$

respectively, in which we have used the equilibrium value of the Lagrange multiplier $\lambda_t = \xi_t U_c(C_t)/P_t$ and equilibrium in goods market $Y_t = C_t$.

The cash-in-advance constraint (3) together with the equilibrium in the goods market implies

$$M_t \geq P_t Y_t, \tag{18}$$

while the complementary slackness condition can be written as

$$i_t(M_t - P_t Y_t) = 0. \tag{19}$$

The bound (5) in equilibrium is equal to

$$E_t\left\{\sum_{T=t}^{\infty}\beta^{T-t}\xi_T U_c(Y_T)\left[Y_{T-1} + \frac{i_T}{1+i_T}Y_T\right]\right\} < \infty,$$

in which we have also used (18) and (19). Note that the above equilibrium condition is always satisfied given the assumption of bounded processes for Y_t and ξ_t.

The transversality condition, with equality, completes the demand side of the model

$$\lim_{T\longrightarrow\infty} E_t\left[\beta^{T-t}\frac{\xi_T U_c(Y_T)}{P_T}\left(M_T + \frac{B_T + X_T}{1+i_T} + Q_T D_T\right)\right] = 0, \tag{20}$$

which is derived from (4), where we have used $R^n_{t,T} = \beta^{T-t}\lambda_T/\lambda_t$, $\lambda_t = \xi_t U_c(C_t)/P_t$, and the goods market equilibrium together with the fact that the state-contingent securities are traded in zero-net supply within the private sector: the transversality condition therefore constrains just the long-run behavior of the "outside" assets held by the households.

The treasury's and central bank's budget constraints are given by

$$Q_t D_t^F + \frac{B_t^F}{1+i_t} = (1-\varkappa_t)(1+\delta Q_t)D_{t-1}^F + B_{t-1}^F - T_t^F - T_t^C \tag{21}$$

and

$$Q_t D_t^C + \frac{B_t^C}{1+i_t} - M_t - \frac{X_t}{1+i_t} = (1-\varkappa_t)(1+\delta Q_t)D_{t-1}^C + B_{t-1}^C - X_{t-1} - M_{t-1} - T_t^C \quad (22)$$

respectively, where equilibrium in the asset markets implies that

$$B_t^F = B_t + B_t^C \quad (23)$$

$$D_t^F - D_t = D_t^C. \quad (24)$$

Note moreover that in (22) we have used the equilibrium conditions that money and reserves issued by the central bank are held by households, $M_t = M_t^C$ and $X_t = X_t^C$.[19]

To complete the characterization of the rational expectations equilibrium we need to specify the monetary/fiscal policy regime. First we note that excluding the complementary-slackness condition (19) and the bound (20) there are seven equilibrium conditions for the thirteen unknown stochastic processes $\{P_t, i_t, Q_t, M_t, X_t, B_t, B_t^C, B_t^F, D_t, D_t^C, D_t^F, T_t^F, T_t^C\}_{t=t_0}^{\infty}$ implying that the monetary/fiscal policy regime should specify six additional equations. In particular, the monetary/fiscal policy regime specifies six of the stochastic processes $\{i_t, M_t, X_t, B_t^C, B_t^F, D_t^C, D_t^F, T_t^F, T_t^C\}_{t=t_0}^{\infty}$, possibly as functions of some other endogenous and/or exogenous variables.

It is out of the scope of this paper to analyze all possible monetary/fiscal policy regimes. Indeed, we restrict attention to a subset of regimes, which is however quite inclusive and broad enough to encompass all interesting cases. The monetary/fiscal policy regimes under consideration can be described by a combination of *conventional monetary policy, transfer policy* and *balance-sheet policy.*

To simplify notation, in what follows, we define the vector $\mathbf{Z}_t \equiv (P_t, i_t, Q_t, M_t)$ and the vector $\mathbf{K}_t \equiv (X_t, B_t, B_t^C, B_t^F, D_t, D_t^C, D_t^F, T_t^F, T_t^C)$ while the vectors $\bar{\mathbf{Z}}_t$ and $\bar{\mathbf{K}}_t$ include $\mathbf{Z}_t$ and $\mathbf{K}_t$, respectively, and their own lags.

To understand what we mean by *conventional monetary policy*, consider the equilibrium conditions (16) to (19). Since (19) is a complementary-slackness condition, there are three equations in the vector of four unknown stochastic processes $\{\mathbf{Z}_t\}_{t=t_0}^{\infty}$, given the exogenous state process $\{Y_t\}_{t=t_0}^{\infty}$. Considered alone this set of equations leaves one degree of freedom to specify one of the endogenous stochastic processes to eventually determine all four endogenous variables.

[19]It is important to note that the transversality condition (20) implies an aggregate transversality condition on the net consolidated liabilities of both treasury and central bank which together with the flow budget constraints (21) and (22) entails a consolidated intertemporal budget constraint. We do not explicitly write this constraint since it is already implied by the set of equations written above.

We call *conventional monetary policy* the specification of one of the stochastic processes $\left\{i_t, M_t\right\}_{t=t_0}^{\infty}$ as a function of the other endogenous variables P, Q and/or exogenous state variables like Y. A type of rule in this class is setting in each contingency i_t as a function $i_t = \mathcal{I}(\bar{\mathbf{Z}}_t,\ \zeta_t)$ where ζ_t is a generic vector of exogenous stochastic disturbances that may include ξ_t, Y_t and $\varkappa_t$ while $\mathcal{I}(\cdot)$ is non-negative for all the values of its arguments, consistently with the zero-lower bound on the short-term nominal interest rate. Another type of rule in this class involves instead setting M_t in each contingency as $M_t = \mathcal{M}(\bar{\mathbf{Z}}_t,\ \zeta_t)$ where $\mathcal{M}(\cdot)$ is positive for all values of its arguments.

An important restriction in the specification of the *conventional monetary policy* is the requirement that the endogenous variables included in the arguments of $\mathcal{I}(\cdot)$ or $\mathcal{M}(\cdot)$ are just those belonging to the vector $\mathbf{Z}_t$ and not to $\mathbf{K}_t$. This represents the conventional way to think about determination of prices in this class of models: specify either an interest-rate rule or a money-supply rule and possibly determine the path of prices, interest rate and money using equations (16) and (18). Equation (17) residually determines the asset price Q_t, if the policy rule does not itself react to Q_t. Therefore, for a given *conventional monetary policy* the set of equations (16) to (19) can in principle determine the path of the vector of stochastic processes $\left\{\mathbf{Z}_t\right\}_{t=t_0}^{\infty}$.

However, it is key to note that this path, to be an equilibrium, needs also to satisfy the other equilibrium conditions together with the additional restrictions coming from the remaining specification of the monetary/fiscal policy regime. In this respect we specify a *transfer policy* in which both the stochastic processes $\left\{T_t^F, T_t^C\right\}$ are functions of the other endogenous and/or exogenous variables. A general *transfer policy* that we assume in this work is the following: *i*) $T_t^F = \mathcal{T}^F(\bar{\mathbf{T}}_t^C, \bar{\mathbf{D}}_{t-1}^F, \bar{\mathbf{B}}_{t-1}^F, \bar{\mathbf{Z}}_t, \zeta_t)$ in which the treasury is setting lump-sum taxes as a function, among other variables, of the current and past levels of central bank's remittances and of treasury's outstanding short and long-term liabilities;[20] *ii*) $T_t^C = \mathcal{T}^C(\bar{\mathbf{N}}_{t-1}^C, \bar{\mathbf{Z}}_t, \zeta_t)$ in which the central bank is setting remittances as a function, among other variables, of the level of its own net worth N_t^C.[21] In what follows we denote compactly the two transfer policies with the two-dimensional vector of functions $\mathcal{T}(\cdot)$. The above policies are not comprehensive of all the possible policies that can be considered but are broad enough to encompass all the relevant cases for our analysis.

We are left with the specification of three of the sequences $\left\{X_t, B_t^C, B_t^F, D_t^C, D_t^F\right\}_{t=t_0}^{\infty}$ to complete the characterization of the monetary/fiscal policy regime. In this work we limit our attention to regimes in which three of the sequences $\left\{B_t^C, B_t^F, D_t^C, D_t^F\right\}_{t=t_0}^{\infty}$ are specified

[20] Consistently with the notation introduced before: $\bar{\mathbf{T}}_t^C \equiv (T_t^C, T_{t-1}^C, ...)$, $\bar{\mathbf{D}}_t^F \equiv (D_t^F, D_{t-1}^F, ...)$, $\bar{\mathbf{B}}_t^F \equiv (B_t^F, B_{t-1}^F, ...)$ and $\bar{\mathbf{N}}_t^C \equiv (N_t^F, N_{t-1}^F, ...)$.

[21] Note that a reaction of remittances to the variables $\bar{\mathbf{N}}_{t-1}^C, \bar{\mathbf{Z}}_t,\ \zeta_t$ also accounts for a response to current and past profits, given the definition (14).

as functions of the other endogenous and/or exogenous variables. This is what we define a *balance-sheet policy.* Moreover, it should be noted from the flow budget constraint (21) that since the monetary/fiscal policy regime specifies already a *conventional monetary policy* and a *transfer policy*, only one of the stochastic processes $\{B_t^F, D_t^F\}_{t=t_0}^{\infty}$ can be chosen independently. Without losing generality, we assume that $\{D_t^F\}_{t=t_0}^{\infty}$ is specified. In what follows we denote the specification of the *balance-sheet policy* with a three-dimensional vector $\mathbf{B}_t \equiv (D_t^F, B_t^C, D_t^C)$ and with the non-negative functional form $\mathcal{B}(\cdot)$ such that $\mathbf{B}_t = \mathcal{B}$ $(\bar{\mathbf{B}}_{t-1}, \bar{\mathbf{Z}}_t, \zeta_t)$, capturing the possibility that a *balance-sheet policy* reacts also to current and past macroeconomic conditions. An *unconventional open-market operation* is a balance-sheet policy in which $D_t^C > 0$ in some contingencies.

Definition 1 *A* ***conventional monetary policy*** *specifies either the stochastic process* $\{M_t\}_{t=t_0}^{\infty}$ *as* $\mathcal{M}(\bar{\mathbf{Z}}_t,\ \zeta_t)$ *where* $\mathcal{M}(\cdot)$ *is positive for all values of its arguments or* $\{i_t\}_{t=t_0}^{\infty}$ *as* $i_t = \mathcal{I}(\bar{\mathbf{Z}}_t,\ \zeta_t)$ *where* $\mathcal{I}(\cdot)$ *is non-negative for all the values of its arguments. A* ***transfer policy*** *specifies the stochastic processes* $\{T_t^F, T_t^C\}_{t=t_0}^{\infty}$ *as* $T_t^F = \mathcal{T}^F(\bar{\mathbf{T}}_t^C, \bar{\mathbf{D}}_{t-1}^F, \bar{\mathbf{B}}_{t-1}^F, \bar{\mathbf{Z}}_t, \zeta_t)$ *and* $T_t^C = \mathcal{T}^C(\bar{\mathbf{N}}_{t-1}^C, \bar{\mathbf{Z}}_t, \zeta_t)$*. A* ***balance-sheet policy*** *specifies the vector of stochastic processes* $\{\mathbf{B}_t\}_{t=t_0}^{\infty} \equiv \{B_t^C, D_t^C, D_t^F\}_{t=t_0}^{\infty}$ *as* $\mathbf{B}_t = \mathcal{B}\ (\bar{\mathbf{B}}_{t-1}, \bar{\mathbf{Z}}_t, \zeta_t)$ *where* $\mathcal{B}(\cdot)$ *is non-negative for all the values of its arguments.*

Given this premise we now introduce the definition of rational expectations equilibrium in which $\mathbf{w}_{t_0-1}$ is a vector that includes $M_{t_0-1}, X_{t_0-1}, B_{t_0-1}^C, B_{t_0-1}^G, D_{t_0-1}^C, D_{t_0-1}^G$ and other initial conditions that could be specified by the monetary/fiscal policy regime.[22]

Definition 2 *Given a* ***conventional monetary policy****, a* ***transfer policy*** *and a* ***balance-sheet policy*** *a rational expectations equilibrium is a collection of stochastic processes* $\{\mathbf{Z}_t^*, \mathbf{K}_t^*\}_{t=t_0}^{\infty}$ *such that* $i_t^* \geq 0$, $P_t^* > 0$, $Q_t^* > 0$, $X_t^* \geq 0$ *and* $B_t^{F*} \geq 0$ *at each time* $t \geq t_0$ *(and in each contingency at* t*) and such that: i)* $\{\mathbf{Z}_t^*\}_{t=t_0}^{\infty}$ *satisfies each of the conditions in equations (16) to (19) at each time* $t \geq t_0$ *(and in each contingency at* t*) and the specification of the* ***conventional monetary policy****, given the stochastic processes for the exogenous disturbances* $\{\zeta_t\}$ *and the initial conditions* $\mathbf{w}_{t_0-1}$*; ii)* $\{\mathbf{K}_t^*\}_{t=t_0}^{\infty}$ *satisfies each of the conditions in equations (20) to (24) at each time* $t \geq t_0$ *(and in each contingency at* t*) and the specification of the* ***transfer policy*** *and* ***balance-sheet policy****, given the vector of stochastic processes* $\{\mathbf{Z}_t^*\}_{t=t_0}^{\infty}$ *of part i), the stochastic processes for the exogenous disturbances* $\{\zeta_t\}$ *and initial conditions* $\mathbf{w}_{t_0-1}$*.*

In the definition, the time-t component of the endogenous stochastic process is meant to be a function of the history of shocks, $\mathbf{s}^t \equiv (\mathbf{s}_t, \mathbf{s}_{t-1}, \mathbf{s}_{t-2}, ..., \mathbf{s}_{t_0})$ and the initial conditions $\mathbf{w}_{t_0-1}$.

[22] We assume that initial conditions are such that $N_{t_0-1} = \bar{N} > 0$.

Therefore $\mathbf{Z}_t^* \equiv \mathbf{Z}^*(\mathbf{s}^t, \mathbf{w}_{t_0-1})$ and $\mathbf{K}_t^* \equiv \mathbf{K}^*(\mathbf{s}^t, \mathbf{w}_{t_0-1})$. The state $\mathbf{s}_t$ is a vector including ξ_t, $\varkappa_t$ and Y_t and other exogenous state variables, which could be specified by the monetary/fiscal policy regime. It can also include sunspot disturbances.[23] In what follows, to simplify notation, we just use $\{\mathbf{Z}_t^*, \mathbf{K}_t^*\}$ in place of $\{\mathbf{Z}_t^*, \mathbf{K}_t^*\}_{t=t_0}^{\infty}$.

3 The Neutrality Property

Taking as a starting point an equilibrium, the main objective of our analysis is to study whether alternative compositions of the central bank's balance sheet can influence equilibrium variables such as prices and interest rates. In particular, Definition 2 allows to make the right comparison in order to rule out a causal relationship between *balance-sheet policies* and prices, when instead something else in the specification of the monetary/fiscal policy regime has also changed, and is actually responsible of the variation in prices observed in equilibrium.

Consider a rational expectations equilibrium $\{\mathbf{Z}_t^*, \mathbf{K}_t^*\}$ and the associated *conventional monetary policy,* $\mathcal{I}(\cdot)$ *or* $\mathcal{M}(\cdot)$*, transfer policy* $\mathcal{T}(\cdot)$, and *balance-sheet policy* $\mathcal{B}(\cdot)$. In particular let us first focus on an equilibrium in which the nominal interest rate is always above the zero-lower bound, $i_t^* > 0$. Next, change the *balance-sheet policy* from $\mathcal{B}(\cdot)$ to $\tilde{\mathcal{B}}(\cdot)$. The alternative *balance-sheet policy* $\tilde{\mathcal{B}}(\cdot)$ is said to be neutral if there is an equilibrium $\{\tilde{\mathbf{Z}}_t, \tilde{\mathbf{K}}_t\}$ with $\tilde{\mathbf{Z}}_t = \mathbf{Z}_t^*$ characterized by the same *conventional monetary policy* and *transfer policy* and the new *balance-sheet policy.* The vector $\mathbf{Z}_t^*$ is therefore invariant to the change in *balance-sheet policy* while keeping the same *conventional monetary policy* and *transfer policy.*

More generally, a Neutrality Property applies if the above argument is valid for *any* appropriately-bounded *balance-sheet policy*. Indeed, it might very well be possible that only some balance-sheet policies are neutral, but not all. For example, temporary purchases of long-term bonds may be neutral while permanent ones may not. In this case, the Neutrality Property does not apply.

The invariance of $\mathbf{Z}_t^*$ following the alternative *balance-sheet policies* captures the defining feature of the neutrality result: that the new *balance-sheet policy* does not induce any wealth effect on the households – at the initial prices – so that no change is implied in either aggregate demand or equilibrium prices.

The case in which the nominal interest rate is not above the zero-lower bound in every contingency deserves special treatment. When $i_t = 0$ money becomes a perfect substitute of reserves. In this case, it can be possible that a *balance-sheet policy* that leads to an

[23] Our analysis is not concerned about the uniqueness (local or global) of the rational expectations equilibrium. See Bassetto (2005) on how to implement desired equilibria through certain strategies followed by the policymakers.

increase in the supply of money M_t could deliver a neutrality result since at the zero-lower bound households are willing to absorb any additional supply of money without changing their portfolio choices and their consumption decisions. Therefore, in the case of equilibria in which the nominal interest rate stays even occasionally at the zero-lower bound, for a Neutrality Property to hold, we should require that, in the contingencies in which $i_t^* = 0$, only the stochastic processes $\{P_t^*, Q_t^*, i_t^*\}$ – rather than the whole vector $\{\mathbf{Z}_t^*\}$ – do not vary under the alternative *balance-sheet policy*, while in the same contingencies, and following the alternative *balance-sheet policy*, M_t can instead take any value $\tilde{M}_t \geq P_t^* Y_t$.[24]

Definition 3 *(Neutrality Property) Consider the set $\mathcal{N}$ of rational expectations equilibria associated with a given* ***transfer policy*** *$\mathcal{T}(\cdot)$ and a rational expectations equilibrium $\{\mathbf{Z}_t^*, \mathbf{K}_t^*\} \in \mathcal{N}$ associated with a* ***conventional monetary policy****, $\mathcal{I}(\cdot)$ or $\mathcal{M}(\cdot)$, and a* ***balance-sheet policy*** *$\mathcal{B}(\cdot)$. Consider an alternative, appropriately bounded,* ***balance-sheet policy*** *$\tilde{\mathcal{B}}(\cdot)$. The* ***balance-sheet policy*** *$\tilde{\mathcal{B}}(\cdot)$ is neutral with respect to $\{\mathbf{Z}_t^*, \mathbf{K}_t^*\}$ if there exists a rational expectations equilibrium $\{\tilde{\mathbf{Z}}_t, \tilde{\mathbf{K}}_t\} \in \mathcal{N}$ associated with the same* ***conventional monetary policy*** *$\mathcal{I}(\cdot)$ or $\mathcal{M}(\cdot)$ and* ***transfer policy*** *$\mathcal{T}(\cdot)$ and with the* ***balance-sheet policy*** *$\tilde{\mathcal{B}}(\cdot)$ where:*

1. *$\tilde{P}_t = P_t^*$, $\tilde{\imath}_t = i_t^*$, and $\tilde{Q}_t = Q_t^*$ in each contingency,*
2. *$\tilde{M}_t = M_t^*$ in each contingency in which $i_t^* > 0$ while $\tilde{M}_t \geq P_t^* Y_t$ in each contingency in which $i_t^* = 0$.*

In the contingencies in which $i_t^* = 0$ (and only in these contingencies) $\tilde{M}_t \geq P_t^* Y_t$ implies a change in ***conventional monetary policy*** *if and only if the latter is specified as $M_t = \mathcal{M}(\bar{\mathbf{Z}}_t, \zeta_t)$. The Neutrality Property holds if the neutrality result applies for any appropriately bounded* ***balance-sheet policy*** *$\tilde{\mathcal{B}}(\cdot)$ and for each equilibrium $\{\mathbf{Z}_t^*, \mathbf{K}_t^*\} \in \mathcal{N}$.*

A key feature of the Neutrality Property is that the specification of the functional forms of the *transfer policy,* $\mathcal{T}(\cdot)$, and of the *conventional monetary policy* – either $\mathcal{I}(\cdot)$ or $\mathcal{M}(\cdot)$ –

[24]It is instead key for a proper definition of neutrality that M_t^* is invariant when the nominal interest rate is positive. See also Eggertsson and Woodford (2003). Auerbach and Obstfeld (2005) show that policies raising the money supply at the zero-lower bound consistently with $M_t > P_t Y_t$ can have an effect on current price level since they affect the price level once the economy exits the zero-lower bound. They can also influence the duration of the trap. However, these effects rely on a change in policy (what we called *conventional monetary policy*) that lasts after the trap ends, i.e. M is changed also after the end of the trap. Therefore, in this case, the change in prices observed in equilibrium is due to the change in *conventional monetary policy.* Instead, the neutrality result holds if after the end of the trap the *conventional monetary policy* is kept unchanged (see also Robatto, 2014). Buiter (2014) shows that an expansion in the stock of base money can have permanent wealth effects even in a permanent liquidity trap provided money is not seen as a liability by the central bank.

is not changed across the comparison (with the caveat mentioned in Definition 3) while what is varied is the functional form of the *balance-sheet policy* $\mathcal{B}(\cdot)$.

What we are going to show in the next section is that the Neutrality Property holds in our model only conditional on some specifications of the *transfer policy*. Indeed, Definition 3 characterizes neutrality starting from a set of equilibria identified by a certain *transfer policy*. This is not surprising: Wallace (1981) proved his irrelevance result of open-market operations in an overlapping-generation monetary model provided only "that lump-sum taxes are adjusted in an appropriate way" where "appropriate means, among other things, that fiscal policy is held constant." Wallace (1981) considers an environment in which fiat currency is not dominated in return. Sargent and Smith (1987) extend the irrelevance theorems to include cases of return dominance. Both Wallace (1981) and Sargent and Smith (1987) consider models in which open-market operations involve only risk-free short-term securities and in which there is a consolidated government's balance sheet, pooling together treasury and central bank.

Eggertsson and Woodford (2003) instead extend the irrelevance result to a model in which the central bank engages in unconventional open-market operations. However, they limit their analysis to the case in which central bank's net worth is zero and its only liability is money. This assumption constrains the set of balance-sheet policies that can be consistent with neutrality. Indeed, policies that enlarge the size of the central bank's balance sheet are neutral in their economy only when the nominal interest is at the zero-lower bound. We will instead show that this kind of policies can be neutral even when the nominal interest rate is positive, provided that the central bank issues interest-bearing reserves.[25] The reason is that the central bank can adjust reserves without necessarily varying *conventional monetary policy*.[26]

Another important difference between our result of neutrality and that of Eggertsson and Woodford (2003) is related to the transfer policy that ensures neutrality. In our case, it consists of two elements, as in Definition 1: *i*) the transfer policy between central bank and treasury and *ii*) that between treasury and the private sector. In their analysis, the key transfer policy to ensure neutrality is only that between the treasury and the private

[25]In this respect, our analysis is instead in line with the more recent literature on the role of reserves in dealing with the expansion of the central bank's balance sheet (see Bassetto and Messer, 2013, Del Negro and Sims, 2015, Hall and Reis, 2015).

[26]In the case in which $X_t = 0$, the first important difference is that the monetary/fiscal policy regime should specify five instead of six additional restrictions. If we maintain the same definitions of *conventional monetary policy* and *transfer policy* as in Definition 1, then *balance-sheet policies* can only specify two of the sequences $\left\{B_t^C, B_t^G, D_t^C, D_t^G\right\}$ as functions of the other endogenous variables and/or of exogenous state variables. This implies that a model without interest-bearing reserves limits substantially the kind of balance-sheet policies that can be considered independently of the specification of the *conventional monetary policy* and *transfer policy.*

sector which, moreover, acts on a consolidated budget constraint pooling together treasury and central bank. We instead keep separate budget constraints in the neutrality analysis and this is also key to address departure from non-neutrality, which is another novelty of our contribution with respect to all the above literature.[27]

Finally, we want to emphasize that the Neutrality Property is specified for generic balance-sheet policies regarding $\{D_t^C\}$, without spelling out whether the issuer is the treasury or the private sector. Indeed, the only feature of the issuer that matters is its credit worthiness, which may determine the size of the wealth effects on households when the central bank purchases long-term assets and experiences income losses.

3.1 Neutrality property holds

We start with the case mostly studied in the literature, in which the Neutrality Property holds conditional on certain *transfer policies*. The literature usually proceeds to make assumptions about the "consolidated" behavior of government, including central bank and treasury.[28] It is instead a key distinction of our analysis to keep the two institutions independent of each other to characterize departures from neutrality. We start by defining a regime in which fiscal policy is *passive*, in line with the literature but focusing only on the behavior of the treasury instead of that of the consolidated government.

Definition 4 *Under a passive fiscal policy the stochastic path of taxes $\{T_t^F\}$ is specified to ensure that the following limiting condition*

$$\lim_{T\longrightarrow\infty} E_t\left\{R_{t,T}\left(Q_T\frac{D_T^F}{P_T}+\frac{1}{1+i_T}\frac{B_T^F}{P_T}\right)\right\}=0 \tag{25}$$

is satisfied looking forward from each date $t\geq t_0$ (and in each contingency at t) together with the sequence of equilibrium conditions (21) for any finite D_{t-1}^F, B_{t-1}^F, any appropriately bounded stochastic process $\{T_T^C\}_{T=t}^{\infty}$ and for any collection of stochastic processes $\{\mathbf{Z}_t^*\}$ satisfying the conditions of part i) of Definition 2, consistently each with a specified conventional monetary policy.

According to Definition 4, lump-sum taxes are set in a way that the expected present discounted real value of treasury liabilities converges to zero for any vector of stochastic

[27] Another difference with Eggertsson and Woodford (2003) is that they assume a more general composition of the assets portfolio of the central bank, while we focus only on two securities. This difference has no consequence for the generality of our results.

[28] This is the case of Wallace (1981), Sargent and Smith (1987).

processes $\{\mathbf{Z}_t^*\}$ satisfying the equilibrium conditions (16) to (19) given a (and for any) *conventional monetary policy.* In equation (25), we have defined the real stochastic discount factor as $R_{t,T} \equiv \beta^{T-t}\xi_T Y_T^{-\rho}/\xi_t Y_t^{-\rho}$ which is only driven by exogenous processes.

Proposition 1 *The fiscal rule*

$$\frac{T_t^F}{P_t} = \bar{T}^F - \gamma_f \frac{T_t^C}{P_t} + \phi_f \left[\frac{(1+r_t)Q_{t-1}D_{t-1}^F + B_{t-1}^F}{P_t} \right] \tag{26}$$

is in the class of passive fiscal policies if and only if $\gamma_f = 1$ and $0 < \phi_f < 2$.

Proof. In the Appendix. ■

In (26), recall that we have defined the gross nominal return on long-term debt as $(1+r_t) \equiv (1+\delta Q_t)(1-\varkappa_t)/Q_{t-1}$. According to the fiscal rule (26), an increase in the outstanding real market value of treasury debt – of whatever maturity – signals an adjustment in the path of real taxes needed to repay it given the requirement that the parameter ϕ_f should be positive and in the range $0 < \phi_f < 2$. Furthermore, the rule is such that the treasury does not have to rely on central bank's remittances to repay its obligations, since the parameter γ_f should be equal to one. Therefore, if the central bank is reducing payments to the treasury, the latter should immediately raise lump-sum taxes on the private sector to "support" the same equilibrium allocation for prices and interest rate. This may seem a strong requirement but it is not just a peculiarity of the above rule. Indeed, we can consider another way to state the above definition of passive fiscal policy by noting that (25) together with (21) and (16)-(17) implies that

$$\frac{B_{t-1}^F}{P_t} + (1+r_t)\frac{Q_{t-1}D_{t-1}^F}{P_t} = E_t \sum_{T=t}^{\infty} R_{t,T} \left[\frac{T_T^F}{P_T} + \frac{T_T^C}{P_T} \right].$$

Under passive fiscal policy, taxes adjust in a way to ensure that the above equation holds for any vector of stochastic processes $\{\mathbf{Z}_t^*\}$ satisfying the equilibrium conditions (16) to (19) at each point in time and contingency given a *conventional* monetary policy and for any appropriately bounded stochastic process $\{T_T^C\}_{T=t}^{\infty}$ and finite D_{t-1}^F, B_{t-1}^F. A reduction in the present discounted value of remittances from the central bank should reflect under a passive fiscal policy a specular increase in the present discounted value of taxes levied on the private sector.

A passive fiscal policy has direct implications for the equilibrium path of central bank's net worth. Indeed, equation (25), together with the equilibrium condition (20) implies that the expected present discounted value of the central bank's real net worth converge to zero in equilibrium:

$$\lim_{T\longrightarrow\infty} E_t \left\{ R_{t,T} \frac{N_T^C}{P_T} \right\} = 0.$$

The latter equilibrium condition together with the flow budget constraint (22) and (16)-(17) now implies the following central bank's intertemporal budget constraint

$$\frac{X_{t-1}}{P_t} + \frac{M_{t-1}}{P_t} - \frac{B^C_{t-1}}{P_t} - (1+r_t)\frac{Q_{t-1}D^C_{t-1}}{P_t} = E_t \sum_{T=t}^{\infty} R_{t,T}\left[\frac{i_T}{1+i_T}\frac{M_T}{P_T} - \frac{T^C_T}{P_T}\right]. \tag{27}$$

The real market value of the outstanding net liabilities of the central bank at a generic time t, which corresponds to the left-hand side of (27), should be backed by the present discounted value of the revenues obtained by issuing money net of the transfers between the central bank and the treasury. Interestingly, there could be rational expectations equilibria in which the left-hand side of (27) is positive (net worth is negative), provided that the incoming seigniorage net of transfers is enough to back the net liabilities of the central bank. However, the key observation is that, in general, (27) can restrict the path of prices, interest rates and other endogenous variables in a way that the vector of stochastic processes $\{\mathbf{Z}^*_t\}$ satisfying the equilibrium conditions (16) to (19) consistently with some *conventional monetary policy* is not part of an equilibrium, unless additional assumptions are added to which we now turn our attention. We define a *passive policy of central bank's remittances*, in a similar way to the above definition of *passive fiscal policy* and irrespective of the latter specification.

Definition 5 *Under a passive policy of central bank's remittances the stochastic path of remittances $\{T^C_t\}$ is specified to ensure that*

$$\lim_{T\longrightarrow\infty} E_t\left\{R_{t,T}\frac{N_T}{P_T}\right\} = 0 \tag{28}$$

is satisfied looking forward from each date $t \geq t_0$ (and in each contingency at t) together with the sequence of equilibrium conditions (22) for any finite $X_{t-1}, B^C_{t-1}, D^C_{t-1}$ and for any sequence of stochastic processes $\{\mathbf{Z}^*_t\}$ satisfying the conditions of part i) of Definition 2, consistently each with a specified conventional monetary policy.

Under a passive remittances' policy, a worsening of the market value of the central bank's net liability position signals an increase in seigniorage revenues or in transfers from the treasury. According to Definition 5, we could design many remittances' policies that can satisfy the definition. One possibility is the following:

Proposition 2 *The remittances' policy*

$$\frac{T^C_t}{P_t} = \bar{T}^C + \gamma_c\frac{\Psi^C_t}{P_t} + \phi_c\frac{N^C_{t-1}}{P_t} \tag{29}$$

is in the class of passive remittances' policies if and only if $0 < \gamma_c < 2$ and $0 < \phi_c < 2$.

Proof. In the Appendix. ■

The above rule shows a positive relationship between the remittances to the treasury and both central bank's profits and past level of net worth. The central bank should avoid that net worth diverges and therefore prevent any wealth effect on households at the equilibrium prices. The reaction to both profits and past net worth ensures the boundedness of net worth which then satisfies condition (28) at any equilibrium prices. It is worth noting that a reaction to current profits implicitly builds also a reaction to the past level of net worth as shown in (14).

The first neutrality result that we discuss holds under a combined regime given by the two passive policies defined above.

Proposition 3 *Under a combined regime of passive fiscal policy and passive policy of central-bank remittances the Neutrality Property holds.*

Proof. In the Appendix. ■

Under the conditions stated in Proposition 3 whether the central bank purchases or not long-term securities, eventually recording losses on these operations, is irrelevant for the equilibrium allocation of prices, interest rates and asset prices.

Rule (29) does not require a one-to-one reaction of remittances to central bank's profits or losses, but the reaction can be smoothed across time through a response to past level of net worth. However, in common central banks' practices, remittances are not linked to past levels of net worth and instead depend more on the current level of profits. One interesting example is what we call *full treasury's support*, which is the case where transfers from central bank to treasury are always equal to profits, $T_t^C = \Psi_t^C$. The central bank remits positive profits to the treasury and, specularly, the treasury is ready to immediately cover the losses of the central bank when they occur.[29] A somewhat surprising result is contained in the following proposition.

Proposition 4 *A regime of full treasury's support, $T_t^C = \Psi_t^C$ at each date t (and in each contingency at t), is not in the class of passive remittances' policies.*[30]

[29] An example is that of the Bank of England which in January 2009 established a wholly-owned subsidiary called Bank of England Asset Purchase Facility Fund Limited with the responsibility of buying private and public long-term securities through funds of the same Bank of England raised through increases in reserves (see Bank of England, 2013). The created company is fully indemnified by the Treasury since any financial losses as a result of the asset purchases are borne by the Treasury and any gains are owed to the Treasury.

[30] Moreover a regime of full treasury's support is clearly not in the class of rules identified by (29). A different definition of treasury's support would be the one in which the transfers to and from the treasury are set to achieve a constant path of real net worth as in the real mark-to-market dividend rule defined by Hall and Reis (2015). This remittances' policy will be in the class of passive remittances' policies. Our definition of treasury's support instead coincides with the nominal mark-to-market dividend rule of Hall and Reis (2015).

Proof. In the Appendix. ■

This result, however, does not imply that the Neutrality Property is violated when $T_t^C = \Psi_t^C$, but only that some stochastic processes $\{\mathbf{Z}_t\}$ satisfying the equilibrium conditions (16) to (19) are ruled out as equilibrium allocations under a regime of full treasury's support. In particular, as it is shown in the proof of Proposition 4, we should exclude equilibria in which the short-term nominal interest rate remains at zero for an infinite period of time. This is, however, an interesting result since it implies that permanent liquidity traps are not equilibria if the treasury follows a passive fiscal policy and at the same time central bank's profits are fully transferred to the treasury or losses are fully covered by the treasury.

Proposition 5 *Under a passive fiscal policy and full treasury's support the Neutrality Property holds.*

Proof. In the Appendix. ■

The proof of the above Proposition sheds light on the key role played by interest-bearing reserves for neutrality cases. Indeed, under Propositions 3 and 5 Neutrality Property holds even if the central bank increases the size of its balance sheet when nominal interest rate is positive. The role of reserves is indeed critical for the validity of this result. A constant nominal net worth, as implied by a regime of full treasury's support, requires that in equilibrium

$$Q_t^* \tilde{D}_t^C + \frac{\tilde{B}_t^C}{1+i_t^*} - M_t^* - \frac{\tilde{X}_t}{1+i_t^*} = \bar{N}.$$

This equation shows that alternative balance-sheet policies $\tilde{\mathcal{B}}(\cdot)$ implying different paths for the asset composition of the central bank $\{\tilde{B}_t^C, \tilde{D}_t^C\}$ can be accommodated by variations in central-bank reserves $\{\tilde{X}_t\}$ at the equilibrium prices $\{i_t^*, Q_t^*\}$ without changing the equilibrium path of money $\{M_t^*\}$.[31] Without interest-bearing reserves, the specification of the *balance-sheet policy* loses one degree of freedom and can only set one of the stochastic processes $\{B_t^C, D_t^C\}$ while the other is endogenously determined by the equilibrium conditions and the remaining specification of the monetary/fiscal policy regime. The central bank can, for example, increase the stock of long-term securities held in its portfolio. When $i_t^* > 0$ and under full *treasury's support* and *passive fiscal policy*, these purchases are neutral because they are followed by a drop in the holdings of short-term securities that keeps invariant the total value of the assets of the central bank. Increases in the total value of the assets, though, are possible but they have to be matched by a higher level of M_t and therefore, given (18),

[31] As detailed in the Appendix, it is also clear that alternative *balance-sheet policies* should be appropriately bounded to satisfy the non-negative requirement on central-bank reserves. In the case of positive nominal interest rates, the value of the total assets purchased by the central bank should be bounded below by the value of non interest-bearing liabilities, *i.e.* $Q_t^* \tilde{D}_t^C + \tilde{B}_t^C/(1+i_t^*) \geq M_t^* + \bar{N}$.

are consistent with a different, and higher, equilibrium price level – a non-neutrality result. Instead, when we allow the central bank to issue interest-bearing reserves, balance-sheet policies that raise the value of the assets of the central bank are neutral even when the nominal interest rate is positive if the conditions of Propositions 3 and 5 are met.[32]

Rules (26) and (29), or $T_t^C = \Psi_t^C$ in place of the latter, clearly illustrate the transfer mechanisms that support the Neutrality Property. For central bank's long-term asset purchases to have an effect, there should be some change in total (financial and human) wealth of households to induce them to vary their consumption choices. The consequent change in aggregate demand, given an exogenous stream of output, would then result in a variation of equilibrium prices. However, rules (26) and (29) make instead sure that there is no such a change in households' total wealth. There are only offsetting adjustments of human and financial wealth. Indeed, if the central bank purchases some risky securities which were before in the hands of the private sector, that risk does not remain in the hands of the central bank since rule (29) ensures that the treasury transfers resources to the central bank in the case the risk materializes in negative profits, while rule (26) ensures that the treasury gets these resources from the private sector through higher lump-sum taxes. At the end, the materialization of risk goes back to the private sector whose total wealth does not change when evaluated at the initial equilibrium prices, therefore equilibrium prices do not need to change.

Alternative transfer policies that break these linkages can challenge the result of neutrality. We turn to this analysis in the next section.

3.2 Neutrality property does not hold

In this section, we discuss three cases for which the Neutrality Property does not hold. In the first two cases, we maintain the assumption that fiscal policy is passive and instead elaborate more on the kind of remittances' policies that can lead to violations of the Neutrality Property. In the third case, we discuss the implications of an active fiscal policy regime.

3.2.1 Absence of treasury's support

In this subsection, we limit the type of remittances' policy between central bank and treasury by assuming that the treasury never transfers resource to the central bank, i.e. $T_t^C \geq 0$. The

[32] When $i_t = 0$ and $X_t = 0$, an increase in the holdings of long-term securities, and therefore an enlargement of the balance-sheet size, can be instead accommodated through an increase in money supply (or reserves), without the need of an offsetting fall in the holdings of short-term securities. The higher supply of money can be absorbed by households without affecting their consumption choices since the opportunity cost of money is zero. Under a *passive fiscal policy* and *treasury's support*, therefore, we obtain a neutrality result similar to that discussed by Eggertsson and Woodford (2003), using a different *transfer policy*.

central bank lacks treasury's support. To intuit the implications of this restriction, consider an allocation $\{\mathbf{Z}_t^*\}$ satisfying the equilibrium conditions (16) to (19) at each point in time and contingency given a *conventional monetary policy*. We evaluate whether this allocation can be an equilibrium considering a passive fiscal policy, a non-negative remittances policy and a generic *balance-sheet policy* $\mathcal{B}(\cdot)$. Given that fiscal policy is still passive, equation (25) together with (20) implies that (28) must hold in equilibrium.

The allocation $\{\mathbf{Z}_t^*\}$ is part of a rational expectations equilibrium, given the specification of the monetary/fiscal policy regime and in particular without treasury's support, if there are stochastic processes $\{X_t^*, T_t^{C*}\}$, with $X_t^*, T_t^{C*} \geq 0$, such that

$$\frac{X_{t-1}^*}{P_t^*} - \frac{B_{t-1}^{C*}}{P_t^*} + \frac{M_{t-1}^*}{P_t^*} - (1 + r_t^*)\frac{Q_{t-1}^* D_{t-1}^{C*}}{P_t^*} = E_t \sum_{T=t}^{\infty} R_{t,T} \left[\frac{i_T^*}{1 + i_T^*} \frac{M_T^*}{P_T^*} - \frac{T_T^{C*}}{P_T^*} \right] \tag{30}$$

holds at all times and in each contingency considering that the stochastic path of $\{B_t^{C*}, D_t^{C*}\}$ is implied by the *balance-sheet policy*.[33] The Neutrality Property holds if the above condition is satisfied for any appropriately bounded *balance-sheet policy* $\mathcal{B}(\cdot)$ at each time t and contingency at t.

We describe now two remittances' policies – satisfying $T_t^C \geq 0$ – that can imply violations of the Neutrality Property. The first is an exogenous real remittances' policy for which $T_t^C/P_t = \bar{T}_t^C$, where we assume that $\bar{T}_t^C$ is a Markov process, with transition density $\pi_C(\bar{T}_{t+1}^C|\bar{T}_t^C)$ and initial distribution f_C. Moreover (π_C, f_C) is such that $\bar{T}_t^C \in [\bar{T}_{\min}^C, \bar{T}_{\max}^C]$ where in particular $\bar{T}_{\min}^C \geq 0$.

Proposition 6 *Under a passive fiscal policy and an exogenous real remittances' policy with* $T_t^C/P_t = \bar{T}_t^C \geq 0$*, the Neutrality Property does not hold.*

The proof follows by substituting the exogenous remittances' policy into (30). Consider a generic rational expectations equilibrium $\{\mathbf{Z}_t^*, \mathbf{K}_t^*\}$ for a given *conventional monetary policy*, a *transfer policy* composed by a passive fiscal policy and an exogenous remittances' policy, and for some *balance-sheet policy* $\mathcal{B}(\cdot)$. As discussed above, to be an equilibrium, it should satisfy (30) where $T_t^{C*}/P_t^* = \bar{T}_t^C$. Maintain the same *conventional monetary policy* and *transfer policy* and change the *balance-sheet policy* to $\tilde{\mathcal{B}}(\cdot)$. Then, for neutrality to hold, $\{\mathbf{Z}_t^*, \tilde{\mathbf{K}}_t\}$ should satisfy the following:

$$\frac{\tilde{X}_{t-1}}{P_t^*} - \frac{\tilde{B}_{t-1}^C}{P_t^*} + \frac{M_{t-1}^*}{P_t^*} - (1 + r_t^*)\frac{Q_{t-1}^* \tilde{D}_{t-1}^C}{P_t^*} = E_t \sum_{T=t}^{\infty} R_{t,T} \left[\frac{i_T^*}{1 + i_T^*} \frac{M_T^*}{P_T^*} - \bar{T}_t^C \right].$$

[33] Note that $R_{t,T}$ is function of exogenous states only.

However, considering that (30) holds, and given that r_t^* is stochastic, the above equation cannot hold in each contingency unless $\tilde{D}_{t-1}^C = D_{t-1}^{C*}$.

The second remittances' policy that we consider is what we call a *"deferred-asset" regime*. This is meant to capture the case of the U.S. Federal Reserve, which does not receive full support from the treasury and, in the case of negative profits, stops making remittances and issues a "deferred asset" that can be paid back by retaining future earnings. Only once the "deferred asset" is paid in full, the central bank resumes remitting positive profits to the treasury.[34]

Definition 6 *A "deferred-asset" policy of central-bank remittances is defined as* $T_t^C = \max(\Psi_t^C, 0)$ *if* $N_{t-1}^C \geq \bar{N} > 0$ *otherwise* $T_t^C = 0$.[35]

Unlike Proposition 6, under a *"deferred-asset" policy* whether or not the Neutrality Properties is violated depends on the actual size of the central bank's income losses.

Proposition 7 *Under a passive fiscal policy and a "deferred-asset" policy of central bank's remittances the Neutrality Property holds if and only if*

$$\frac{\tilde{N}_t^C}{P_t^*} > -E_t \sum_{T=t}^{\infty} R_{t,T} \left(\frac{i_T^*}{1+i_T^*} \frac{M_T^*}{P_T^*} \right) \tag{31}$$

in equilibrium at each time t (and in each contingency at t).[36]

Proof. In the Appendix. ■

We leave the proof to the Appendix but here we provide some intuition for condition (31). Note that an alternative way to write (30), using the definitions of central bank's net worth and profits, is:

$$\frac{N_t^C}{P_t^*} + E_t \sum_{T=t}^{\infty} R_{t,T} \left(\frac{i_T^*}{1+i_T^*} \frac{M_T^*}{P_T^*} \right) = E_t \sum_{T=t+1}^{\infty} R_{t,T} \left(\frac{T_T^C}{P_T^*} \right). \tag{32}$$

The left hand side of the above equation is the value of the central bank (in real terms) given by the sum of its real net worth and the value of current and future resources that

[34] See Carpenter et al. (2015). In our analysis we are abstracting from operating costs and standard dividends to member banks subscribing the capital of the central bank.

[35] Writing explicitly a "deferred asset" in the problem like a negative liability, as it is done in practice to avoid that the accounting value of net worth declines (see also Hall and Reis, 2015), does not really matter for the analysis since values of net worth below the threshold $\bar{N}$ would correspond to periods in which the deferred asset is positive. In both cases, positive income will be retained by the central bank either to increase net worth or to pay the "deferred asset". Note that we are adopting a nominal mark-to-market dividend rule according to the definition given by Hall and Reis (2015).

[36] Note that (31) should be evaluated at $\{\mathbf{Z}_t^*\}$ of the "starting" equilibrium $\{\mathbf{Z}_t^*, \mathbf{K}_t^*\}$ of Definition 3 where $\tilde{N}_t^C$ instead is the value of net worth reached under the alternative balance-sheet policy $\tilde{\mathcal{B}}(\cdot)$.

can be obtained from the monopoly power of issuing money. In equilibrium, given a *passive fiscal policy*, the value of the central bank should be equal to the expected present discounted value of real transfers to and from the treasury. Under *full treasury's support*, no matter what is the equilibrium value of the central bank, the treasury is ready to back it. If the left-hand side is negative because net worth has reached a too-low level in equilibrium, it is indeed the treasury that transfers resources to the central bank, thus adjusting the present discounted value of net transfers. In absence of treasury's support, however, the right-hand side of (32) cannot be negative, thus imposing a lower bound on the level that net worth can reach (equation 31) consistently with the allocation $\left\{\mathbf{Z}_t^*\right\}$.[37]

In absence of treasury's support, purchases of long-term securities can lead to violations of (31) at $\left\{\mathbf{Z}_t^*\right\}$. Indeed, income losses translate directly into declining net worth

$$N_t^C = N_{t-1}^C + \Psi_t^C - T_t^C < N_{t-1}^C$$

which in turn can be inconsistent with (31) or (32). This inconsistency implies that $\left\{\mathbf{Z}_t^*\right\}$ is not part of a rational expectations equilibrium given the *balance-sheet policies* undertaken, meaning that $\left\{\mathbf{Z}_t\right\}$ should change in a way that the central bank remains solvent at the level of net worth N_t^C reached. In practice, this can be accomplished by an increase in the present discounted value of seigniorage revenues and/or an increase in the current price level.

Moreover, when (31) is satisfied in each contingency then (32) implies that the expected present discounted value of remittances to the treasury is positive, which ensures that there are contingencies where the central bank records positive profits and rebuilds net worth back to $\bar{N}$ (see Definition 6). Indeed, we now show two special cases in which the necessary and sufficient condition for neutrality is precisely that net worth returns back to $\bar{N}$ in a finite period of time. In the first, we consider that the exogenous stochastic processes have an absorbing state after some finite period of time. In the second, we allow for only temporary central bank's purchases of long-term securities.[38]

Proposition 8 *Consider either the case i) in which all the exogenous stochastic disturbances have an absorbing state starting from time τ or the case ii) in which $D_t^C = 0$ for each $t \geq \tau$. Under a passive fiscal policy and a "deferred-asset" policy of central bank's remittances the*

[37]The solvency condition (32) has been already emphasized in the works of Bassetto and Messer (2013), Del Negro and Sims (2015) and Hall and Reis (2015). In particular Del Negro and Sims (2015) have discussed that violations of the solvency conditions without treasury support could lead to a change in the interest-rate policy while Hall and Reis (2015) have instead focused on the possible non-stationarity of the path of reserves, since they consider equilibrium prices as given.

[38]Note that allowing for only temporary central bank's purchases of long-term securities is in contrast with the generality of Definition 3. However, one could define weaker forms of Neutrality Property restricting the set of balance-sheet policies.

Neutrality Property holds if and only if $\tilde{N}_t^C = \bar{N} > 0$ *in equilibrium for each* $t \geq \tau_1$ *with* $\tau_1 \geq \tau$.[39]

Proof. In the Appendix. ■

The condition of the above proposition underlines a return to profitability, which is the only way through which the central bank can recover from income losses without treasury's support. Under the conditions of Proposition 8 and after period τ, central bank's profits (15) are given by $\Psi_t^C = i_{t-1}(N_{t-1}^C + M_{t-1}^C)$. In particular an alternative *balance-sheet policy* $\tilde{\mathcal{B}}(\cdot)$ implies neutrality if $\tilde{N}_t + M_t^* > 0$ for each $t \geq \tau$, which is equivalent to require that assets should be above interest-bearing liabilities.[40] If $\tilde{N}_t + M_t^* > 0$ for each $t \geq \tau$, indeed, profits are positive, which allow the central bank to restore the initial net worth in a finite period of time by retaining earnings. In particular, the sufficient condition $\tilde{N}_t + M_t^* > 0$ for each $t \geq \tau$ imposes a bound on the level of net worth ($\tilde{N}_t > -M_t^*$) and, moreover, also shows that if net worth falls below the bound then the way to restore profitability is to increase the supply of money, thus breaking neutrality. We will elaborate more on this result in Section 4 when discussing the numerical analysis.

3.2.2 Financial independence

In this subsection, we still maintain the assumption of passive fiscal policy but assume that even if treasury's support is available, the central bank wants to avoid it to a certain extent. We call this regime *financial independence* and there are several ways through which it can be formalized. A simple (and strong form) of *financial independence* is assuming that the central bank wants to completely avoid periods of negative profits. Milder forms could allow for decline in net worth for a limited duration. In what follows, we assume that the central bank transfers its income to the treasury, $T_t^C = \Psi_t^C$, and faces the *financial-independence* restriction that profits should be non-negative, $\Psi_t^C \geq 0$, thus avoiding the support of the treasury.

An interesting monetary economics trilemma arises among choosing freely the *conventional monetary policy* and the *balance-sheet policy* while maintaining *financial independence*. Proposition 3 shows one leg of this trilemma: a central bank which engages in any arbitrary *balance-sheet policy* and is committed to a certain *conventional monetary policy* needs some support from the treasury and therefore cannot be *financially independent*. Transfers from

[39] Note that $\tilde{N}_t^C = \bar{N}$ in equilibrium means that net worth under an alternative balance-sheet policy $\tilde{\mathcal{B}}(\cdot)$ should reach $\bar{N}$ conditional on $\{\mathbf{Z}_t^*\}$ of the "starting" equilibrium $\{\mathbf{Z}_t^*, \mathbf{K}_t^*\}$ of Definition 3.

[40] Moreover note that the proof of Proposition 8 shows that the allocation in which interest rates are zero for an infinite period is not an equilibrium given the transfer policy.

the central bank to the treasury should be negative under some conditions, i.e. $T_t^C < 0$ for some t. As well, a central bank committed to a certain *conventional monetary policy* that always wants to maintain *financial independence* has to restrict the type of *balance-sheet policies.* For example, it should limit purchases only to riskless short-term securities as in the conventional open-market operations. Finally, a central bank that chooses an arbitrary *balance-sheet policy* and aims at remaining *financially independent* cannot freely choose the *conventional monetary policy*.

We elaborate now on the latter leg of the trilemma. Under *financial independence*, $T_t^C = \Psi_t^C \geq 0$, central bank's net worth is always constant and positive $N_t^C = N_{t_0-1}^C = \bar{N} > 0$ for each t. Consider an equilibrium $\{\mathbf{Z}_t^*, \mathbf{K}_t^*\}$ given some *balance-sheet policy* $\mathcal{B}(\cdot)$, consistent with *financial independence* and recall the definition of profits (15)

$$\Psi_t^{C*} = i_{t-1}^*(\bar{N} + M_{t-1}^{C*}) + (r_t^* - i_{t-1}^*)Q_{t-1}^* D_{t-1}^{C*} \geq 0.$$

Consider now an alternative balance-sheet policy $\tilde{\mathcal{B}}(\cdot)$ that changes, among other variables, D_{t-1}^{C*} to $\tilde{D}_{t-1}^C$. It is easy to see that in the contingencies in which the excess return on long-term securities is negative ($r_t^* < i_{t-1}^*$) a balance-sheet policy implying higher holdings of long-term securities, $\tilde{D}_{t-1}^C > D_{t-1}^{C*}$, can turn profits into the negative territory requiring then a change of *conventional monetary policy* to meet the non-negative profit condition. The central bank could change the path of interest rates and/or money supply or tilt monetary policy in a way that the return on long-term securities doesn't fall much, for whatever is possible. It is worth emphasizing that it is not necessary that the non-negative constraint on profit binds today to lead to a change in policy, but it is just sufficient the expectation that the constraint is going to bind in some future contingency.[41]

The above reasoning leads to the following Proposition.

Proposition 9 *Under a passive fiscal policy and financial independence of the central bank the Neutrality Property does not hold.*

[41] Bhattaraj et al. (2015) also analyze a case in which unconventional balance-sheet policies can signal a change in the conventional monetary policy but in a context in which the policymaker cannot commit as opposed to our framework. Moreover, they assume a loss function in which the central bank faces a quadratic penalty of non-zero remittances, which treats symmetrically positive or negative values and implies non-neutrality of any balance-sheet policy. Instead, our definition of *financial independence* penalizes only central bank's income losses (and not positive remittances) which are therefore the only driver of non-neutrality in our case.

3.2.3 Active fiscal policy

We now relax the assumption of passive fiscal policy and limit the ability of the treasury to tax or transfer resources as needed by assuming an active policy of the form

$$\frac{T_t^F}{P_t} = \bar{T}_t^F, \tag{33}$$

where $\bar{T}_t^F$ is a Markov process, with transition density $\pi_F(\bar{T}_{t+1}^F|\bar{T}_t^F)$ and initial distribution f_F. We assume that (π_F, f_F) is such that $\bar{T}_t^F \in [\bar{T}_{\min}^F, \bar{T}_{\max}^F]$.

To get the intuition straight in this section we assume that there is *full treasury's support*, i.e. $T_t^C = \Psi_t^C$ at each t. As previously shown, this implies a constant central bank's net worth $N_t^C = N_{t_0-1} = \bar{N} > 0$. By assuming *full treasury's support*, we are putting ourselves in the case in which, were fiscal policy passive, Neutrality Property would hold. But fiscal policy is now active.

Consider an equilibrium $\{\mathbf{Z}_t^*, \mathbf{K}_t^*\}$ under the fiscal rule (33) and *full treasury's support*, given a *conventional monetary policy* and a *balance-sheet policy*. The set of equilibrium conditions (20) to (24) implies a consolidated intertemporal budget constraint

$$\frac{X_{t-1}^*}{P_t^*} + \frac{M_{t-1}^*}{P_t^*} + \frac{B_{t-1}^*}{P_t^*} + (1+r_t^*)\frac{Q_{t-1}^* D_{t-1}^*}{P_t^*} = E_t \sum_{T=t}^{\infty} R_{t,T}\left[\frac{i_T^*}{1+i_T^*}\frac{M_T^*}{P_T^*} + \bar{T}_T^F\right]$$

showing that the overall liabilities of the whole government should be backed by the expected present discounted value of seigniorage revenues and primary surpluses. Under *full treasury's support,* we can further write the above intertemporal budget constraint as

$$\frac{B_{t-1}^{F*}}{P_t^*} + (1+r_t^*)\frac{Q_{t-1}^* D_{t-1}^{F*}}{P_t^*} - \frac{\bar{N} + \Psi_t^{C*}}{P_t^*} = E_t \sum_{T=t}^{\infty} R_{t,T}\left[\frac{i_T^*}{1+i_T^*}\frac{M_T^*}{P_T^*} + \bar{T}_T^F\right], \tag{34}$$

having used the definitions of central bank's net worth and profits. Consider now an alternative *balance-sheet policy* that just changes the holdings of long-term securities of the central bank at time $t-1$ to $\tilde{D}_{t-1}^C \neq D_{t-1}^{C*}$ assuming $r_t^* \neq i_{t-1}^*$. It simply follows that $\tilde{\Psi}_t^C \neq \Psi_t^{C*}$ which implies that $\{\mathbf{Z}_t^*\}$ could no longer be consistent with (34) and therefore cannot be part of an equilibrium under the new *balance-sheet policy.* This is again a result of non-neutrality.

If the central bank purchases long-term securities and the treasury does not pass the gains or losses to the private sector, there can be a reallocation of risk in the economy inducing wealth effects on households that move consumption, aggregate demand and then prices.

Proposition 10 *Under an active fiscal rule of the form (33) and for any specification of central-bank remittances' policy, the Neutrality Property does not hold.*

4 Numerical evaluation

This section presents numerical exercises on the quantitative effects of the non-neutrality results that we discussed in the previous section. To this end, we generalize the model to include supply of labor by households and endogenous production by firms in a market characterized by monopolistic competition and nominal rigidities in the form of Calvo's price-setting mechanism. The stochastic structure is enriched by including also productivity and mark-up stochastic disturbances. Details of the model are presented in the Appendix.

Using this more general model, we offer two types of numerical analysis. First, we characterize the neutrality or non-neutrality of alternative *balance-sheet policies* for the optimal allocation conditional on a specific *transfer policy*. In particular, we study the dynamic response of variables of interest to realizations of either interest-rate or credit risks, in isolation.

Second, we produce simulated time series to study the evolution of variables of interest conditional on a sequence of shocks that shares key features with the U.S. data in the recent financial crisis. In particular we compare the outcome of a suboptimal monetary policy – a Taylor Rule – with two alternatives: the constrained first best and the optimal monetary policy conditional on the three non-neutrality cases discussed in the previous section.

4.1 Impulse-Response Analysis

In this section, we show how alternative balance-sheet compositions affect the response of output, inflation and interest rate to exogenous shocks materializing either interest-rate or credit risk, under optimal monetary policy conditional on given transfer policies. Regarding the transfer policy, we consider four cases that we label: i) *neutral transfer policy*, combining a *passive fiscal policy* and a *passive remittances' policy* as in Proposition 3; ii) *"lack of treasury's support" regime*, consisting of a *passive fiscal policy* and a *"deferred-asset" remittances' policy* as in Proposition 7; iii) *financial independence*, combining *passive fiscal policy* and *central bank's financial independence* as in Proposition 9; iv) *active fiscal policy*, consisting of an *active fiscal policy* and a *"deferred-asset" remittances' policy* as in Proposition 10.

Optimal monetary policy maximizes the utility of the representative household. In a second-order approximation around a non-stochastic steady state, this amounts to minimizing, at time t_0, the following quadratic loss function

$$L_{t_0} = \frac{1}{2} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[\tilde{Y}_t^2 + \lambda_\pi (\pi_t - \bar{\pi})^2 \right] \tag{35}$$

where λ_π is the relative weight (defined in the appendix) attached to inflation costs, in which we have defined $\pi_t \equiv \ln \Pi_t$ and $\bar{\pi} \equiv \ln \bar{\Pi}$. Moreover, $\tilde{Y}_t$ denotes the log of the output gap with respect to the efficient equilibrium, defined as

$$\tilde{Y}_t \equiv \ln Y_t - \frac{1+\eta}{\rho+\eta} \ln A_t$$

where ρ is the relative risk-aversion coefficient, η is the inverse of the Frisch-elasticity of labor supply, and A_t is a stochastic index of labor productivity.[42]

We start our analysis with the regime of *neutral transfer policy*.[43] The optimal monetary policy in this case can be understood as that minimizing the loss function (35) subject to a minimal set of constraints, which only includes an aggregate-supply and an aggregate-demand equation. The (modified) New-Keynesian Aggregate-Supply equation is

$$\pi_t - \bar{\pi} = \kappa \tilde{Y}_t + \zeta \hat{\imath}_t + \beta E_t(\pi_{t+1} - \bar{\pi}) + u_t \tag{36}$$

where κ and ζ are defined in the appendix, $\hat{\imath}_t \equiv \frac{i_t - \bar{\imath}}{1+\bar{\imath}}$, and u_t captures *exogenous* cost-push shocks. Notice, moreover, that the financial friction – which is usually absent in standard versions of the New-Keynesian Aggregate-Supply equation and is here captured by fluctuations in the nominal interest rate – acts as an *endogenous* cost-push disturbance in the AS equation. The Aggregate-Demand equation is given by

$$\rho E_t(\tilde{Y}_{t+1} - \tilde{Y}_t) = \hat{\imath}_t - r_t^n - E_t(\pi_{t+1} - \bar{\pi}) \tag{37}$$

where the natural rate of interest is defined as

$$r_t^n \equiv \rho \frac{1+\eta}{\rho+\eta} E_t \Delta \ln A_{t+1} - E_t \Delta \ln \xi_{t+1}$$

which varies because of preference and productivity shocks.[44]

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[42] An interesting result, derived in the Appendix, is that the efficient steady state of the model is reached by setting the employment subsidy to $\varrho \equiv 1-(1-1/\theta)/(1+\bar{\imath})$ where $\bar{\imath}$ is the steady-state level of the nominal interest rate. One needs to use only one instrument of policy to offset both the monopolistic distortion and the financial friction, since both create an inefficient wedge between the marginal rate of substitution between leisure and consumption and the marginal product of labor. Moreover, given this result, the steady-state level of the nominal interest rate can be different from zero, while the inflation rate can be set at the target $\bar{\Pi}$. The steady-state version of equation (8) relates the nominal interest rate to the inflation rate $\beta(1+\bar{\imath}) = \bar{\Pi}$. This result crucially depends on the assumption that all consumption requires cash. It would fail in a model with cash and credit goods.

[43] In particular, we use rule (26), with response coefficients $\gamma_f = 1$ and $\phi_f = .01$, and rule (29), with response coefficients $\gamma_c = \phi_c = 1$.

[44] In the simulations below, the stochastic shocks evolve according to the following processes

$$\begin{aligned} \Delta \ln \xi_t &= \rho_\xi \Delta \ln \xi_{t-1} + \sigma_\xi \varepsilon_{\xi,t} \\ \ln A_t &= \rho_A \ln A_{t-1} + \sigma_A \varepsilon_{A,t} \\ u_t &= \rho_u u_{t-1} + \sigma_u \varepsilon_{u,t} \end{aligned}$$

Under neutrality, therefore, the path of output, inflation and nominal interest rate that solves the minimization of (35) under constraints (36)–(37), given $i_t \geq 0$ at each time $t \geq t_0$, is independent of the balance-sheet policy, and in particular of the amount of long-term assets (D^C) held at time t_0 by the central bank.[45]

Figures 1 and 2 show the dynamic response of selected variables to shocks materializing interest-rate risk and credit risk, respectively, under two alternative balance-sheet policies: $D^C = 0$ or $\tilde{D}^C = 72\%$.[46]

With respect to interest-rate risk, we run the following experiment. We simulate an economy which at time $t_0 - 1$ is already in a liquidity trap, because of a preference shock (ξ) that hit sometime in the past and turned the natural interest rate negative. At time t_0 the central bank first chooses whether to stick to its past balance-sheet policy ($D^C = 0$) or to engage in large-scale asset purchases ($D^C > 0$) and then commits to a state-contingent path for the endogenous variables from t_0 onward, conditional on the chosen balance-sheet policy. One year later (at time $t_0 + 4$) an unexpected preference shock hits, turning the natural interest rate positive again. At this time, the path of current and future short-term rates changes, producing an unexpected fall in the price of long-term securities and therefore implying income losses for the central bank, in the case it holds long-term assets.

With respect to credit risk, we consider an economy starting at steady state, and a credit event hitting at time t_0, which implies default on a share $\varkappa$ of long-term debt. After period t_0 no other credit event or other shocks are either expected or actually occur. As clear from equation (14), when a credit event occurs, the central bank might experience a loss on its balance sheet if it holds long-term securities.

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with $\rho_\xi = .93$, $\rho_A = .9$, $\rho_u = .9$, $\sigma_\xi = .001$, $\sigma_A = .001$, $\sigma_u = .01$, and $\varepsilon_{i,t} \sim iid(0,1)$ for $i = \xi$, A, u.

[45] To account for the non-negative constraints possibly affecting the evolution of the nominal interest rate and central bank's remittances, we use a piecewise linear approximation of the first-order conditions of the optimal policy problem. See Guerrieri and Iacoviello (2015).

[46] The model is calibrated (quarterly) as follows. We set the steady-state inflation rate and nominal interest rate on short-term bonds to 2% and 3.5%, respectively and in annualized terms; accordingly, we set $\beta = (1 + \bar{\pi})/(1 + \bar{\imath})$. We calibrate the composition of central bank's balance sheet considering as initial steady state the situation in 2009Q3, when the economy had already been in a liquidity trap for about three quarters. Accordingly we set the share of money to total liabilities equal to 53%, the share of net worth to total liabilities to 1%, and the share of long-term asset to total assets to 72%. This calibration implies that the steady-state quarterly remittances to the treasury are equal to about 0.6% of the central bank's assets and that the central bank's position on short-term interest-bearing liabilities (central bank reserves) amounts to 46% of the central bank's balance sheet. The duration of long-term assets is set to ten years (accordingly, $\delta = .9896$). Finally, following Benigno et al. (2016), we set the relative risk-aversion coefficient to $\rho = 1/.66$, the inverse of the Frisch-elasticity of labor supply to $\eta = 1$, the elasticity of substitution across goods to $\theta = 7.88$, the parameter α capturing the degree of nominal rigidity in the model implies an average duration of consumer prices of four quarters ($\alpha = 0.75$). As a result, the slope of the Phillips Curve is $\kappa = .024$. To calibrate the initial level of the natural interest rate, we follow Benigno et al. (2016), who show that the extent of households' debt deleveraging observed since 2008 in the U.S. is consistent with a fall of the natural interest rate to about -6% from a steady-state level of 1.5%. See also Gust et al. (2016), who provide consistent empirical evidence.

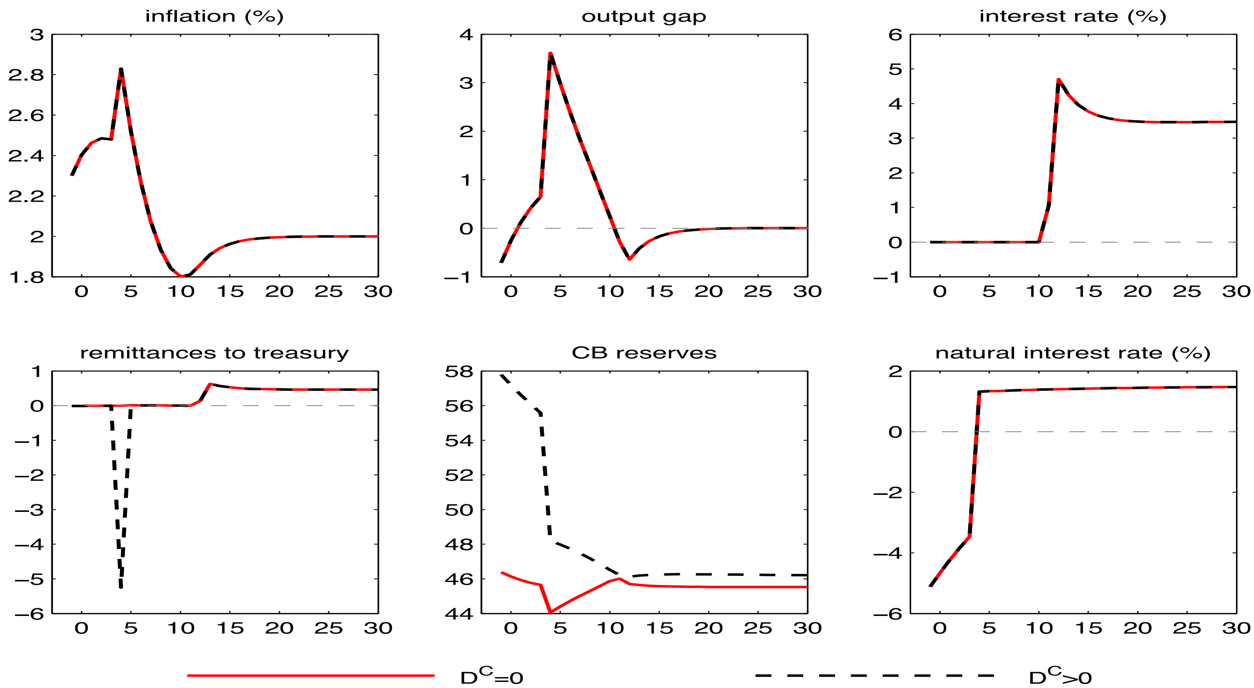

Figure 1: Equilibrium dynamics of selected variables under optimal monetary policy facing interest-rate risk. Regime i): *neutral transfer policy*. The economy starts in a liquidity trap with a negative natural rate of interest; the latter turns positive unexpectedly after one year. Red solid line: central bank holds only short-term assets. Black dashed line: central bank holds also long-term assets. X-axis displays quarters.

The top panels of Figure 1 display the path of inflation, the output gap and the nominal interest rate, and show the familiar result, already discussed in Eggertsson and Woodford (2003), that committing to a higher inflation for the periods after the liftoff of the natural rate of interest allows to limit the deflationary impact of the negative shock, despite the nominal interest rate cannot be cut as much as needed because of the zero floor. This commitment translates into maintaining the policy rate at the zero bound for several periods after the natural rate has turned back positive (in the specific case of Figures 1, for six quarters more).

The bottom panels show instead the evolution of two key variables related to the balance sheet of the central bank – as well as the path of the natural interest rate: the quarterly real remittances to the treasury T_t^C/P_t and the central bank's real reserves X_t/P_t, all expressed as a share of the steady-state balance sheet of the central bank. Consistently with Proposition 3, the central bank's real net worth remains constant at its initial level of 1% (not shown) and the dynamics of profits (and remittances) reflect the specific composition of the central bank's balance sheet. When the central bank has only short-term assets, remittances are non-negative while with long-term assets they mainly follow their return. As the natural rate unexpectedly turns positive, the expectation that the nominal interest rate will jump

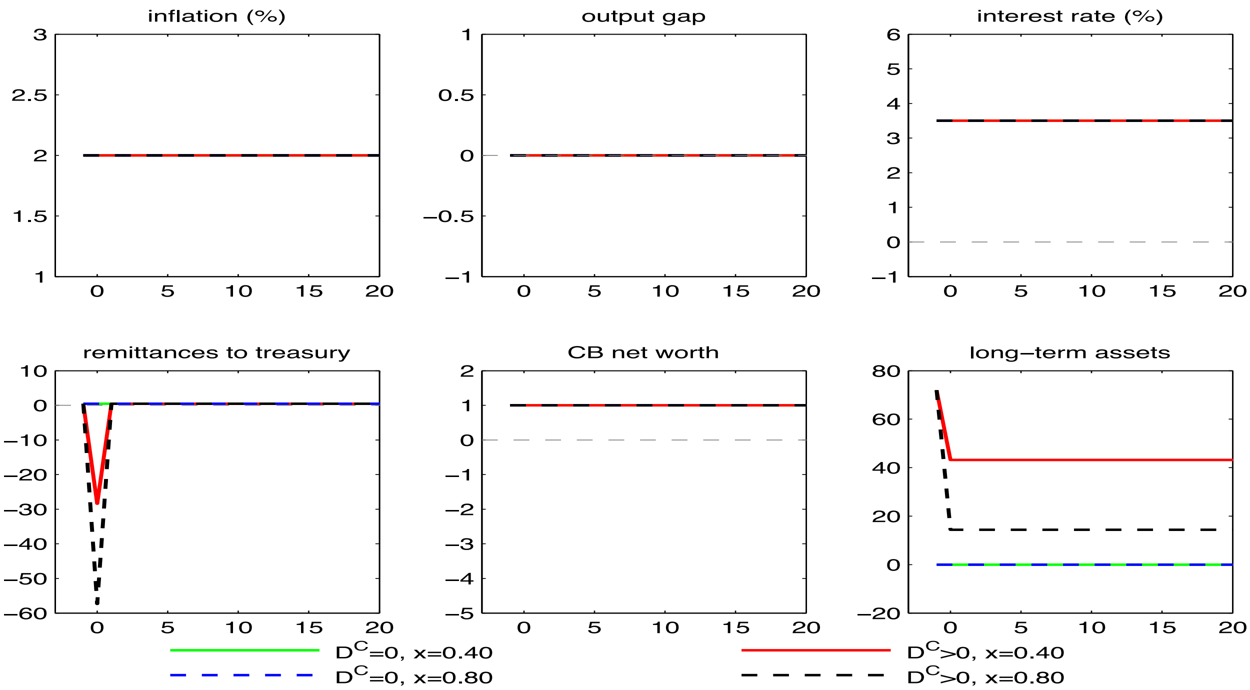

Figure 2: Response of selected variables, under optimal monetary policy, to a one-period credit event of alternative sizes, under alternative balance-sheet policies. Regime i): *neutral transfer policy*. Green solid line: Credit event implies default on 40% of long-term assets, central bank holds only short-term assets. Red solid line: Credit event implies default on 40% of long-term assets, central bank holds also long-term assets. Blue dashed line: Credit event implies default on 80% of long-term assets, central bank holds only short-term assets. Black dashed line: Credit event implies default on 80% of long-term assets, central bank holds also long-term assets. X-axis displays quarters.

up a few periods later is enough to bring down long-term asset prices and their return, thereby implying negative profits for the central bank. Under *passive remittances' policy*, negative profits trigger a transfer of resources from the treasury to the central bank (negative remittances), so that net worth does not move. Central bank's reserves instead fall as a consequence of the lower valuation of the long-term assets.

In Figure 2, under the same calibration, we consider a mild and a strong credit event with default rate respectively of 40% and 80% (i.e. $\varkappa = 0.40$ or $\varkappa = 0.80$, displayed by the continuous and dashed lines in Figure 2). The top panels show that the optimal monetary policy requires to completely stabilize inflation, output and interest rate at their targets. Indeed, the shock $\varkappa$ does not appear in (35) and (36)–(37). Given the *transfer policy* assumed, the optimal monetary policy is also not affected by the alternative *balance-sheet policy*. The difference is in the remittances to the treasury. In the case of a standard composition of the balance-sheet ($D_t^C = 0$), profits and remittances are always positive while when the central bank holds long-term securities losses are covered by the treasury, given *passive remittances' policy*, and the more so the higher the default rate.

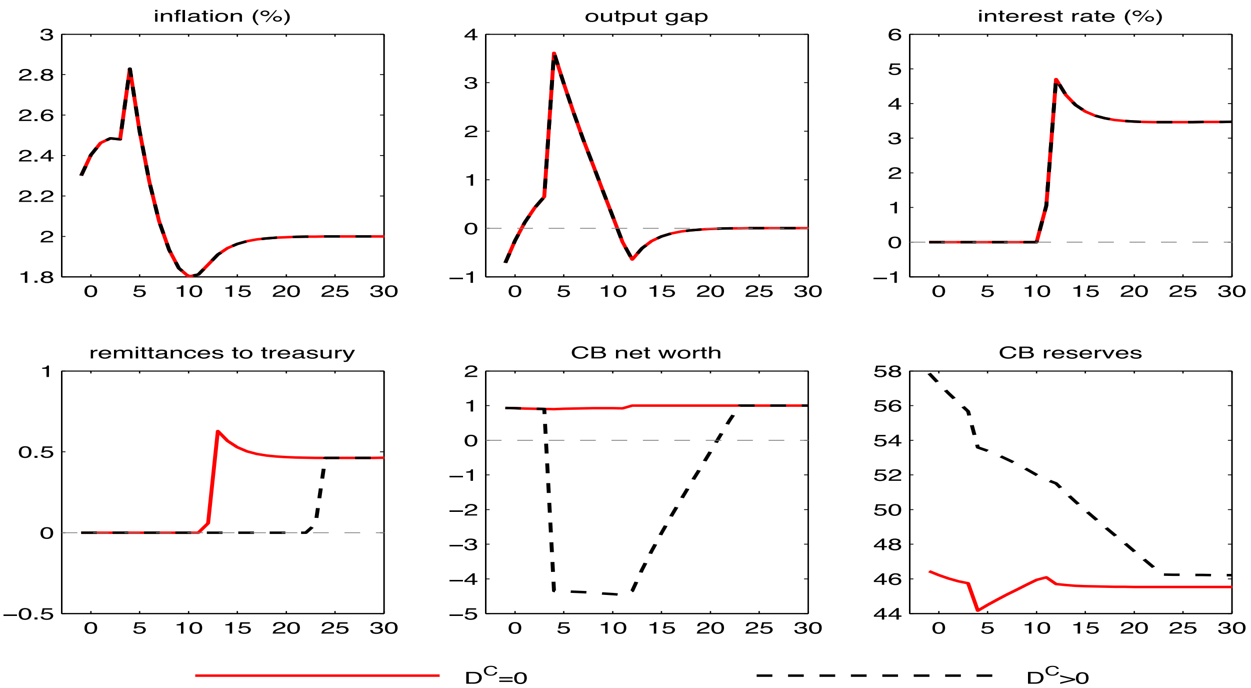

Figure 3: Equilibrium dynamics of selected variables under optimal monetary policy facing interest-rate risk. Regime ii): *"lack of treasury's support"*. The economy starts in a liquidity trap with a negative natural rate of interest; the latter turns positive unexpectedly after one year. Red solid line: central bank holds only short-term assets. Black dashed line: central bank holds also long-term assets. X-axis displays quarters.

We consider now the second case of a *"deferred-asset" regime*. Figures 3 through 5 analyze the same scenarios as Figures 1 and 2, respectively, maintaining the assumption of *passive fiscal policy*, the same *balance-sheet policies* but changing the remittances' policy to a *"deferred-asset" regime* analogous to the one specified in Definition 6.[47]

With only interest-rate risk, as shown in Figure 3, the responses of inflation, output and interest rate do not change across the two alternative balance-sheet policies. This case is indeed consistent with the necessary and sufficient conditions for neutrality of Propositions 7 and 8. Indeed, losses are not large enough to impair the profitability of the central bank ($N_t^C + M_t^* > 0$ under the optimal monetary policy). As central bank's profits turn negative, remittances to the treasury fall to zero and stay at this level even when central bank's profits start to be positive as long as real net worth is below its long-run level, thereby allowing the latter to converge back to 1% of the balance sheet within a few quarters. After net worth is back at the initial value of 1%, central bank's profits are again rebated to the treasury. The implication is that central bank's reserves are temporarily higher than under passive remittances' policy, and are paid back by next-period profits.

[47]In particular, we adapt the rules introduced in the previous section to ensure a stationary real net worth (rather than nominal). This adjustment will also apply later when we deal with the case of financial independence.

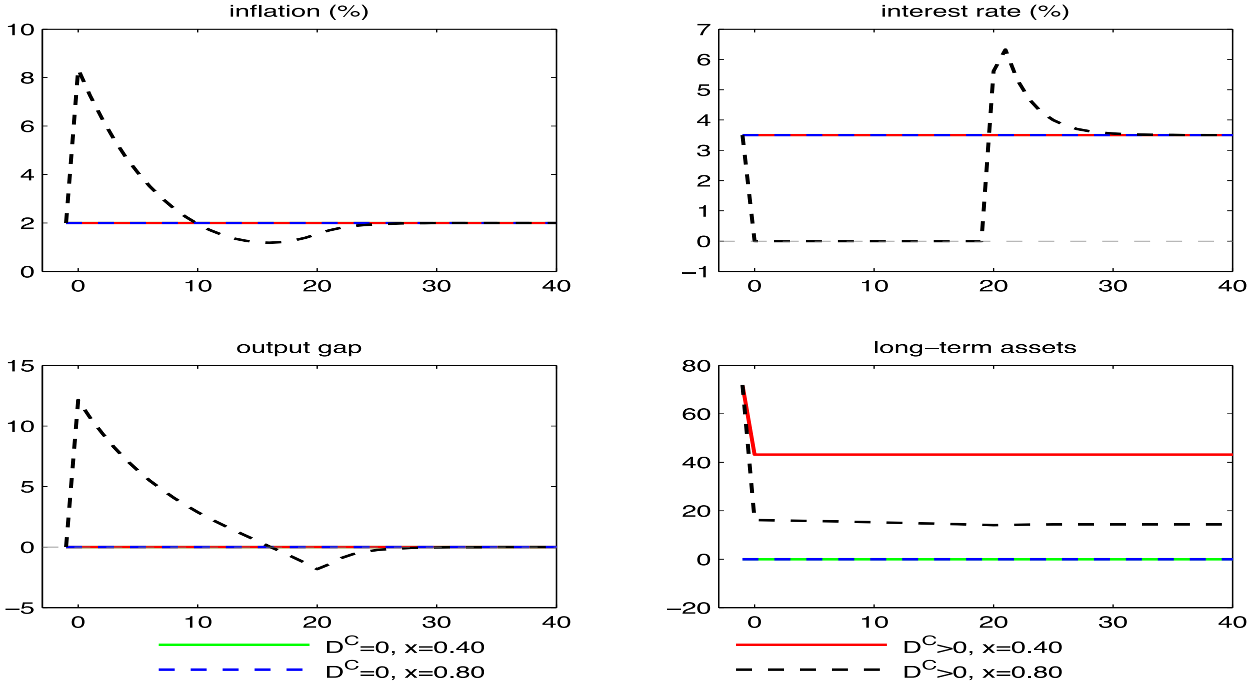

Figure 4: Response of selected variables, under optimal monetary policy, to a one-period credit event of alternative sizes, under alternative balance-sheet policies. Regime ii): *"lack of treasury's support"*. Green solid line: Credit event implies default on 40% of long-term assets, central bank holds only short-term assets. Red solid line: Credit event implies default on 40% of long-term assets, central bank holds also long-term assets. Blue dashed line: Credit event implies default on 80% of long-term assets, central bank holds only short-term assets. Black dashed line: Credit event implies default on 80% of long-term assets, central bank holds also long-term assets. X-axis displays quarters.

Figure 4, in the case of credit risk, shows instead a non-neutrality result when the credit event is significant (i.e. $\varkappa = 0.80$) and the central bank holds long-term assets ($\tilde{D}^C > 0$). Indeed, in this case losses are strong enough to impair the profitability of the central bank: without a change in prices and output with respect to the case $D_t^C = 0$, profits would remain indefinitely negative. The conditions for neutrality of Propositions 7 and 8 are violated. Instead, if the credit event is not too strong (i.e. $\varkappa = 0.40$), neutrality emerges and the central bank is therefore able to return to the steady-state level of net worth in a finite period of time without changing equilibrium prices and output with respect to the case in which $D_t^C = 0$, as shown in the Figure.

Figure 5 further shows the path of remittances, nominal money supply and central bank's net worth under the mild and strong credit events of Figure 4 given the two balance-sheet policies $D_t^C = 0$ and $\tilde{D}^C > 0$. The solid line, capturing the mild-credit event (when $\tilde{D}^C > 0$), shows that the fall in net worth, as a consequence of the income loss at t_0, is not enough to impair the ability of the central bank to produce positive gains from seigniorage in the future (i.e. $N_t^C + M_t^* > 0$ for each $t \geq \tau$). Such positive profits, therefore, will be possible without

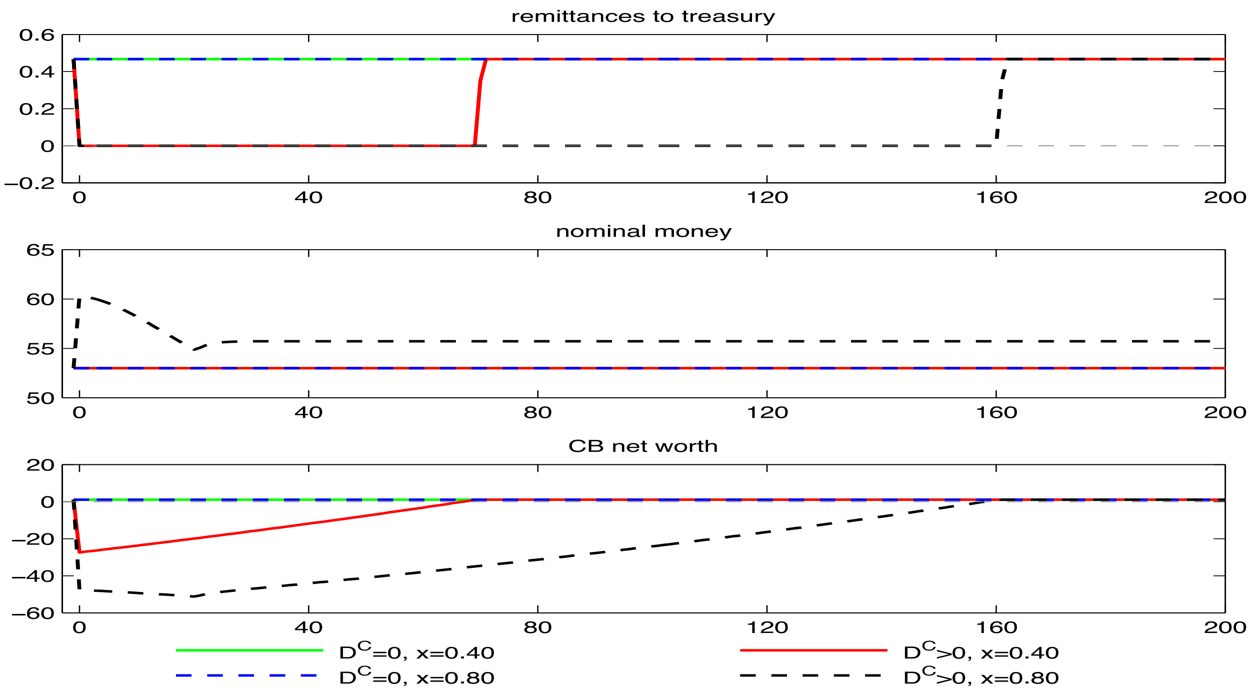

Figure 5: Response of selected variables, under optimal monetary policy, to a one-period credit event of alternative sizes, under alternative balance-sheet policies. Regime ii): *"lack of treasury's support"*. Green solid line: Credit event implies default on 40% of long-term assets, central bank holds only short-term assets. Red solid line: Credit event implies default on 40% of long-term assets, central bank holds also long-term assets. Blue dashed line: Credit event implies default on 80% of long-term assets, central bank holds only short-term assets. Black dashed line: Credit event implies default on 80% of long-term assets, central bank holds also long-term assets. X-axis displays quarters.

the need for the path of nominal money supply to deviate from the equilibrium associated with $D^C = 0$ (second panel of Figure 5). Moreover, these gains will be used to repay the deferred asset over a period in which remittances are zero and net worth can be rebuilt (first and third panels of Figure 5, respectively).

Results substantially change if the credit event is strong. In this case, the nominal stock of non-interest bearing liabilities, $N_t^C + M_t^*$, if evaluated at the inflation rate of the equilibrium with $D_t^C = 0$, would turn negative within the first quarters and violate afterward the solvency condition of the central bank at the initial equilibrium prices. The dashed lines in Figure 5 shows how to optimally deal with a shock of this size. The central bank should commit to substantially raise the stock of nominal money supply in the short-run – to compensate for the fall in nominal net worth – and set it at a permanently higher level in the long-run. Such commitment will ensure that the stock of non-interest bearing liabilities eventually reverts to positive values and produces the profits needed to repay the deferred asset and rebuild net worth (although over an extremely long time). To generate such a path of nominal money supply, the central bank should be accommodative enough to push up prices and inflation.

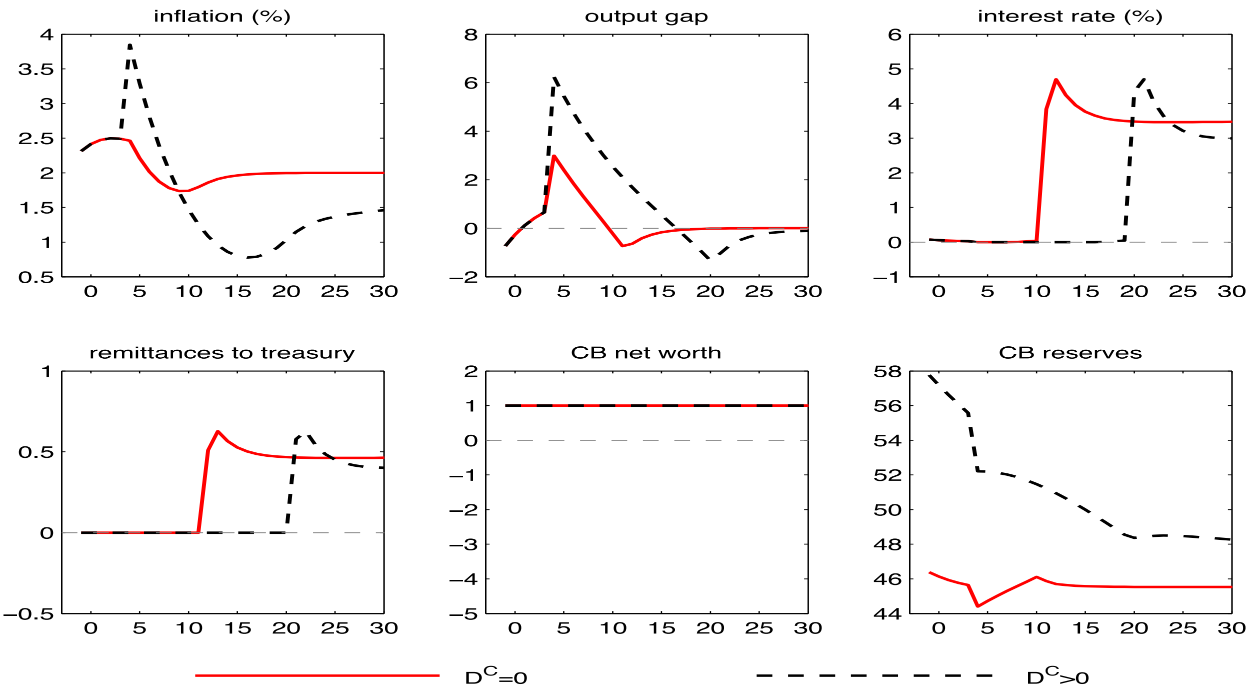

Figure 6: Equilibrium dynamics of selected variables under optimal monetary policy facing interest-rate risk. Regime iii): *financial independence*. The economy starts in a liquidity trap with a negative natural rate of interest; the latter turns positive unexpectedly after one year. Red solid line: central bank holds only short-term assets. Black dashed line: central bank holds also long-term assets. X-axis displays quarters.

In particular, as the dashed line in Figure 4 shows, inflation and output should go well above their target on impact, which in turn requires the nominal interest rate to fall down to the zero-lower bound. In the specific case displayed in Figures 4 and 5, it takes about 30 quarters for real variables to converge back to the path they would follow under neutrality, and for nominal money supply to stabilize on a new, higher, level.

We turn now to third regime of *financial independence*. Figure 6 shows the case of interest-rate risk under the same calibration of Figure 1 but with a different *transfer policy*: now a combination of *passive fiscal policy* and a remittances' policy ensuring *financial independence* of the central bank.

The Figure shows that the central bank, when it holds long-term securities and in order to satisfy the non-negative constraint on its remittances, has to engineer a dynamic path for asset prices such that the long-term return does not display the sharp drop when the preference shock brings the natural rate of interest back in the positive region. This requires committing to an interest rate path that remains at the zero-lower bound substantially longer than before, and implies (when r_t^n actually returns positive) that inflation rises above the 2% target about three times more than in the case $D^C = 0$, and an output boom about twice as strong. The central bank's profits (and the corresponding remittances to the treasury),

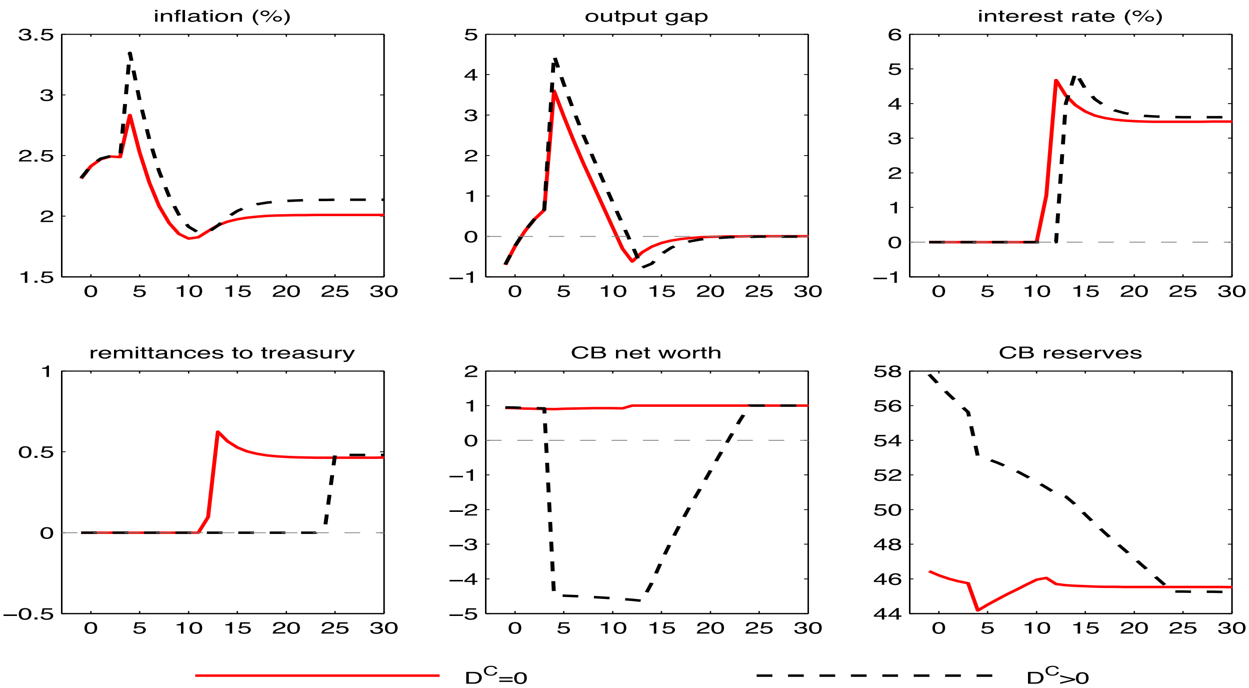

Figure 7: Equilibrium dynamics of selected variables under optimal monetary policy facing interest-rate risk. Regime iv): *active fiscal policy.* The economy starts in a liquidity trap with a negative natural rate of interest; the latter turns positive unexpectedly after one year. Red solid line: central bank holds only short-term assets. Black dashed line: central bank holds also long-term assets. X-axis displays quarters.

as a result, are zero and stay at zero several periods longer than the duration of the shock, following thereafter the path of the nominal interest rate with one period delay. Nominal money supply temporarily increases, when the natural interest rate turns back positive, to accommodate the surge in inflation and the output boom. To ensure that net worth stays constant, the central bank's reserves progressively decrease to their steady-state level, though along a substantially smoother path compared to the case of *passive transfer policies* (see Figure 1). In a liquidity trap, therefore, a central bank committed to financial independence signals, when purchasing long-term securities, a change in its desired monetary policy stance towards temporarily higher inflation and a delayed exit strategy.

Finally, we analyze the fourth regime of *active fiscal policy.* Figure 7 considers interest-rate risk and the same calibration as Figure 1 but assumes a *transfer policy* given by a combination of an *active fiscal policy* and a *"deferred-asset" remittances' policy.*[48] When the natural rate turns back positive, a central bank that holds long-term assets suffers losses

[48]Consistently with the calibration discussed earlier, we set the ratio of long-term public debt to GDP in the initial steady state equal to $\bar{Q}\bar{D}^G/(4\bar{Y}\bar{P}) = 0.35$, in annual terms, as reported by the US Bureau of Public Debt for 2009Q3. In particular, we consider the stock of publicly-held marketable government debt including securities with maturity above one year. The remittances' rule (29) is parameterized as before, while the tax rule (26) has now coefficients $\gamma_f = \phi_f = 0$.

which are kept in the treasury's balance sheet. The private sector experiences a positive wealth gain that pushes up inflation, both on impact and in the medium run, supported by a longer stay of the nominal interest rate at the zero-lower bound.

4.2 Stochastic Simulations

In this section we simulate a scenario that resembles the response of output and inflation to the recent U.S. financial crisis. Figure 8 shows the simulated path of preference, productivity and cost-push shocks for a sample of our interest, as well as the implied natural interest rate. Within this sample, as shown in the figure, we generate a financial crisis by adding – *ad hoc* – two large unexpected preference shocks during times where productivity was above steady state.[49] The financial crisis hits at period 12, because of a large preference shock that brings the natural interest rate at around -6%, in annual terms; a second preference shock hits at period 26, and brings the natural rate back around its steady-state value of 1.5%. We highlight the implied period of negative natural rate with a shaded area.

Given this sequence of shocks, we first simulate the evolution of macroeconomic variables under a standard monetary/fiscal policy regime: the central bank holds only short-term riskless assets, it follows a Taylor Rule, fiscal policy is passive and the remittances' policy follows the "deferred-asset" regime of the Fed. As shown by the blue solid line in Figure 9, the output gap and inflation experience a drop, in response to the financial crisis, of the same magnitude as that observed in the U.S. data.[50] The output gap falls to about -5% on impact and the economy experiences a moderate deflation, while the nominal interest is cut down to the zero-lower bound. Both output and inflation remain well below their target through most of the period in which the natural interest rate is negative. The nominal interest rate exits the zero-lower bound in the same quarter in which the natural rate turns back positive.

In this scenario, we first ask what is the constrained first-best policy with respect to all the dimensions of the monetary/fiscal policy regime. The key insight is that, in order to minimize the number of constraints restricting the optimal policy – thus maximizing welfare – it is sufficient that the Ramsey planner set *passive transfer policies*. Generalizing our previous results (see Proposition 6), alternative compositions of the central bank's balance sheet are completely irrelevant under the constrained first best.

Figure 9 shows the evolution of macroeconomics variables under the constrained first best (black dashed line) in comparison with the Taylor Rule. Before the crisis, the output

[49]This scenario is meant to capture some important features of U.S. data at the beginning of the Great Recession, where a fall in demand and output was associated with a relatively good performance of productivity. See also Fernandez-Villaverde et al. (2015) and Gust et al. (2016). Calibration of structural parameters is the same as in the previous section.

[50]See Benigno et al. (2016). The parameters of the Taylor Rule are calibrated appropriately.

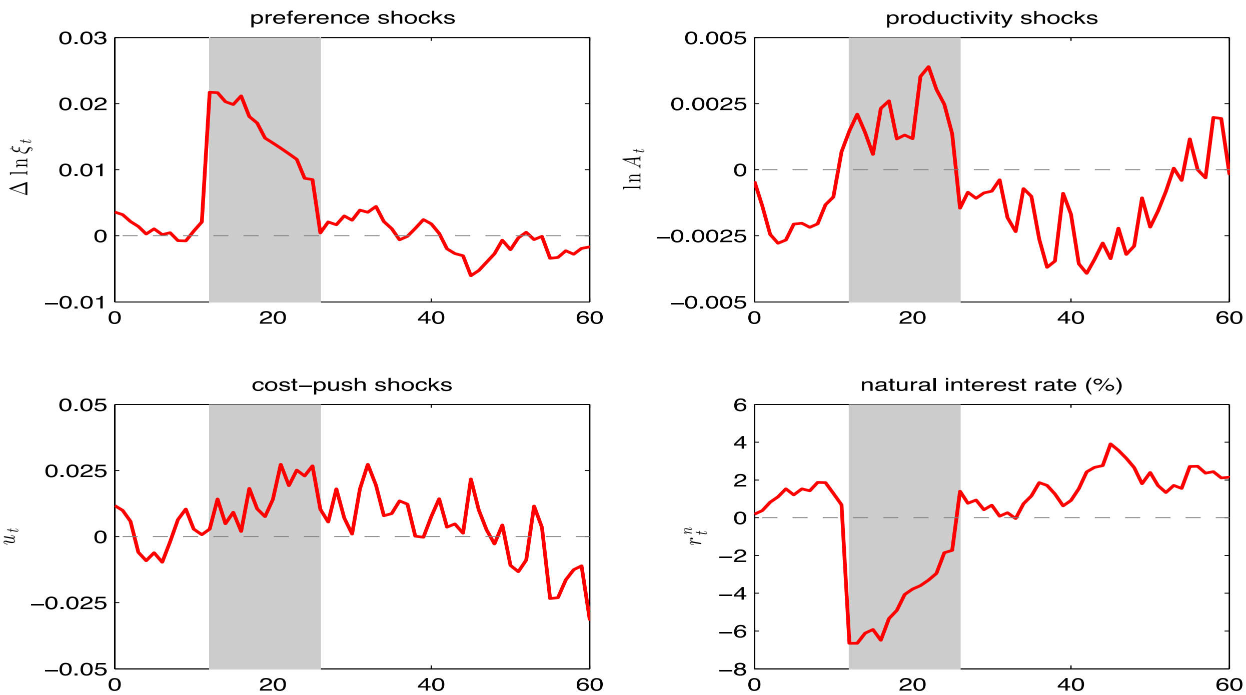

Figure 8: Evolution of stochastic shocks in the sample of interest. X-axis displays quarters.

gap and inflation rate follow similar paths. As the natural rate turns negative, however, the two policies depart in a significant way: under the constrained first best, the economy avoids much of the recessionary and deflationary effects on impact, and inflation and output converge toward their targets about twice as fast. This is achieved because the private sector expects the nominal interest rate to stay at the zero-lower bound substantially longer than under the Taylor Rule. Therefore, one way to interpret the observed poor macroeconomic performance, in the face of the financial crisis, is the lack of commitment on the part of the central bank to keep the nominal interest rate at zero longer than the duration of the negative natural rate.

The equilibrium allocation implied by the suboptimal policy could also be improved in the aftermath of the financial crisis by switching to a monetary policy regime consistent with the commitment that characterizes the constrained first best.[51] Figure 10 shows the case in which such a switch occurs two quarters into the liquidity trap (red solid line), and compares it with the suboptimal policy (blue thin line) and the constrained first best (black dashed line). The vertical dashed line highlights the time of the change in policy. As the figure shows, committing to a longer stay at the zero-lower bound even during the financial crisis can reflate the economy and bring it closer to the constrained first best.

[51] In particular, we consider a monetary/fiscal policy regime where fiscal policy is passive, the remittances' policy follows the "deferred-asset" regime of the Fed, the central bank holds only short-term risk-less assets and it follows optimal monetary policy under commitment conditional on the current state of the economy.

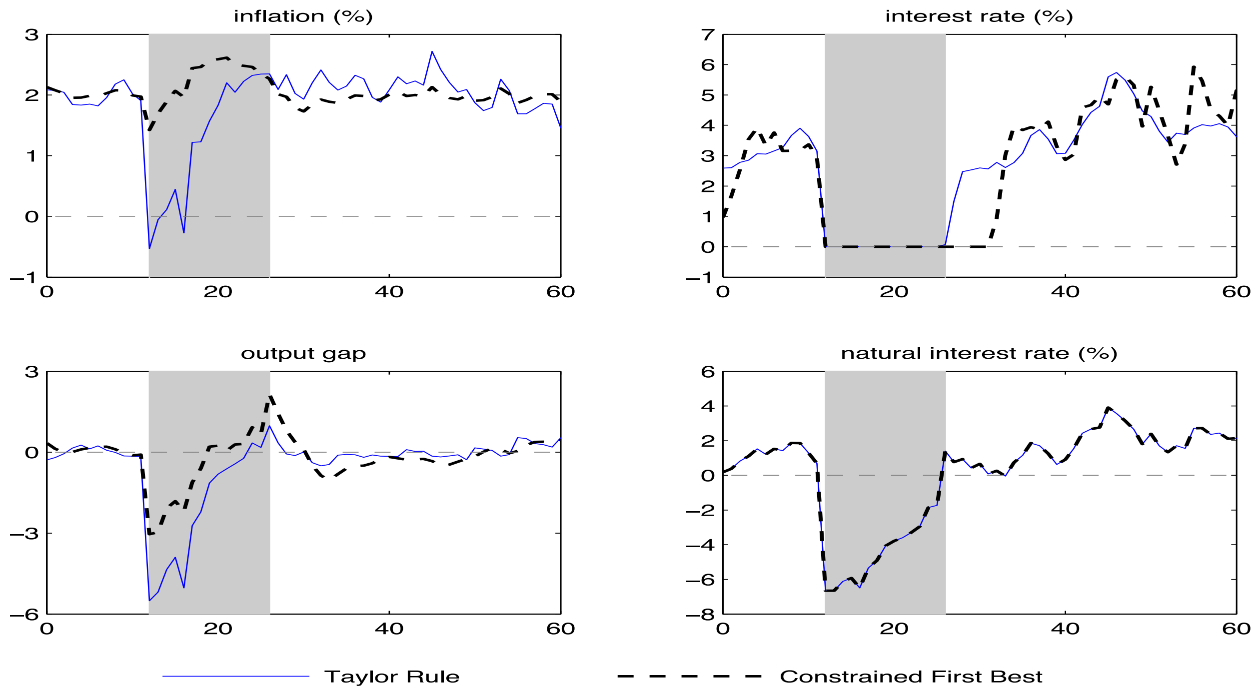

Figure 9: Stochastic simulation of selected variables. Comparison between suboptimal monetary policy (Taylor Rule, red solid line) and Constrained First Best (black dashed line). Shaded area: the natural interest rate is negative. X-axis displays quarters.

Once in a liquidity trap, however, gaining credibility on such a commitment may be hard, because both the Taylor Rule and the optimal policy imply the same outcome for the current nominal interest rate. The key insight is to note that the theoretical analysis of Section 3.2 suggests a way for the policymaker to credibly move out from the suboptimal policy. Indeed, by using *non-neutral* unconventional open-market operations, the policymaker can make the Taylor Rule inconsistent with the attainment of a (locally stable) rational expectations equilibrium. Thus, the announcement of unconventional asset purchases with non-neutral features is able to change private sector's expectations toward a shift in conventional monetary policy, and in particular a move out from the Taylor Rule followed in the past. Krishnamurthy and Vissing-Jorgensen (2011) and Woodford (2012) have argued that unconventional monetary policy – regardless of neutrality – can be effective as a generic signal toward a shift in the monetary policy stance. We instead identify a specific channel through which *non-neutral* unconventional open-market operations can signal a change in the monetary policy stance: by completely ruling out the continuation of previous policy.

In our economy, this can be done in three ways.

First, the central bank can purchase assets of dubious credit worthiness whose losses would lead to its insolvency under the previous rule (we label this case *"lack of treasury's support"* regime, consistently with the previous sections). Second, the central bank can purchase long-

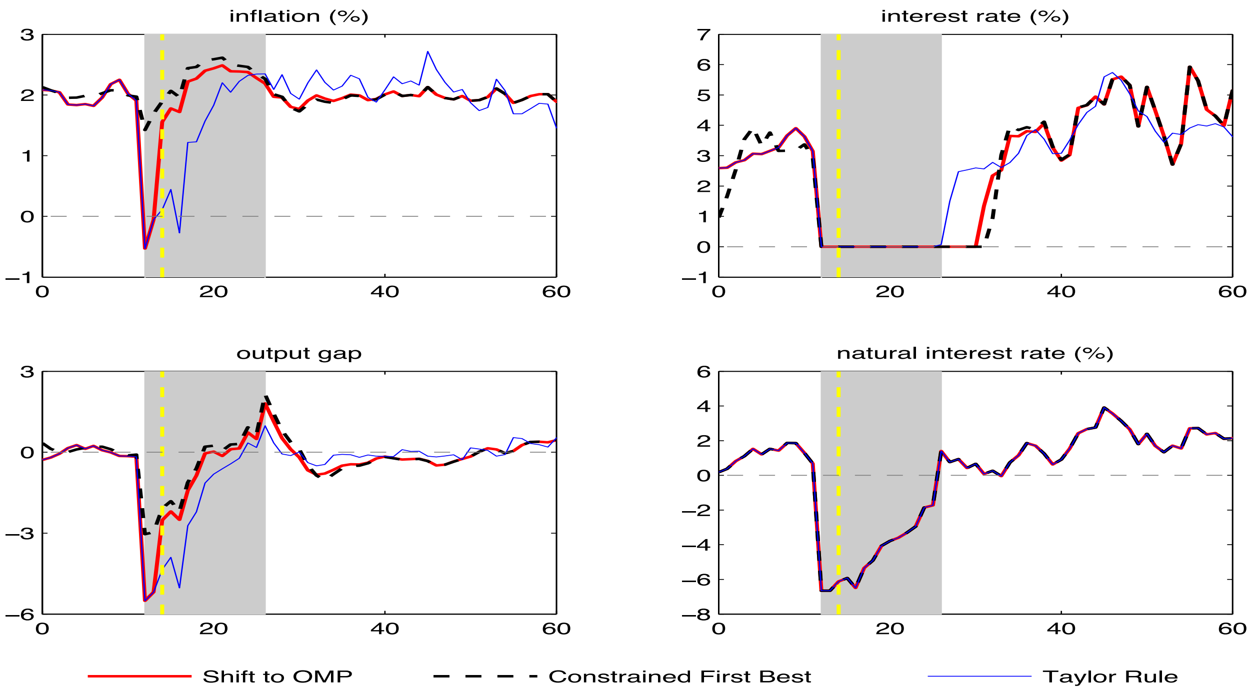

Figure 10: Stochastic simulation of selected variables. Comparison among suboptimal monetary policy (Taylor Rule, blue thin line), Constrained First Best (black dashed line) and shift to optimal monetary policy (OMP, red solid line). Vertical line indicates the time of the shift. Shaded area: the natural interest rate is negative. X-axis displays quarters.

term assets and commit to be financially independent (*"financial independence"* regime). In both these cases fiscal policy remains passive. Third, the overall government can implement "helicopter money" by having the treasury shift to active fiscal policy and the central bank purchase long-term assets (*"active fiscal policy"* regime).[52]

Among the possible monetary policy rules that can be used in replacement of the previous policy, we assume that, in each of the three cases underlined above, the central bank follows the optimal monetary policy under commitment given the respective transfer and balance-sheet policies, and conditional on the state of the economy at that point.

Figure 11 considers the *"lack of treasury's support"* regime. In particular, two quarters into the liquidity trap the central bank implements a large-scale asset purchase that brings the share of long-term risky assets to 72% of the steady-state balance sheet.[53] These assets are unexpectedly seized by about two thirds in the quarter following the liftoff of the natural interest rate (which itself implies downward pressures on the return on long-term assets and therefore on central bank's net worth).[54] Under this new regime the economy will reflate

[52]In all cases the remittances' policy follows the "deferred-asset" regime of the Fed.

[53]In the numerical simulations we assume that the new balance-sheet policy lasts for the entire sample of our interest, which spans about ten years since the start.

[54]Note that to have non-neutrality is not necessary that the credit event materializes in the sample, but at

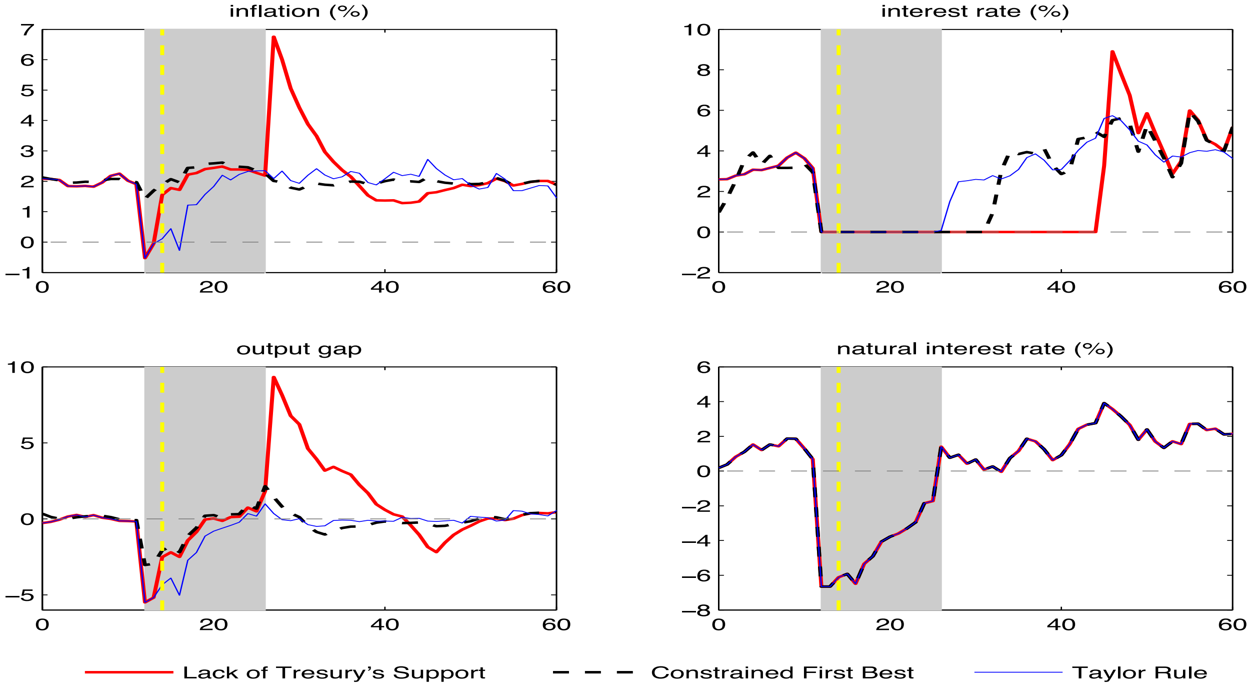

Figure 11: Stochastic simulation of selected variables. Comparison among suboptimal monetary policy (Taylor Rule, blue thin line), Constrained First Best (black dashed line) and shift to *"lack of treasury's support"* regime (red solid line). Vertical line indicates the time of the shift. Shaded area: the natural interest rate is negative. X-axis displays quarters.

during the liquidity trap at the cost of a substantially higher inflation when the credit event hits because, as discussed in Sections 3.2.1 and 4.1, this is the way for the central bank to restore its long-run profitability. To this aim, the nominal interest rate is kept at the zero-lower bound substantially longer than otherwise.

Figure 12 considers the *"financial independence"* regime, in which the size of the asset purchase is the same as in Figure 11, but the securities are not defaulted on in the sample.[55] Still, in order to maintain financial independence, the central bank is required to engineer a non-negative path for the excess return on long-term assets, such that it never suffers any income loss. In the short run, this implies that the central bank commits to delay the exit from the zero-lower bound policy, and inflation rises even above the constrained first best to support a stronger output boom. In the medium run, however, inflation falls below target and converges back slowly.

Another interesting result shown by Figure 12 is that, out of the liquidity trap, keeping

least in some contingency. In the numerical example of Figure 11 we analyze the case in which default occurs during the sample. Note also that non-neutrality under this regime may arise as well because of interest-rate risk, if the central bank is leveraged enough.

[55]The definition of financial independence used in the simulations is such that real net worth remains constant at its steady-state level of 1% of the steady-state balance sheet.

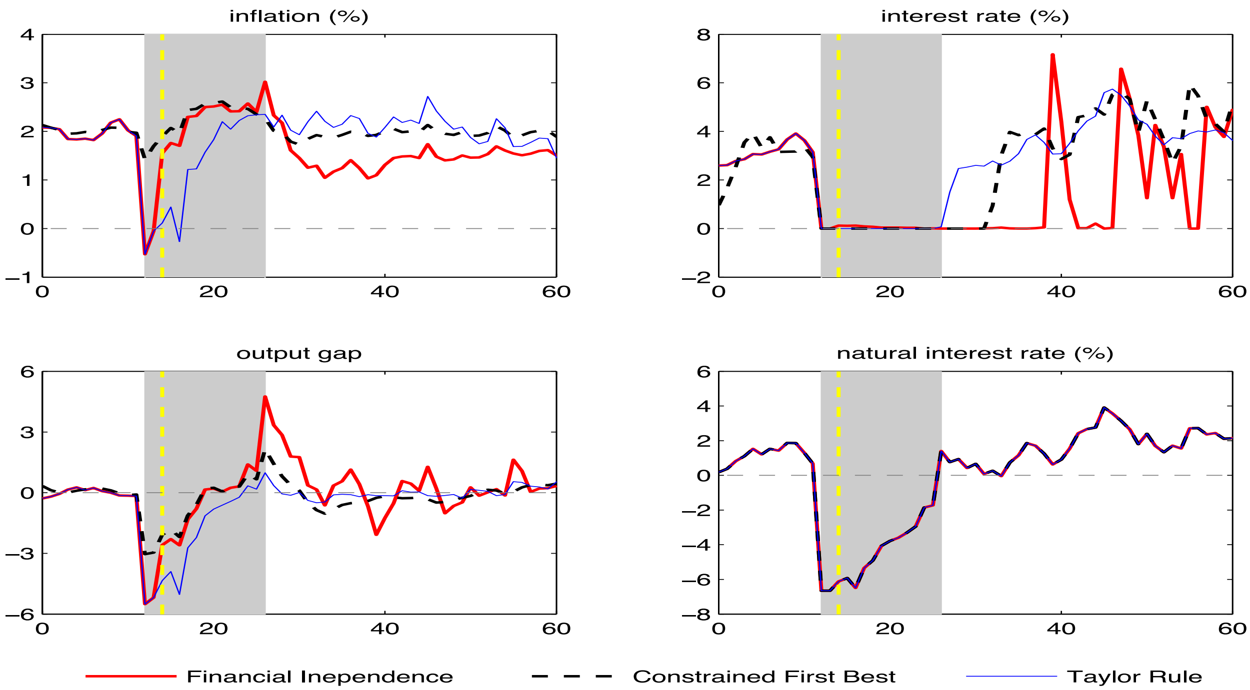

Figure 12: Stochastic simulation of selected variables. Comparison among suboptimal monetary policy (Taylor Rule, blue thin line), Constrained First Best (black dashed line) and shift to *"financial independence"* regime (red solid line). Vertical line indicates the time of the shift. Shaded area: the natural interest rate is negative. X-axis displays quarters.

a smooth path of long-term asset prices, to avoid income losses, comes at the cost of more volatile nominal interest rate, output and inflation.

Figure 13 focuses on the *"active fiscal policy"* regime under the same balance-sheet policy as in the previous case. Unconventional open-market operations are able to jump start the economy in the short run at the cost of a higher inflation in the medium run. Indeed, because of the active fiscal policy, central bank's asset purchases imply a transfer of risk on the balance sheet of the government. The private sector therefore experiences a positive wealth effect that pushes up nominal spending and inflation in the medium run. Moreover, once the economy is out of the liquidity trap, the volatility of the return on long-term assets – which under *passive fiscal policy* would be absorbed by fluctuations in the primary surplus – is now reflected in the evolution of households' wealth and nominal spending. Given the relative cost of inflation versus output stabilization, implied by our calibration, this volatility is mostly mirrored by the nominal interest rate and real output.

Table 1 compares the welfare losses implied by the alternative regimes considered in this section. Engaging in unconventional open-market operations, in any of the three non-neutral regimes that we have analyzed, performs better than following the old suboptimal policy regime (Taylor Rule). The "*financial independence*" regime, in particular, creates lower

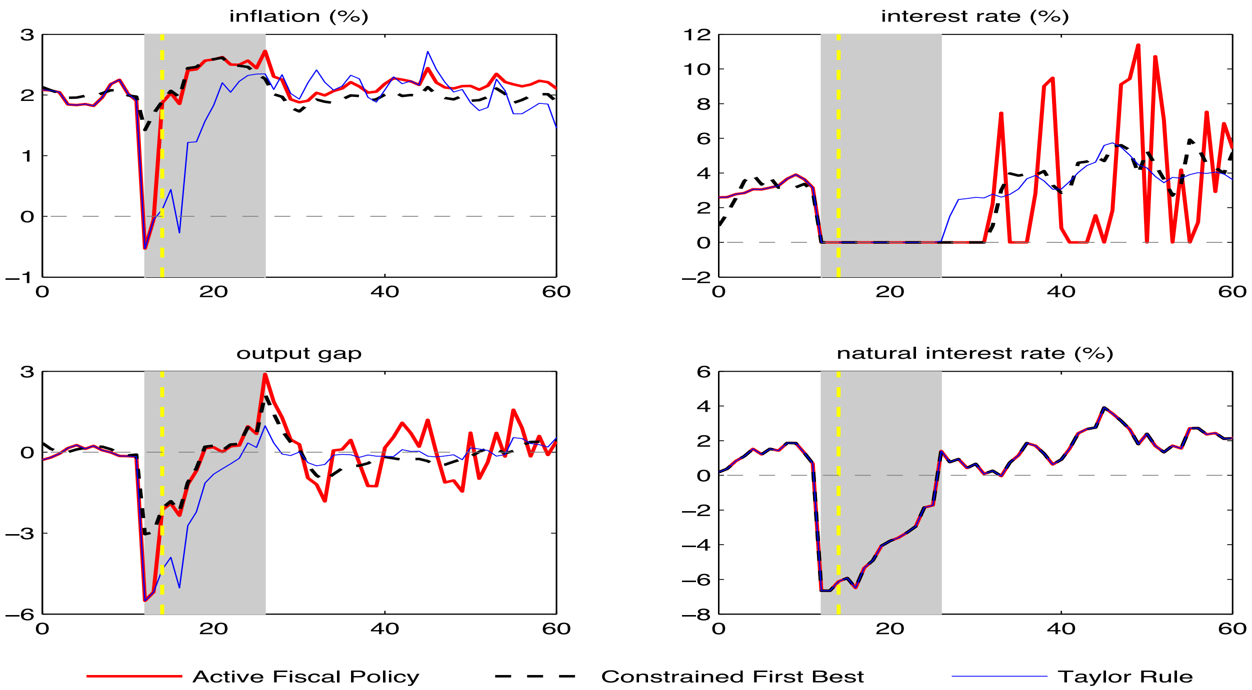

Figure 13: Stochastic simulation of selected variables. Comparison among suboptimal monetary policy (Taylor Rule, blue thin line), Constrained First Best (black dashed line) and shift to *"active fiscal policy"* regime (red solid line). Vertical line indicates the time of the shift. Shaded area: the natural interest rate is negative. X-axis displays quarters.

losses with respect to the other non-neutral policies, and implies similar welfare costs as switching to the optimal monetary policy during the crisis.

Figure 14 explores more in detail the effects on inflation of the three cases of non-neutrality outlined in this section, for alternative sizes of balance-sheet policies. In particular we vary the share of long-term assets on total assets from zero to 100%. For each share and each regime, we focus on the inflation response at a short horizon of six quarters, at a medium horizon of twelve quarters and at a long horizon of fifty quarters.

The figure compares the three non-neutral regimes that we discussed with three benchmarks: the constrained first best, the suboptimal Taylor Rule, the shift to optimal monetary policy during the crisis.

Table 1: Welfare Losses

Benchmark Regimes		**Non-Neutral OMO Regimes**	
Constrained First Best	0.92	Lack of Treasury's Support	5.62
Taylor Rule	6.92	Financial Independence	3.93
Switch to Optimal Monetary Policy	3.47	Active Fiscal Policy	4.64

Note: Welfare losses are basis points of steady-state consumption. OMO stands for Open Market Operations.

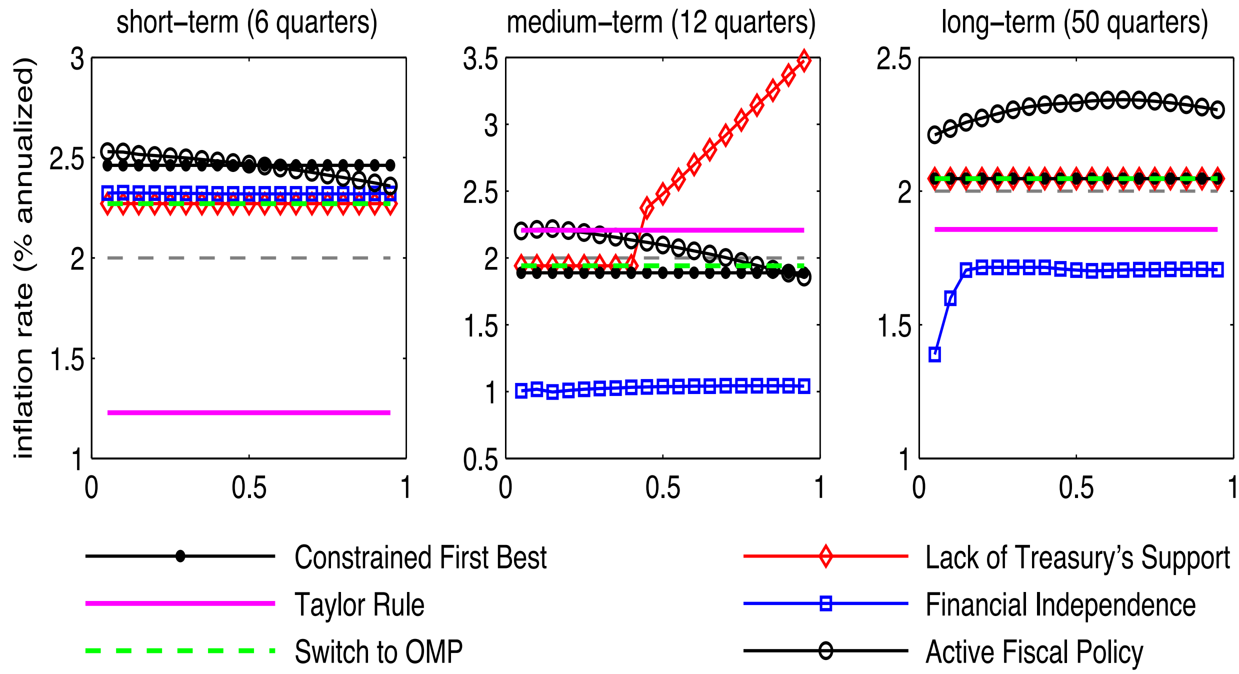

Figure 14: Short-term (4 quarters) and medium-term (40 quarters) effects on inflation of alternative balance-sheet policies. Dashed gray line: inflation target (2%). Dotted black line: Constrained First Best. Solid magenta line: Taylor Rule. Dashed green line: switch to Optimal Monetary Policy (OMP). Red diamonds: case *i*); absence of treasury's support. Blue squares: case *ii*); financially independent central bank. Black circles: case *iii*); active fiscal policy. X-axis: size of long-term asset purchases, as a share of steady-state balance sheet. Y-axis: inflation rate in percentage points (annualized).

Implementing unconventional open market operations, under all the non-neutral regimes considered, is able to push up inflation substantially, in the short run, with respect to the Taylor Rule. Over the medium and long run, instead, the three regimes show different implications. Under "*lack of treasury's support*", the medium-run effects on inflation are larger for sizeable asset purchases, following the credit event, while small purchases produce losses that can be simply absorbed by retaining earnings without any effect on the inflation rate. In the long run, however, the effects on inflation are in line with those under the constrained first best. Under "*financial independence*" inflation is subdued in the medium and long run, and mainly independent of the size of asset purchase: the objective of the central bank in this case is to avoid income losses altogether, regardless of how many long-term assets it holds. The "*active fiscal policy*" regime, instead, implies in the long run an upward bias in inflation, relative to all benchmarks, and the effects on inflation tend to increase with the amount of asset purchases. The implicit wealth transfer to the private sector is higher in this case, implying a larger increase in nominal spending.

5 Conclusions

This work has studied monetary/fiscal policy regimes which may or may not support neutrality results following central bank's generic open-market operations. To preserve tractability, we kept the environment as simple as possible, at the cost of disregarding some important features that we now discuss more extensively.

In our model, long-term assets have only a pecuniary return and therefore the focus of our analysis has been restricted on the equilibrium consequences of central bank's losses due to long-term assets' purchases. The literature has instead stressed the importance of considering also non-pecuniary benefits of debt securities, to characterize departures from irrelevance of central bank's asset purchase programs. There are mainly two classes of such models. The first includes limits to arbitrage in the private financial intermediation of certain securities which translates into credit or term premia (see among others Curdia and Woodford, 2011). These excess returns can be relaxed by central bank intervention in these markets through its ability to finance the purchases more easily than the private sector by expanding reserves. In the second class, long-term *assets* have also a non-pecuniary value like, for example, that of relaxing collateral constraints (see among others Araújo et al., 2015, and Reis, 2016). Along this direction central bank's purchases can produce effects on the economy.

Another strand of literature has emphasized the benefits of balance-sheet policies on the ground that the *liabilities* of the central bank can have an advantage in relaxing some liquidity (or collateral) constraints that bind the action of private agents.[56] But in this case the benefits obtained by increasing the central bank's liabilities are the same regardless of whether the central bank purchases short or long-term securities.

Central banks around the world have very different accounting practices, capital requirements and transfer policies. A comprehensive analysis of all the various possibilities is out of the scope of this paper, though some alternative assumptions not made here could affect the results. One that deserves particular attention is the way purchases of long-term securities are accounted in the balance sheet and therefore in the profit-loss statement. We have evaluated them at the market value but some central banks do it at the historical value, like the Federal Reserve.[57]

A more relevant extension for emerging-market economies could be the modelling of reserves in foreign currency. In this case, capital losses can be consequence of exchange rate movements and can affect the conduct of monetary policy also for what concerns its effects on

[56] See Benigno and Nisticò (2017) and Reis (2016).

[57] In general the ECB uses a mark-to-market procedure with the possibility of inputting precautionary reserves in the case of gains. However, in the recent purchases of covered bonds and sovereign debt, it moved to an accounting system at historical costs.

the exchange rate (see Jeanne and Svensson, 2007). In this respect, Adler et al. (2012) have shown that for emerging market economies deviations from standard interest-rate policies can be explained by concerns about the weakness of the central bank's balance sheet.

We have discussed our theoretical results in a cash-in-advance model *à la* Lucas and Stokey (1987) where the asset market opens before the goods market. The results of this paper are robust to other ways of modeling the liquidity friction like money in the utility function or through a cash-in-advance constraint in which the goods market opens before the asset markets.[58] The analysis can be extended also to cashless-limiting economies or to overlapping-generation monetary models.

In our model, the velocity of money is constant and unitary. Qualitative results can be robust to environments in which the velocity is endogenous. An interesting extension is to relate it to the balance-sheet position of the central bank. The possibility that a currency can be substituted with other means of payments, when the balance sheet deteriorates, can impair the long-run profitability of the central bank and leave the currency unbacked if there is no fiscal support.[59]

Finally, our analysis has emphasized the importance of the interaction between monetary and fiscal policy. In practice, it is hard to exactly tell when a regime of full treasury's support is in place or even when fiscal policy is passive. It should be interesting to consider the implications of uncertainty on the monetary and fiscal regimes, like in Leeper (2013), or the political dimension of the strategic interaction between treasury and central bank.[60]

[58]Details of the latter model are available upon request.

[59]Quinn and Roberds (2014) discuss the disappearance of the Florin as an international reserve currency in the late 1700s as a consequence of the central bank's income losses on non-performing loans. In Del Negro and Sims (2015), a specific transaction cost of holding money balances delivers a money demand function elastic with respect to the nominal interest rate. In their analysis real money balances can become zero for an interest rate above a certain finite threshold.

[60]See for an interesting avenue of research Gonzalez-Eiras and Niepelt (2015).

References

[1] Adler, Gustavo, Pedro Castro, and Camilo E.Tovar (2012). Does Central Bank Capital Matter for Monetary Policy? IMF Working Paper WP/12/60.

[2] Araújo, Alosio, Susan Schommer and Michael Woodford (2015). Conventional and Unconventional Monetary Policy with Endogenous Collateral Constraints. American Economic Journal: Macroeconomics, 7(1): 1-43.

[3] Archer, David and Paul Moser-Boehm (2013). Central Bank Finances. BIS Paper No. 71.

[4] Auerbach, Alan J. and Maurice Obstfeld (2005). The Case for Open-Market Purchases in a Liquidity Trap. American Economic Review, vol. 95(1): 110-137.

[5] Bank of England (2013). Annual Report 2012/13: Bank of England Asset Purchase Facility Fund Limited. Bank of England publication, 28 February 2013.

[6] Bassetto, Marco (2005). Equilibrium and Government Commitment. Journal of Economic Theory, vol. 124(1): 79-105.

[7] Bassetto, Marco and Todd Messer (2013). Fiscal Consequence of Paying Interest on Reserves. Federal Reserve Bank of Chiacago, w.p. No. 2013-04.

[8] Benigno, Pierpaolo, Gauti Eggertsson and Federica Romei (2016). Dynamic Debt Deleveraging and Optimal Monetary Policy. CEPR Discussion Paper No. 11180.

[9] Benigno, Pierpaolo and Salvatore Nisticò (2017). Safe Assets, Liquidity and Monetary Policy. The American Economic Journal: Macroeconomics, Vol. 9(2)

[10] Berriel, Tiago C. and Saroj Bhattarai (2009). Monetary Policy and Central Bank Balance Sheet Concerns. The B.E. Journal of Macroeconomics, Vol. 9(1), Contributions, Article 1.

[11] Bhattarai, Saroj, Gauti Eggertsson and Bulat Gafarov (2015). Time Consistency and the Duration of Government Debt: A Signalling Theory of Quantitative Easing. NBER Working Paper No. 21336.

[12] Buiter, William H. (2014). The Simple Analytics of Helicopter Money: Why It Works — Always. Economics: The Open-Access, Open-Assessment E-Journal, 8: 1-51.

[13] Carpenter, Seth B., Jane E. Ihrig, Elizabeth C. Klee, Daniel W. Quinn, and Alexander H. Boote (2015). The Federal Reserve's Balance Sheet and Earnings: A Primer and Projections. International Journal of Central Banking, 11(2): 237-284.

[14] Chamley, Christophe and Herakles Polemarchakis (1984). Assets, General Equilibrium and the Neutrality of Money. Review of Economic Studies 51(1): 129-138.

[15] Christensen, Jens H.E., Jose A. Lopex and Glenn D. Rubebusch (2015). A Probability-Based Stress Test of Federal Reserve Assets and Income. Journal of Monetary Economics, 73: 26-43.

[16] Cúrdia, Vasco and Michael Woodford (2011). The Central-Bank Balance Sheet as An Instrument of Monetary Policy. Journal of Monetary Economics, 58(1): 54-79.

[17] Del Negro, Marco and Cristopher Sims (2015). When Does a Central Bank's Balance Sheet Require Fiscal Support? Journal of Monetary Economics, 73: 1-19.

[18] Eggertsson, Gauti and Michael Woodford (2003). The Zero Bound on Interest Rates and Optimal Monetary Policy. Brookings Papers on Economic Activity 1: 212-219.

[19] Fernandez-Villaverde, Jesus, Pablo Guerrón and Grey Gordon (2015). Nonlinear Adventures at the Zero Lower Bound. Journal of Economic Dynamics and Control, 57, 182-204.

[20] Gonzalez-Eiras, Martin and Dirk Niepelt (2015). Politico-economic Equivalance. Review of Economic Dynamics 18: 843-862.

[21] Greenlaw, David, James D. Hamilton, Peter Hooper, and Frederic S. Mishkin (2013). Crunch Time: Fiscal Crises and the Role of Monetary Policy. Mimeo, University of California at San Diego.

[22] Guerrieri, Luca, and Matteo Iacoviello (2015). Occbin: A Toolkit to Solve Models with Occasionally Binding Constraints Easily, Journal of Monetary Economics, 70: pp. 22-38

[23] Gust, Christopher J., Edward P. Herbst, J. David Lopez-Salido, and Matthew E. Smith (2016). The Empirical Implications of the Interest-Rate Lower Bound. Finance and Economics Discussion Series 2012-83r. Board of Governors of the Federal Reserve System

[24] Hall, Robert. and Ricardo Reis (2015). Maintaining Central-Bank Solvency Under New-Style Central Banking. NBER Working Paper No. 21173.

[25] Jeanne, Olivier and Lars E.O. Svensson (2007). Credible Commitment to Optimal Escape from a Liquidity Trap: The Role of the Balance Sheet of an Independent Central Bank. American Economic Review 97: 474-490.

[26] Krishnamurthy, Arvind and Annette Vissing-Jorgensen (2011). The Effects of Quantitative Easing on Interest Rates: Channels and Implications for Policy. Brookings Papers on Economic Activity, 43(2): 215-287.

[27] Leeper, Eric (1991). Equilibria Under 'Active'and 'Passive' Monetary and Fiscal Policies. Journal of Monetary Economics 27(1): 129-147.

[28] Leeper, Eric (2013). Fiscal Limits and Monetary Policy. NBER Working Paper No. 18877.

[29] Lucas, Robert E. Jr. (1984). Money in a Theory of Finance. Carnegie-Rochester Conference Series on Public Policy, 21, 9-46.

[30] Lucas, Robert E. Jr. and Nancy L. Stokey (1987). Money and Interest in a Cash-in-Advance Economy. Econometrica, 55(3): 491-513.

[31] Park, Seok Gil (2015). Central Banks' Quasi-Fiscal Policies and Inflation. International Journal of Central Banking, 11(2): 199-236.

[32] Quinn, Stephen and William Roberds (2014). Death of a Reserve Currency. Atlanta Fed Working Paper 2014-17.

[33] Reis, Ricardo (2013). The Mystique Surrounding the Central Bank's Balance Sheet, Applied to the European Crisis. The American Economic Review, 103 (3): 135-140.

[34] Reis, Ricardo (2015). Comment on: When does a central banks balance sheet require fiscal support? by Marco Del Negro and Christopher A. Sims, Journal of Monetary Economics 73: 20-25.

[35] Reis, Ricardo (2016). QE in the Future: The Central Bank's Balance Sheet in a Fiscal Crisis. Mimeo, London School of Economics.

[36] Robatto, Roberto (2014). Open Market Operations and Money Supply at Zero Nominal Interest Rates. Mimeo, Chicago University.

[37] Sargent, Thomas (1982). Beyond Demand and Supply Curves in Macroeconomics. American Economic Review, 72: 382-389.

[38] Sargent, Thomas (1987). Dynamic Macroeconomic Theory. Cambridge: Harvard University Press.

[39] Sargent, Thomas (2011). Where to Draw Lines: Stability Versus Efficiency. Economica, 78: 197-214.

[40] Sargent, Thomas (2014). Robert E. Lucas, Jr.'s Collected Papers on Monetary Theory. Mimeo, New York University.

[41] Sargent, Thomas and Bruce D. Smith (1987). Irrelevance of Open Market Operations in Some Economies with Government Currency Being Dominated in Rate of Return. The American Economic Review, 77: 78-92.

[42] Sargent, Thomas and Neil Wallace (1981). Some Unpleasant Monetarist Arithmetic. Federal Reserce Bank of Minneapolis Quarterly Revies, 5(1): 1-17.

[43] Sims, Christopher (1994). A Simple Model for Study the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy. Economic Theory 4(3): 381-399.

[44] Sims, Christopher (2000). Fiscal Aspects of Central Bank Independence. Unpublished manuscript, Princeton University.

[45] Sims, Christopher (2005). Limits to Inflation Targeting. In Ben S. Bernanke and Michael Woodford (Eds.) *National Bureau of Economic Research Studies in Business Cycles,* vol. 32, ch. 7, Chicago: University of Chicago Press, pp 283-310.

[46] Sims, Christopher (2013). Paper Money. American Economic Review 103(2): 563-584.

[47] Smith, Adam (1806). An Inquiry into the Nature and Causes of the Wealth of Nations. Vol.II. Edinnburgh: William Greech.

[48] Stella, Peter (1997). Do Central Banks Need Capital? IMF Working Paper WP/97/83.

[49] Stella, Peter (2005). Central Bank Financial Strength, Transparency, and Policy Credibility. IMF Staff Papers Vol. 52(2):

[50] Wallace, Neil (1981). A Modigliani-Miller Theorem for Open-Market Operations. The American Economic Review, 71(3): 267-274.

[51] Woodford, Michael (1994). Monetary Policy and Price Level Determinacy in a Cash-in-Advance Economy. Economic Theory 4: 345-380.

[52] Woodford, Michael (2001). Fiscal Requirements for Price Stability. Journal of Money, Credit and Banking 33(1): 669-728.

[53] Woodford, Michael (2012). Methods of Policy Accommodation at the Interest-Rate Lower Bound. Economic Policy Symposium at Jackson Hole: The Changing Policy Landscape.

[54] Zhu, Feng (2004). Central Bank Balance Sheet Concerns, Monetary and Fiscal Rules, and Macroeconomic Stability. Unpublished Manuscript, Bank for International Settlements.

A Appendix

We collect in this appendix some derivations and proofs.

A.1 Proofs

A.1.1 Proof of Proposition 1

The fiscal rule

$$\frac{T_t^F}{P_t} = \bar{T}^F - \gamma_f \frac{T_t^C}{P_t} + \phi_f \left[\frac{(1+r_t)Q_{t-1}D_{t-1}^F + B_{t-1}^F}{P_t} \right] \tag{A.1}$$

is in the class of passive fiscal policies if and only if $\gamma_f = 1$ *and* $0 < \phi_f < 2$.

Proof. We find the conditions under which the rule (A.1) satisfies the requirements of passive fiscal policy given by Definition 4. First use the flow budget constraint (21) and write it in real terms:

$$Q_t \frac{D_t^F}{P_t} + \frac{1}{1+i_t} \frac{B_t^F}{P_t} = \frac{(1+r_t)Q_{t-1}D_{t-1}^F + B_{t-1}^F}{P_t} - \frac{T_t^F}{P_t} - \frac{T_t^C}{P_t}. \tag{A.2}$$

Using (A.1) into (A.2), we obtain

$$Q_t \frac{D_t^F}{P_t} + \frac{1}{1+i_t} \frac{B_t^F}{P_t} = (1-\phi_f) \left[\frac{(1+r_t)Q_{t-1}D_{t-1}^F + B_{t-1}^F}{P_t} \right] - \bar{T}^F - (1-\gamma_f) \frac{T_t^C}{P_t}. \tag{A.3}$$

Therefore we can write

$$\begin{aligned} E_{T-1} \left\{ R_{T-1,T} \left(Q_T \frac{D_T^F}{P_T} + \frac{1}{1+i_T} \frac{B_T^F}{P_T} \right) \right\} = \\ (1-\phi_f) E_{T-1} \left\{ R_{T-1,T} \frac{(1+r_T)Q_{T-1}D_{T-1}^F + B_{T-1}^F}{P_T} \right\} + \\ - E_{T-1} \left\{ R_{T-1,T} \bar{T}^F \right\} - (1-\gamma_f) E_{T-1} \left\{ R_{T-1,T} \frac{T_T^C}{P_T} \right\}. \end{aligned} \tag{A.4}$$

Note first that the equilibrium conditions (8) and (11) imply

$$E_{T-1} \left\{ R_{T-1,T} \frac{(1+r_T)}{P_T} \right\} = \frac{1}{P_{T-1}}$$

and

$$E_{T-1} \left\{ R_{T-1,T} \frac{1}{P_T} \right\} = \frac{1}{(1+i_{T-1})P_{T-1}},$$

since $R_{T-1,T}/\Pi_T = R^n_{T-1,T}$. We can therefore write (A.4) as

$$E_{T-1}\left\{R_{T-1,T}\left(Q_T\frac{D^F_T}{P_T}+\frac{1}{1+i_T}\frac{B^F_T}{P_T}\right)\right\} = (1-\phi_f)\left(Q_{T-1}\frac{D^F_{T-1}}{P_{T-1}}+\frac{1}{1+i_{T-1}}\frac{B^F_{T-1}}{P_{T-1}}\right) - E_{T-1}\left\{R_{T-1,T}\bar{T}^F\right\} - (1-\gamma_f)E_{T-1}\left\{R_{T-1,T}\frac{T^C_T}{P_T}\right\}. \quad (A.5)$$

Pre-multiplying equation (A.5) by $R_{T-2,T-1}$ and taking the expectation at time $T-2$, we can write (A.5) as

$$E_{T-2}\left\{R_{T-2,T}\left(Q_T\frac{D^F_T}{P_T}+\frac{1}{1+i_T}\frac{B^F_T}{P_T}\right)\right\} = (1-\phi_f)E_{T-2}\left\{R_{T-2,T-1}\left(Q_{T-1}\frac{D^F_{T-1}}{P_{T-1}}+\frac{1}{1+i_{T-1}}\frac{B^F_{T-1}}{P_{T-1}}\right)\right\} - E_{T-2}\left\{R_{T-2,T}\right\}\bar{T}^F - (1-\gamma_f)E_{T-2}\left\{R_{T-2,T}\frac{T^C_T}{P_T}\right\}$$

in which we can substitute on the right hand side equation (A.5) lagged one period to get

$$E_{T-2}\left\{R_{T-2,T}\left(Q_T\frac{D^F_T}{P_T}+\frac{1}{1+i_T}\frac{B^F_T}{P_T}\right)\right\} = (1-\phi_f)^2\left(Q_{T-2}\frac{D^F_{T-2}}{P_{T-2}}+\frac{1}{1+i_{T-2}}\frac{B^F_{T-2}}{P_{T-2}}\right) - (1-\phi_f)(1-\gamma_f)E_{T-2}\left\{R_{T-2,T-1}\frac{T^C_{T-1}}{P_{T-1}}\right\} - (1-\gamma_f)E_{T-2}\left\{R_{T-2,T}\frac{T^C_T}{P_T}\right\} - (1-\phi_f)E_{T-2}\left\{R_{T-2,T-1}\right\}\bar{T}^F - E_{T-2}\left\{R_{T-2,T}\right\}\bar{T}^F.$$

After reiterating the substitution back to time t, we get

$$E_t\left\{R_{t,T}\left(Q_T\frac{D^F_T}{P_T}+\frac{1}{1+i_T}\frac{B^F_T}{P_T}\right)\right\} = (1-\phi_f)^{T-t}\left(Q_t\frac{D^F_t}{P_t}+\frac{1}{1+i_t}\frac{B^F_t}{P_t}\right) - \bar{T}^F E_t\left\{\sum_{j=t+1}^{T}(1-\phi_f)^{T-j}R_{t,j}\right\} - (1-\gamma_f)E_t\left\{\sum_{j=t+1}^{T}(1-\phi_f)^{T-j}R_{t,j}\frac{T^C_j}{P_j}\right\}. \quad (A.6)$$

We now study under which conditions (A.6) converges to zero in the limit $T \to \infty$. Consider the first-term on the right-hand side. This clearly converges to zero if and only if $0 < \phi_f < 2$. Now consider the term

$$E_t\left\{\sum_{j=t+1}^{T}(1-\phi_f)^{T-j}R_{t,j}\right\}, \quad (A.7)$$

which can be written as

$$E_t\left\{\sum_{j=t+1}^{T}(1-\phi_f)^{T-j}R_{t,j}\right\}=(1-\phi_f)^T E_t\left\{\sum_{j=t+1}^{T}\left(\frac{\beta}{1-\phi_f}\right)^j\frac{\xi_j Y_j^{-\rho}}{\xi_t Y_t^{-\rho}}\right\}$$

and which converges to zero as $T\to\infty$, provided $|\beta/(1-\phi_f)|\leq 1$, if and only if $0<\phi_f<2$, considering that the stochastic processes ξ_t, Y_t are bounded.

Equivalently, the term (A.7) can be written as

$$E_t\left\{\sum_{j=t+1}^{T}(1-\phi_f)^{T-j}R_{t,j}\right\}=\beta^{T-t}E_t\left\{\sum_{j=0}^{T-t-1}\left(\frac{\beta}{1-\phi_f}\right)^{-j}\frac{\xi_{T-j}Y_{T-j}^{-\rho}}{\xi_t Y_t^{-\rho}}\right\}$$

which converges to zero as $T\to\infty$, provided $|\beta/(1-\phi_f)|\geq 1$ since $0<\beta<1$, considering that the stochastic processes ξ_t, Y_t are bounded. Therefore $0<\phi_f<2$ is also necessary and sufficient for the second-term on the right-hand side of (A.6) to converge to zero.

Now consider the last term on the right-hand side of (A.6) which can be written as

$$(1-\gamma_f)E_t\left\{\sum_{j=t+1}^{T}(1-\phi_f)^{T-j}R_{t,j}\frac{T_j^C}{P_j}\right\}=(1-\gamma_f)E_t\left\{\sum_{j=t+1}^{T}(1-\phi_f)^{T-j}\beta^j\frac{\xi_j Y_j^{-\rho}}{\xi_t Y_t^{-\rho}}\frac{T_j^C}{P_j}\right\}$$

Considering bounded processes for T_t^C, the above term should also converge to zero as $T\to\infty$ for any stochastic processes $\{P_t, i_t, Q_t, M_t\}$ satisfying the conditions of part i) of Definition 2 consistently each with a specified conventional monetary policy.

The sufficiency of $\gamma_f=1$ to grant this convergence is self evident.

To prove also necessity, note that if $i_j=0$ for each $j\geq t$, equation (16) implies $E_t\beta^j\xi_j Y_j^{-\rho}\xi_t^{-1}Y_t^{\rho}P_j^{-1}=P_t^{-1}$. Let us assume that T_t^C has a deterministic path, then if $i_j=0$ for each $j\geq t$ the above expression can be written as

$$(1-\gamma_f)E_t\left\{\sum_{j=t+1}^{T}(1-\phi_f)^{T-j}R_{t,j}\frac{T_j^C}{P_j}\right\}=(1-\gamma_f)\frac{1}{P_t}\left\{\sum_{j=t+1}^{T}(1-\phi_f)^{T-j}T_j^C\right\}.$$

Note that in this case, even if the deterministic process for $\{T_t^C\}$ implies that T_T^C converges to zero as $T\to\infty$, the sum would in general converge (if at all) to a finite number, not necessarily zero. This proves necessity of $\gamma_f=1$.

Therefore $\gamma_f=1$ and $0<\phi_f<2$ are necessary and sufficient conditions for (A.1) to be in the class of passive fiscal policies. ■

A.1.2 Proof of Proposition 2

The remittances' policy

$$\frac{T_t^C}{P_t} = \bar{T}^C + \gamma_c \frac{\Psi_t^C}{P_t} + \phi_c \frac{N_{t-1}^C}{P_t} \tag{A.8}$$

is in the class of passive remittances' policies if and only if $0 < \gamma_c < 2$ *and* $0 < \phi_c < 2$.

Proof. Recall the law of motion of net worth (13) and substitute in the remittances policy. It implies

$$\frac{N_T^C}{P_T} = (1-\phi_c)\frac{N_{T-1}^C}{P_T} + (1-\gamma_c)\frac{\Psi_T^C}{P_T} - \bar{T}^C.$$

Using the definition of profits (15) we can obtain

$$\begin{aligned}\frac{N_T^C}{P_T} &= (1-\phi_c+(1-\gamma_c)i_{T-1})\frac{N_{T-1}^C}{P_T} + (1-\gamma_c)i_{T-1}\frac{M_{T-1}}{P_T} + \\ &\quad +(1-\gamma_c)(r_T - i_{T-1})\frac{Q_{T-1}D_{T-1}^C}{P_T} - \bar{T}^C.\end{aligned}$$

By substituting on the left hand side of the above equation the law of motion for N_{T-1}^C we get

$$\begin{aligned}\frac{N_T^C}{P_T} &= \prod_{j=T-2}^{T-1}(1-\phi_c+(1-\gamma_c)i_j)\frac{N_{T-2}^C}{P_T} + (1-\gamma_c)i_{T-1}\frac{M_{T-1}}{P_T} + \\ &\quad (1-\phi_c+(1-\gamma_c)i_{T-1})(1-\gamma_c)i_{T-2}\frac{M_{T-2}}{P_T} + (1-\gamma_c)(r_T - i_{T-1})\frac{Q_{T-1}D_{T-1}^C}{P_T} \\ &\quad (1-\phi_c+(1-\gamma_c)i_{T-1})(1-\gamma_c)(r_{T-1}-i_{T-2})\frac{Q_{T-2}D_{T-2}^C}{P_T} - \bar{T}^C \\ &\quad -(1-\phi_c+(1-\gamma_c)i_{T-1})\frac{P_{T-1}}{P_T}\bar{T}^C.\end{aligned}$$

Repeating the substitution for N_{T-2}^C and then recursively back to time t we get

$$\begin{aligned}\frac{N_T^C}{P_T} &= \left(\prod_{j=t}^{T-1}\Psi_j\right)\frac{N_t^C}{P_T} + (1-\gamma_c)\sum_{j=t}^{T-1}\left(\prod_{i=j+1}^{T-1}\Psi_i\right)i_j\frac{M_j}{P_T} \\ &\quad +(1-\gamma_c)\sum_{j=t}^{T-1}\left(\prod_{i=j+1}^{T-1}\Psi_i\right)(r_{j+1}-i_j)\frac{Q_jD_j^C}{P_T} - \bar{T}^C\sum_{j=t}^{T-1}\left(\prod_{i=j+1}^{T-1}\Psi_i\frac{P_i}{P_{i+1}}\right)\end{aligned}$$

in which we have defined

$$\Psi_t \equiv (1-\phi_c+(1-\gamma_c)i_t).$$

We can now pre-multiply the above equation by $R_{t,T}$ and take the expectation at time t to get

$$E_t\left\{R_{t,T}\frac{N_T^C}{P_T}\right\} = E_t\left\{R_{t,T}\left(\prod_{j=t}^{T-1}\Psi_j\right)\frac{N_t^C}{P_T}\right\}+(1-\gamma_c)E_t\left\{R_{t,T}\sum_{j=t}^{T-1}\left(\prod_{i=j+1}^{T-1}\Psi_i\right)i_j\frac{M_j}{P_T}\right\}$$
$$+(1-\gamma_c)E_t\left\{R_{t,T}\sum_{j=t}^{T-1}\left(\prod_{i=j+1}^{T-1}\Psi_i\right)(r_{j+1}-i_j)\frac{Q_jD_j^C}{P_T}\right\}$$
$$-\bar{T}^C E_t\left\{R_{t,T}\sum_{j=t}^{T-1}\left(\prod_{i=j+1}^{T-1}\Psi_i\frac{P_i}{P_{i+1}}\right)\right\}. \tag{A.9}$$

Now consider the first term on the right-hand side, we need to prove that

$$\lim_{T\to\infty}E_t\left\{R_{t,T}\left(\prod_{j=t}^{T-1}\Psi_j\right)\frac{N_t^C}{P_T}\right\}=0 \tag{A.10}$$

for any vector of stochastic processes $\{\mathbf{Z}_t\}$ satisfying the conditions of part i) of Definition 2 consistently each with a specified conventional monetary policy. We show that a necessary and sufficient condition is that there exists a $\varepsilon > 0$ and a corresponding time T_1 such that $|\Psi_t| \leq (1-\varepsilon)(1+i_t)$ for each $t \geq T_1$. To see that it is a sufficient condition note that in this case

$$E_t\left\{R_{t,T}\left(\prod_{j=t}^{T-1}\Psi_j\right)\frac{N_t^C}{P_T}\right\}$$
$$\leq E_t\left\{R_{t,T}\left(\prod_{j=t}^{T_1-1}\Psi_j\right)\left(\prod_{j=T_1}^{T-1}(1+i_j)\right)(1-\varepsilon)^{T-T_1}\frac{N_t^C}{P_T}\right\}$$
$$=(1-\varepsilon)^{T-T_1}E_t\left\{R_{t,T_1}\left(\prod_{j=t}^{T_1-1}\Psi_j\right)\frac{N_t^C}{P_{T_1}}\right\}$$

and

$$E_t\left\{R_{t,T}\left(\prod_{j=t}^{T-1}\Psi_j\right)\frac{N_t^C}{P_T}\right\}\geq -(1-\varepsilon)^{T-T_1}E_t\left\{R_{t,T_1}\left(\prod_{j=t}^{T_1-1}\Psi_j\right)\frac{N_t^C}{P_{T_1}}\right\}.$$

Therefore the first term on the right-hand side converges to zero as $T\to\infty$ since the upper and lower bounds converge to zero. To prove the necessity of the condition, consider now the allocation $P_T = R_{t,T}P_t$, $i_T = 0$ for each $t \geq T$ with M_t that satisfies $M_t \geq P_tY_t$ and $Q_t = E_t\left\{\sum_{j=0}^{\infty}\delta^j(1-\varkappa_{t+j})\right\}$. This allocation satisfies the conditions of part i) of Definition 2. Therefore if (A.10) holds, evaluated at the above allocation, it should be also the case that

$$\lim_{T\to\infty} E_t\left\{\prod_{j=t}^{T-1}\Psi_j\right\}=0$$

which requires that $|\Psi_t|<1$ infinitely many times. Therefore the condition "there exists a $\varepsilon>0$ and a corresponding time T_1 such that $|\Psi_t|\leq(1-\varepsilon)(1+i_t)$ for each $t\geq T_1$" is a necessary condition since it implies $|\Psi_t|<1$ infinitely many times. Note indeed that in the allocation used $i_t=0$ for each $t\geq T$.

We now evaluate under which restriction on the parameters ϕ_c and γ_c the inequality is satisfied

$$|\Psi_t|=|1-\phi_c+(1-\gamma_c)i_t|\leq(1-\varepsilon)(1+i_t)$$

for any sequence of stochastic processes $\left\{\mathbf{Z}_t^*\right\}$ satisfying the conditions of part i) of Definition 2 consistently each with a specified conventional monetary policy. Note that when $i_t=0$, the above condition is satisfied when $\varepsilon\leq\phi_c\leq 2-\varepsilon$ while when i_t is unboundedly large, the above condition is satisfied when $\varepsilon\leq\gamma_c\leq 2-\varepsilon$. These are both necessary conditions. Note that given positive ϕ_c and γ_c, ε can be chosen positive, small enough and such that $\varepsilon<\min(\phi_c,\gamma_c)$. Therefore the necessary conditions are that $0<\phi_c<2$ and $0<\gamma_c<2$. These are also sufficient conditions given an $\varepsilon<\min(\phi_c,\gamma_c)$. Note also that under the necessary and sufficient conditions $0<\phi_c<2$ and $0<\gamma_c<2$, the condition "there exists a $\varepsilon>0$ and a corresponding time T_1 such that $|\Psi_t|\leq(1-\varepsilon)(1+i_t)$ for each $t\geq T_1$" is equivalent to "$|\Psi_t|\leq(1-\varepsilon)(1+i_t)$ at all times and contingencies."

Consider now the second term on the right-hand side of (A.9). We now show that $|\Psi_t|\leq(1-\varepsilon)(1+i_t)$ at all times is a sufficient condition for its convergence to zero. Using $|\Psi_j|\leq(1-\varepsilon)(1+i_j)$ for each j given a positive and small ε, we can write

$$(1-\gamma_c)E_t\left\{R_{t,T}\sum_{j=t}^{T-1}\left(\prod_{i=j+1}^{T-1}\Psi_i\right)i_j\frac{M_j}{P_T}\right\}$$
$$\leq(1-\gamma_c)E_t\left\{R_{t,T}\sum_{j=t}^{T-1}\left(\prod_{i=j+1}^{T-1}(1+i_i)\right)(1-\varepsilon)^{T-1-j}i_j\frac{M_j}{P_T}\right\}$$

and

$$(1-\gamma_c)E_t\left\{\sum_{j=t}^{T-1}R_{t,j}\left(\prod_{i=j+1}^{T-1}\Psi_i\right)i_j\frac{M_j}{P_j}\right\}$$
$$\geq-(1-\gamma_c)E_t\left\{R_{t,T}\sum_{j=t}^{T-1}\left(\prod_{i=j+1}^{T-1}(1+i_i)\right)(1-\varepsilon)^{T-1-j}i_j\frac{M_j}{P_T}\right\}$$

Consider moreover that

$$E_t\left\{R_{t,T}\sum_{j=t}^{T-1}\left(\prod_{i=j+1}^{T-1}(1+i_i)\right)(1-\varepsilon)^{T-1-j}i_j\frac{M_j}{P_T}\right\} = E_t\left\{\sum_{j=t}^{T-1}R_{t,j}(1-\varepsilon)^{T-1-j}\frac{i_j}{1+i_j}\frac{M_j}{P_j}\right\}$$
$$= E_t\left\{\sum_{j=t}^{T-1}\beta^j(1-\varepsilon)^{T-1-j}\frac{\xi_jY_j^{-\rho}}{\xi_tY_t^{-\rho}}\frac{i_j}{1+i_j}Y_j\right\}$$

where in the first line we have used repeatedly (16) while in the second line we have substituted in the expression for $R_{t,j}$ and used (19).

We can now write

$$E_t\left\{R_{t,T}\sum_{j=t}^{T-1}\left(\prod_{i=j+1}^{T-1}(1+i_i)\right)\kappa^{T-1-j}i_j\frac{M_j}{P_T}\right\} = (1-\varepsilon)^{T-1}E_t\left\{\sum_{j=t}^{T-1}\left(\frac{\beta}{1-\varepsilon}\right)^j\frac{\xi_jY_j^{-\rho}}{\xi_tY_t^{-\rho}}\frac{i_j}{1+i_j}Y_j\right\}$$

whenever $|\beta/(1-\varepsilon)|\leq 1$ and

$$E_t\left\{R_{t,T}\sum_{j=t}^{T-1}\left(\prod_{i=j+1}^{T-1}(1+i_i)\right)\kappa^{T-1-j}i_j\frac{M_j}{P_T}\right\}$$
$$= \beta^{T-1}E_t\left\{\sum_{j=1}^{T-t}\left(\frac{\beta}{1-\varepsilon}\right)^{-j}\frac{\xi_{T-j}Y_{T-j}^{-\rho}}{\xi_tY_t^{-\rho}}\frac{i_{T-j}}{1+i_{T-j}}Y_{T-j}\right\}$$

whenever $|\beta/(1-\varepsilon)|\geq 1$ which both converge to zero as $T\to\infty$ given that $\varepsilon>0$ and $0<\beta<1$ and all stochastic processes within the curly bracket are bounded. Therefore the existence of a positive and small ε such that $|\Psi_t|\leq(1-\varepsilon)(1+i_t)$ at all times is sufficient for the second term on the right-hand side of (A.9) to converge to zero.

Consider now the third term on the right-hand side of (A.9) under the condition that $|\Psi_t|\leq(1-\varepsilon)(1+i_t)$ at all times for a positive ε

$$(1-\gamma_c)E_t\left\{R_{t,T}\sum_{j=t}^{T-1}\left(\prod_{i=j+1}^{T-1}\Psi_i\right)(r_{j+1}-i_j)\frac{Q_jD_j^C}{P_T}\right\}$$
$$\leq(1-\gamma_c)E_t\left\{R_{t,T}\sum_{j=t}^{T-1}\left(\prod_{i=j+1}^{T-1}(1+i_i)\right)(1-\varepsilon)^{T-1-j}(r_{j+1}-i_j)\frac{Q_jD_j^C}{P_T}\right\},$$

$$(1-\gamma_c)E_t\left\{R_{t,T}\sum_{j=t}^{T-1}\left(\prod_{i=j+1}^{T-1}\Psi_i\right)(r_{j+1}-i_j)\frac{Q_jD_j^C}{P_T}\right\}$$
$$\geq -(1-\gamma_c)E_t\left\{R_{t,T}\sum_{j=t}^{T-1}\left(\prod_{i=j+1}^{T-1}(1+i_i)\right)(1-\varepsilon)^{T-1-j}(r_{j+1}-i_j)\frac{Q_jD_j^C}{P_T}\right\}.$$

Now the terms on the right-hand side of the above inequalities are zero after noting that (16) and (17) imply

$$E_t\left\{R_{t,t+1}\frac{(r_{t+1}-i_t)}{\Pi_{t+1}}\right\}=0.$$

Therefore the existence of a positive ε such that $|\Psi_t|\leq(1-\varepsilon)(1+i_t)$ at all times is sufficient for the third term on the right-hand side of (A.9) to converge to zero.

Finaly, consider the fourth term on the right-hand side of (A.9) under the condition that $|\Psi_t|\leq(1-\varepsilon)(1+i_t)$ at all times for a positive ε

$$\bar{T}^CE_t\left\{R_{t,T}\sum_{j=t}^{T-1}\left(\prod_{i=j+1}^{T-1}\Psi_i\frac{P_i}{P_{i+1}}\right)\right\}\leq\bar{T}^CE_t\left\{R_{t,T}\sum_{j=t}^{T-1}\left(\prod_{i=j+1}^{T-1}(1+i_i)\frac{P_i}{P_{i+1}}\right)(1-\varepsilon)^{T-1-j}\right\},$$

$$\bar{T}^CE_t\left\{R_{t,T}\sum_{j=t}^{T-1}\left(\prod_{i=j+1}^{T-1}\Psi_i\frac{P_i}{P_{i+1}}\right)\right\}\geq-\bar{T}^CE_t\left\{R_{t,T}\sum_{j=t}^{T-1}\left(\prod_{i=j+1}^{T-1}(1+i_i)\frac{P_i}{P_{i+1}}\right)(1-\varepsilon)^{T-1-j}\right\}.$$

Note that

$$\begin{aligned}E_t\left\{R_{t,T}\sum_{j=t}^{T-1}\left(\prod_{i=j+1}^{T-1}(1+i_i)\frac{P_i}{P_{i+1}}\right)(1-\varepsilon)^{T-1-j}\right\} &= E_t\left\{\sum_{j=t+1}^{T}R_{t,j}(1-\varepsilon)^{T-1-j}\right\}\\ &= E_t\left\{\sum_{j=t+1}^{T}\beta^j(1-\varepsilon)^{T-1-j}\frac{\xi_jY_j^{-\rho}}{\xi_tY_t^{-\rho}}\right\}\end{aligned}$$

which converges to zero following previous reasonings. Therefore the existence of a positive ε such that $|\Psi_t|\leq(1-\varepsilon)(1+i_t)$ at all times is sufficient for the fourth term on the right-hand side of (A.9) to converge to zero.

Therefore $0<\phi_c<2$ and $0<\gamma_c<2$ are necessary and sufficient conditions for (A.9) to converge to zero. ■

A.1.3 Proof of Proposition 3

Under a combined regime of passive fiscal policy and passive policy of central-bank remittances the Neutrality Property holds.

Proof. Consider the set $\mathcal{P}$ of rational expectations equilibria characterized by a *transfer* *policy* $\mathcal{T}(\cdot)$ set consistently with the respectively-defined passive policies. Consider a rational expectations equilibrium $\{\mathbf{Z}_t^*,\ \mathbf{K}_t^*\} \in \mathcal{P}$ characterized by a *conventional monetary policy* $\mathcal{I}(\cdot)$ or $\mathcal{M}(\cdot)$ and a *balance-sheet policy* $\mathcal{B}(\cdot)$ on top of the *transfer policy* identifying the set $\mathcal{P}$. Fix the *conventional monetary policy* and the *transfer policy* and consider an alternative appropriately-bounded *balance-sheet policy* $\tilde{\mathcal{B}}(\cdot)$. The existence of an alternative appropriately-bounded *balance-sheet policy* $\tilde{\mathcal{B}}(\cdot)$ will be clear from the proof that follows. Given this monetary/fiscal policy regime, the vector of stochastic processes $\{\mathbf{Z}_t^*\}$ still satisfies the equilibrium conditions (16) to (19) and the *conventional monetary policy*. Moreover the stochastic process $\{M_t^*\}$ can also take any other value $\tilde{M}_t \geq P_t^* Y_t$ in each contingency in which $i_t^* = 0$. In these contingencies (and only in these contingencies) $\tilde{M}_t \geq P_t^* Y_t$ implies a change in *conventional monetary policy* if and only if the latter is specified in terms of $\mathcal{M}(\cdot)$. Under the passive *transfer policy* $\mathcal{T}(\cdot)$ is such that (25) and (28) hold looking forward from each contingency $t \geq t_0$, together with the sequence of equilibrium conditions (21), (22) given the vector of stochastic processes $\{\mathbf{Z}_t^*\}$ and finite $D_{t-1}^G, B_{t-1}^G, X_{t-1}, B_{t-1}^C, D_{t-1}^C$. Consider passive *transfer policies* of the form $T_t^F = \mathcal{T}^F(\bar{\mathbf{T}}_t^C, \bar{\mathbf{D}}_{t-1}^F, \bar{\mathbf{B}}_{t-1}^F, \bar{\mathbf{Z}}_t, \zeta_t)$ and $T_t^C = \mathcal{T}^C(\bar{\mathbf{N}}_{t-1}^C, \bar{\mathbf{Z}}_t, \zeta_t)$.[61] Evaluate them in a generic contingency at time t_0 given that in this contingency the vector $\mathbf{Z}$ takes the value $\mathbf{Z}^*(\mathbf{s}_{t_0}, \mathbf{w}_{t_0-1})$ for initial conditions $\mathbf{w}_{t_0-1}$. Therefore it is possible to obtain $\tilde{T}_{t_0}^F \equiv \tilde{T}^F(\mathbf{s}_{t_0}, \mathbf{w}_{t_0-1})$ and $\tilde{T}_{t_0}^C \equiv \tilde{T}^C(\mathbf{s}_{t_0}, \mathbf{w}_{t_0-1})$. Similarly given the *balance-sheet policy* $\mathbf{B}_t = \tilde{\mathcal{B}}(\bar{\mathbf{B}}_{t-1}, \bar{\mathbf{Z}}_{\mathbf{t}}, \zeta_t)$ it is possible to obtain $\tilde{D}_{t_0}^C \equiv \tilde{D}^C(\mathbf{s}_{t_0}, \mathbf{w}_{t_0-1})$, $\tilde{B}_{t_0}^C \equiv \tilde{B}^C(\mathbf{s}_{t_0}, \mathbf{w}_{t_0-1})$ and $\tilde{D}_{t_0}^F \equiv \tilde{D}^F(\mathbf{s}_{t_0}, \mathbf{w}_{t_0-1})$. Consider the flow budget constraint (22) in the same contingency at time t_0 and use these results to evaluate it and get $\tilde{X}_{t_0} \equiv \tilde{X}(\mathbf{s}_{t_0}, \mathbf{w}_{t_0-1})$

$$Q_{t_0}^* \tilde{D}_{t_0}^C + \frac{\tilde{B}_{t_0}^C}{1+i_{t_0}^*} - \tilde{M}_{t_0} - \frac{X_{t_0}}{1+i_t^*} = (1-\varkappa_t)(1+\delta Q_{t_0}^*) D_{t_0-1}^C + B_{t_0-1}^C - X_{t_0-1} - M_{t_0-1} - \tilde{T}_{t_0}^C$$

where in particular $\tilde{M}_{t_0} = M_{t_0}^*$ if $i_{t_0}^* > 0$ and $\tilde{M}_{t_0}$ can take any other value $\tilde{M}_{t_0} \geq P_{t_0}^* Y_{t_0}$ if $i_{t_0}^* = 0$.[62]

Consider now the flow budget constraint (21) and evaluate it

$$Q_{t_0}^* \tilde{D}_{t_0}^F + \frac{\tilde{B}_{t_0}^F}{1+i_{t_0}^*} = (1-\varkappa_t)(1+\delta Q_{t_0}^*) D_{t_0-1}^F + B_{t_0-1}^F - \tilde{T}_{t_0}^F - \tilde{T}_{t_0}^C.$$

[61] We know from Propositions 1 and 2 that this is a non-empty set of passive *transfer policies.*

[62] Note that the requirement of the rational expectations equilibrium that $\tilde{X}_{t_0} \geq 0$ is equivalent to $Q_{t_0}^*(\tilde{D}_{t_0}^C - D_{t_0}^{C*}) + (\tilde{B}_{t_0}^C - B_{t_0}^{C*})/(1+i_{t_0}^*) - (\tilde{M}_{t_0} - M_{t_0}^*) + X_{t_0}^*/(1+i_{t_0}^*) \geq 0$ which imposes a bound on the alternative balance-sheet policies that can be considered.

The above equation can then be used to determine $\tilde{B}^F_{t_0}$.[63] Repeating the above steps sequentially and taking care at each step of the constraints $\tilde{X}_t \geq 0$ and $\tilde{B}^F_t \geq 0$, it is possible to build stochastic processes $\{\tilde{X}_t, \tilde{B}_t, \tilde{B}^C_t, \tilde{B}^F_t, \tilde{D}_t, \tilde{D}^C_t, \tilde{D}^F_t, \tilde{T}^F_t, \tilde{T}^C_t\}$ under the new appropriately-bounded *balance-sheet policy* $\tilde{\mathcal{B}}(\cdot)$ that satisfy (21) and (22) at each point in time t and contingency and moreover satisfy (25) and (28), given $\{\mathbf{Z}^*_t\}$ and initial conditions $\mathbf{w}_{t_0-1}$.

Note that the sum of (25) and (28) given (23) and (24) implies (20). It follows that there is a vector of stochastic processes $\{\tilde{\mathbf{K}}_t\}$ that satisfies each of the conditions in equations (20) to (24) at each time $t \geq t_0$ (and in each contingency at t) given $\{\mathbf{Z}^*_t\}$ and initial conditions $\mathbf{w}_{t_0-1}$. Therefore $\{\mathbf{Z}^*_t, \tilde{\mathbf{K}}_t\}$ is a rational expectations equilibrium which also belongs to $\mathcal{P}$ with $\{M^*_t\}$ that can take any value $\tilde{M}_t \geq P^*_t Y_t$ in each contingency in which $i^*_t = 0$. Moreover, in these contingencies (and only in these contingencies) $\tilde{M}_t \geq P^*_t Y_t$ implies a change in *conventional monetary policy* if the latter is specified in terms of $\mathcal{M}(\cdot)$. It is clear that the same construction can be repeated for any other appropriately-bounded *balance-sheet policy* and for any other equilibrium belonging to $\mathcal{P}$ The Neutrality Property holds. ■

A.1.4 Proof of Proposition 4

A regime of full treasury's support, $T^C_t = \Psi^C_t$ at each date t (and in each contingency at t), is not in the class of passive remittances' policies.

Proof. Consider the allocation $P_t = \beta^{t-t_0}P$, $i_t = 0$, $Q_t = 1/(1-\delta)$, and $M_t \geq \beta^t PY$. It is easy to verify that this allocation satisfies the equilibrium conditions (16) to (19) as required by Definition 5 when $\xi_t = \xi$, $Y_t = Y$ and $\varkappa_t = 0$, given some non-negative P and considering the *conventional monetary policy* $i_t = 0$. Moreover, a regime of *full treasury's support* implies that net worth is constant at $N^C_t = N^C_{t_0-1} = \bar{N} > 0$ as shown by the law of motion (13). However, (28) is not satisfied because $\lim_{T\longrightarrow\infty} E_t\left[R_{t,T}N^C_T/P_T\right] = \bar{N}/P_t$, which is not necessarily zero, unless $\bar{N} = 0$.[64] Therefore a regime of full treasury's support is not in the class of passive remittances' policies. ■

A.1.5 Proof of Proposition 5

Under a passive fiscal policy and full treasury's support the Neutrality Property holds.

Proof. Consider the set $\mathcal{F}$ of rational expectations equilibria with a *transfer policy* $\mathcal{T}(\cdot)$ set consistently with a *passive fiscal policy* and *full treasury's support.* Consider a rational

[63] Note that the requirement of the rational expectations equilibrium that $\tilde{B}^F_{t_0} \geq 0$ is equivalent to $Q^*_{t_0}(\tilde{D}^F_{t_0} - D^{F*}_{t_0}) - B^{F*}_{t_0}/(1+i^*_{t_0}) \geq 0$ which imposes a bound on the alternative balance-sheet policies that can be considered.

[64] Recall that $R_{t,T} \equiv \beta^{T-t}\xi_T Y_T^{-\rho}/\xi_t Y_t^{-\rho}$ which is equal to β^{T-t} in this case.

expectations equilibrium $\{\mathbf{Z}_t^*, \mathbf{K}_t^*\} \in \mathcal{F}$ characterized by a *conventional monetary policy* $\mathcal{I}(\cdot)$ or $\mathcal{M}(\cdot)$ and a *balance-sheet policy* $\mathcal{B}(\cdot)$ on top of the *transfer policy* identifying the set $\mathcal{F}$. Note that for an equilibrium to be in this set it is necessarily the case that

$$\lim_{T\longrightarrow\infty} E_t \left[\beta^{T-t}\frac{\xi_T Y_T^{-\rho} P_t^*}{\xi_t Y_t^{-\rho} P_T^*}\right] = 0. \tag{A.11}$$

Indeed, given a passive fiscal policy (28) should necessarily hold while full treasury's support implies $N_t = N_{t_0-1} = \bar{N} > 0$. Therefore (A.11) necessarily holds. Fix the *conventional monetary policy* and the *transfer policy* and consider an alternative appropriately-bounded *balance-sheet policy* $\tilde{\mathcal{B}}(\cdot)$. The existence of an alternative appropriately-bounded *balance-sheet policy* $\tilde{\mathcal{B}}(\cdot)$ will be clear from the proof that follows. Given this monetary/fiscal policy regime, the vector of stochastic processes $\{\mathbf{Z}_t^*\}$ still satisfies the equilibrium conditions (16) to (19) and the *conventional monetary policy.* Moreover the stochastic process $\{M_t^*\}$ can also take any other value $\tilde{M}_t \geq P_t^* Y_t$ in each contingency in which $i_t^* = 0$. In these contingencies (and only in these contingencies) $\tilde{M}_t \geq P_t^* Y_t$ implies a change in *conventional monetary policy* if and only if the latter is specified in terms of $\mathcal{M}(\cdot)$.

Consider passive *transfer policies* of the form $T_t^F = \mathcal{T}^F(\bar{\mathbf{T}}_t^C, \bar{\mathbf{D}}_{t-1}^F, \bar{\mathbf{B}}_{t-1}^F, \bar{\mathbf{Z}}_t, \zeta_t)$.[65] Evaluate it in a generic contingency at time t_0 given that in this contingency the vector $\mathbf{Z}$ takes the value $\mathbf{Z}^*(\mathbf{s}_{t_0}, \mathbf{w}_{t_0-1})$ given initial conditions $\mathbf{w}_{t_0-1}$. Therefore it is possible to obtain $\tilde{T}_{t_0}^F \equiv \tilde{T}^F(\mathbf{s}_{t_0}, \mathbf{w}_{t_0-1})$. Similarly given the *balance-sheet policy* $\mathbf{B}_t = \tilde{\mathcal{B}}(\bar{\mathbf{B}}_{t-1}, \bar{\mathbf{Z}}_t, \zeta_t)$ it is possible to obtain $\tilde{D}_{t_0}^C \equiv \tilde{D}^C(\mathbf{s}_{t_0}, \mathbf{w}_{t_0-1})$, $\tilde{B}_{t_0}^C \equiv \tilde{B}^C(\mathbf{s}_{t_0}, \mathbf{w}_{t_0-1})$ and $\tilde{D}_{t_0}^F \equiv \tilde{D}^F(\mathbf{s}_{t_0}, \mathbf{w}_{t_0-1})$. Full treasury's support implies that at each point in time and contingency $T_t^C = \Psi_t^C$ and $N_t^C = N_{t_0-1}^C = \bar{N} > 0$. Starting from a generic contingency at time t_0, it is possible to determine $\tilde{T}_{t_0}^C \equiv \tilde{T}^C(\mathbf{s}_{t_0}, \mathbf{w}_{t_0-1})$ as

$$\tilde{T}_{t_0}^C = i_{t_0-1}(N_{t_0-1}^C + M_{t_0-1}) + (r_{t_0}^* - i_{t_0-1})Q_{t_0-1}D_{t_0-1}^C.$$

Moreover $N_{t_0} = \bar{N}$ implies that we can determine $\tilde{X}_{t_0} \equiv \tilde{X}(\mathbf{s}_{t_0}, \mathbf{w}_{t_0-1})$ from

$$Q_{t_0}^* \tilde{D}_{t_0}^C + \frac{\tilde{B}_{t_0}^C}{1+i_{t_0}^*} - \tilde{M}_{t_0} - \frac{\tilde{X}_{t_0}}{1+i_{t_0}^*} = \bar{N},$$

where in particular $\tilde{M}_{t_0} = M_{t_0}^*$ if $i_{t_0}^* > 0$ or $\tilde{M}_{t_0}$ can take any other value $\tilde{M}_{t_0} \geq P_{t_0}^* Y_{t_0}$ if

[65]We know from Proposition 1 that this is a non-empy set.

$i^*_{t_0} = 0.$[66] Consider now the flow budget constraint (21) and evaluate it

$$Q^*_{t_0}\tilde{D}^F_{t_0} + \frac{\tilde{B}^F_{t_0}}{1+i^*_{t_0}} = (1-\varkappa_t)(1+\delta Q^*_{t_0})D^F_{t_0-1} + B^F_{t_0-1} - \tilde{T}^F_{t_0} - \tilde{T}^C_{t_0}.$$

The above equation can then be used to determine $\tilde{B}^F_{t_0}$.[67] Repeating the above steps sequentially and taking care at each step of the constraints $\tilde{X}_t \geq 0$ and $\tilde{B}^F_t \geq 0$, it is possible to build stochastic processes $\{\tilde{X}_t, \tilde{B}_t, \tilde{B}^C_t, \tilde{B}^F_t, \tilde{D}_t, \tilde{D}^C_t, \tilde{D}^F_t, \tilde{T}^F_t, \tilde{T}^C_t\}$ under the new appropriately-bounded *balance-sheet policy* $\tilde{\mathcal{B}}(\cdot)$ that satisfy (21) and (22) at each point in time t and contingency and moreover satisfy (25) and (28), given $\{\mathbf{Z}^*_t\}$ and initial conditions $\mathbf{w}_{t_0-1}$. Note that (28) is satisfied because it is the case that $\tilde{N}^C_t = \bar{N}$ at each t and contingency while (A.11) holds given $\{\mathbf{Z}^*_t\}$. Finally, note that the sum of (25) and (28) given (23) and (24) implies (20). It follows that there is a vector of stochastic processes $\{\tilde{\mathbf{K}}_t\}$ that satisfies each of the conditions in equations (20) to (24) at each time $t \geq t_0$ (and in each contingency at t) given $\{\mathbf{Z}^*_t\}$ and initial conditions $\mathbf{w}_{t_0-1}$. Therefore $\{\mathbf{Z}^*_t, \tilde{\mathbf{K}}_t\}$ is a rational expectations equilibrium which also belongs to $\mathcal{F}$ with $\{M^*_t\}$ that can take any value $\tilde{M}_t \geq P^*_t Y_t$ in each contingency in which $i^*_t = 0$. Moreover, in these contingencies (and only in these contingencies) $\tilde{M}_t \geq P^*_t Y_t$ implies a change in *conventional monetary policy* if and only if the latter is specified in terms of $\mathcal{M}(\cdot)$. It is clear that the same construction can be repeated for any other appropriately-bounded *balance-sheet policy* and for any other equilibrium belonging to $\mathcal{F}$. The Neutrality Property holds. ■

A.1.6 Proof of Proposition 7

Under a passive fiscal policy and a deferred-asset policy of central bank's remittances the Neutrality Property holds if and only if

$$\frac{\tilde{N}^C_t}{P^*_t} > -E_t \sum_{T=t}^{\infty} R_{t,T}\left(\frac{i^*_T}{1+i^*_T}\frac{M^*_T}{P^*_T}\right) \tag{A.12}$$

in equilibrium at each time t (and in each contingency at t).

Proof. Consider the set $\mathcal{G}$ of rational expectations equilibria with a *transfer policy* $\mathcal{T}(\cdot)$ set consistently with a passive fiscal policy and a deferred-asset policy of central bank's

[66] Note that the requirement of the rational expectations equilibrium that $\tilde{X}_{t_0} \geq 0$ is equivalent to $Q^*_{t_0}\tilde{D}^C_{t_0} + \tilde{B}^C_{t_0}/(1+i^*_{t_0}) - \tilde{M}_{t_0} - \bar{N} \geq 0$ which imposes a bound on the alternative balance-sheet policies that can be considered.

[67] Note that the requirement of the rational expectations equilibrium that $\tilde{B}^F_{t_0} \geq 0$ is equivalent to $Q^*_{t_0}(\tilde{D}^F_{t_0} - D^{F*}_{t_0}) - B^{F*}_{t_0}/(1+i^*_{t_0}) \geq 0$ which imposes a bound on the alternative balance-sheet policies that can be considered.

remittances.

We first prove the necessary condition. Assume that the Neutrality Property holds. Consider a rational expectations equilibrium $\left\{\mathbf{Z}_t^*, \mathbf{K}_t^*\right\} \in \mathcal{G}$ characterized by a *conventional monetary policy* $\mathcal{I}(\cdot)$ or $\mathcal{M}(\cdot)$ and a *balance-sheet policy* $\mathcal{B}(\cdot)$ on top of the *transfer policy* identifying the set $\mathcal{G}$. Consider the equilibrium $\left\{\mathbf{Z}_t^*, \tilde{\mathbf{K}}_t\right\}$ for which the Neutrality Property holds given an alternative *balance-sheet policy* $\tilde{\mathcal{B}}(\cdot)$ with respect to $\left\{\mathbf{Z}_t^*, \mathbf{K}_t^*\right\}$. Assume that there is a contingency in which

$$\frac{\tilde{N}_t^C}{P_t^*} \leq -E_t \sum_{T=t}^{\infty} R_{t,T} \left(\frac{i_T^*}{1+i_T^*} \frac{M_T^*}{P_T^*} \right) \tag{A.13}$$

at a generic time t. First note that given a passive fiscal policy, (28) holds looking forward from each t and in each contingency at the equilibrium stochastic processes $\left\{\mathbf{Z}_t^*, \tilde{\mathbf{K}}_t\right\}$. Therefore the intertemporal budget constraint should hold in equilibrium

$$\frac{\tilde{N}_t^C}{P_t^*} + E_t \sum_{T=t}^{\infty} R_{t,T} \left(\frac{i_T^*}{1+i_T^*} \frac{M_T^*}{P_T^*} \right) = E_t \sum_{T=t+1}^{\infty} R_{t,T} \left(\frac{\tilde{T}_T^C}{P_T^*} \right) \tag{A.14}$$

in each contingency and in particular in the same contingency at time t in which (A.13) holds. In particular, we choose the contingency at time t in a way that the history up to time t shows at least one contingency at a generic time j, with $t_0 \leq j < t$, in which $r_j^* - i_{j-1}^* < 0$. Comparison of (A.13) and (A.14) taking into account the non-negativeness of $\tilde{T}_t^C$ under the deferred-asset regime shows that (A.13) can only hold with equality in equilibrium. Note moreover that the law of motion of net worth implies that

$$\tilde{N}_t^C = \tilde{N}_{t-1}^C + i_{t-1}^*(\tilde{N}_{t-1}^C + M_{t-1}^*) + (r_t^* - i_{t-1}^*) Q_{t-1}^* D_{t-1}^{C*} - \tilde{T}_t^C,$$

which can be solved backward given initial conditions. Use now the fact that Neutrality Property holds. Start from the equilibrium $\left\{\mathbf{Z}_t^*, \tilde{\mathbf{K}}_t\right\}$ for a balance-sheet policy $\tilde{\mathcal{B}}(\cdot)$, given $\mathcal{T}(\cdot)$ and $\mathcal{I}(\cdot)$ (or $\mathcal{M}(\cdot)$). Consider another balance-sheet policy $\hat{\mathcal{B}}(\cdot)$ which just changes the long-term asset holdings to $\left\{\hat{D}_i^C\right\}_{i=t_0}^{t-1}$ with $\hat{D}_i^C \geq 0$, in a way that the implied $\hat{N}_t^C$ given the same history $\mathbf{s}^t$ and same realized values of the stochastic processes $\mathbf{Z}^*$ up to time t is such that $\hat{N}_t^C < \tilde{N}_t^C$.[68] However, under this alternative balance-sheet policy $\hat{\mathcal{B}}(\cdot)$, $\hat{N}_t^C$ cannot satisfy (A.14), and therefore $\left\{\mathbf{Z}_t^*, \hat{\mathbf{K}}_t\right\}$ is not an equilibrium contradicting the fact

[68] Since at time j $r_j^* - i_{j-1}^* < 0$ choose $\hat{D}_{j-1}^C$ so that central-bank profits at time j are lower than what implied by $\tilde{D}_{j-1}^C$ and moreover negative while $\hat{D}_i^C = \tilde{D}_i^C$ for all other i with $t_0 \leq i \leq t-1$. For profits to be negative at time j, $\hat{D}_{j-1}^C$ should be chosen sufficiently high. Under these assumptions it is also the case that $\hat{T}_j^C \leq \tilde{T}^C$. Moreover note that under this construction $\hat{X}_i \geq \tilde{X}_i$ for $t_0 \leq i \leq t-1$.

that the Neutrality Property holds for appropriately-bounded *balance-sheet policies*. We have a contradiction and therefore (A.12) should necessarily hold.

To prove the sufficient condition, consider a rational expectations equilibrium $\{\mathbf{Z}_t^*, \mathbf{K}_t^*\} \in \mathcal{G}$ characterized by a *conventional monetary policy* $\mathcal{I}(\cdot)$ or $\mathcal{M}(\cdot)$ and a *balance-sheet policy* $\mathcal{B}(\cdot)$ on top of the *transfer policy* identifying the set $\mathcal{G}$. Assume that

$$\frac{N_t^{*C}}{P_t^*} > -E_t \sum_{T=t}^{\infty} R_{t,T} \left(\frac{i_T^*}{1+i_T^*} \frac{M_T^*}{P_T^*} \right) \tag{A.15}$$

holds in the equilibrium $\{\mathbf{Z}_t^*, \mathbf{K}_t^*\}$ at each point in time and contingency. Therefore N_t^{C*}/P_t^* is bounded below since the right-hand side of (A.15) is bounded given (5). Moreover under a *deferred-asset regime* N_t^{C*} is bounded above by $\bar{N}$. Finally, note that under a passive fiscal policy

$$\lim_{T \longrightarrow \infty} E_t \left[R_{t,T} \frac{P_t^*}{P_T^*} N_T^{C*} \right] = 0 \tag{A.16}$$

holds in equilibrium.

Consider now an alternative *balance-sheet policy* $\tilde{\mathcal{B}}(\cdot)$. We need to prove that under the condition (A.12), there is a rational expectation equilibrium associated with the same *conventional monetary policy* $\mathcal{I}(\cdot)$ or $\mathcal{M}(\cdot)$, *transfer policy* $\mathcal{T}(\cdot)$, and alternative appropriately-bounded *balance-sheet policy* $\tilde{\mathcal{B}}(\cdot)$. Given that the *conventional monetary policy* has not changed, the vector of stochastic processes $\{\mathbf{Z}_t^*\}$ still satisfies the equilibrium conditions (16) to (19) and the *conventional monetary policy*. Moreover the stochastic process $\{M_t^*\}$ can also take any other value $\tilde{M}_t \geq P_t^* Y_t$ in each contingency in which $i_t^* = 0$. In these contingencies (and only in these contingencies) $\tilde{M}_t \geq P_t^* Y_t$ implies a change in *conventional monetary policy* if and only if the latter is specifed as $\mathcal{M}(\cdot)$.

Consider passive *transfer policy* of the form $T_t^F = \mathcal{T}^F(\bar{\mathbf{T}}_t^C, \bar{\mathbf{D}}_{t-1}^F, \bar{\mathbf{B}}_{t-1}^F, \bar{\mathbf{Z}}_t, \zeta_t)$.[69] Evaluate it in a generic contingency at time t_0 given that in this contingency the vector $\mathbf{Z}$ takes the value $\mathbf{Z}^*(\mathbf{s}_{t_0}, \mathbf{w}_{t_0-1})$ considering initial conditions $\mathbf{w}_{t_0-1}$. Therefore it is possible to obtain $\tilde{T}_{t_0}^F \equiv \tilde{T}^F(\mathbf{s}_{t_0}, \mathbf{w}_{t_0-1})$. Similarly given the *balance-sheet policy* $\mathbf{B}_t = \tilde{\mathcal{B}}(\bar{\mathbf{B}}_{t-1}, \bar{\mathbf{Z}}_t, \zeta_t)$ it is possible to obtain $\tilde{D}_{t_0}^C \equiv \tilde{D}^C(\mathbf{s}_{t_0}, \mathbf{w}_{t_0-1})$, $\tilde{B}_{t_0}^C \equiv \tilde{B}^C(\mathbf{s}_{t_0}, \mathbf{w}_{t_0-1})$ and $\tilde{D}_{t_0}^F \equiv \tilde{D}^F(\mathbf{s}_{t_0}, \mathbf{w}_{t_0-1})$. Under a deferred-asset regime $T_{t_0}^C = \Psi_{t_0}^C$ if $\Psi_{t_0}^C \geq 0$ otherwise $T_{t_0}^C = 0$ and therefore it is possible to write $\tilde{T}_{t_0}^C \equiv \tilde{T}^C(\mathbf{s}_{t_0}, \mathbf{w}_{t_0-1})$ where $\tilde{T}^C(\cdot)$ is a non-negative function.

Moreover equation (22) shows that whenever $N_{t-1}^C = \bar{N}$ and $\Psi_t^C \geq 0$, in which case

[69] We know from Propositions 1 that this is a non-empy set.

$T_t^C = \Psi_t^C$, reserves X_t are going to be determined by

$$\frac{X_t - B_t^C}{1+i_t} = \frac{(X_{t-1} - B_{t-1}^C)}{1+i_{t-1}} + (Q_t D_t^C - Q_{t-1} D_{t-1}^C) - (M_t - M_{t-1}),$$

while whenever $N_{t-1}^C < \bar{N}$, in which case $T_t^C = 0$, reserves X_t are going to be determined by

$$\frac{X_t - B_t^C}{(1+i_t)} = (X_{t-1} - B_{t-1}^C) + Q_t D_t^C - (1+r_t) Q_{t-1} D_{t-1}^C - (M_t - M_{t-1}).$$

It is then possible to apply one of the above two conditions in the contingency at time t_0, depending on whether $\Psi_{t_0}^C \geq 0$ or $\Psi_{t_0}^C < 0$ and evaluate it using $\tilde{D}_{t_0}^C$, $\tilde{B}_{t_0}^C$ and $Q_{t_0}^*, i_{t_0}^*, r_{t_0}^*$ while $\tilde{M}_{t_0}$ is such that $\tilde{M}_{t_0} = M_{t_0}^*$ if $i_{t_0}^* > 0$ or can take any other value $\tilde{M}_{t_0} \geq P_{t_0}^* Y_{t_0}$ if $i_{t_0}^* = 0$. The above equations (one of the two depending on the case) can be used to obtain $\tilde{X}_{t_0} \equiv \tilde{X}(\mathbf{s}_{t_0}, \mathbf{w}_{t_0-1})$.[70]

Consider now the flow budget constraint (21) and evaluate it in the same contingency at time t_0 using previous results to obtain $\tilde{B}_{t_0}^F$. Repeating the above steps sequentially, it is possible to build stochastic processes $\{\tilde{X}_t, \tilde{B}_t, \tilde{B}_t^C, \tilde{B}_t^F, \tilde{D}_t, \tilde{D}_t^C, \tilde{D}_t^G, \tilde{T}_t^F, \tilde{T}_t^C\}$ under the new appropriately-bounded *balance-sheet policy* $\tilde{\mathcal{B}}(\cdot)$ that satisfy (21) and (22) at each point in time t and contingency and moreover satisfy (25) given initial conditions $\mathbf{w}_{t_0-1}$ and $\{\mathbf{Z}_t^*\}$. It is also required that the implied $\tilde{N}_t^C$ satisfies

$$\frac{\tilde{N}_t^C}{P_t^*} > -E_t \sum_{T=t}^{\infty} R_{t,T} \left(\frac{i_T^*}{1+i_T^*} \frac{M_T^*}{P_T^*} \right)$$

at each point in time and contingency. Since under the deferred-asset regime $\tilde{N}_t^C \leq \bar{N}$, it follows that $\tilde{N}_t^C$ satisfies exactly the same upper and lower bounds of N_t^{C*} and should therefore also satisfy (A.16). We have therefore proved that if (A.12) holds at each point in time and contingency, there is a vector of stochastic processes $\{\tilde{\mathbf{K}}_t\}$ that satisfies each of the conditions in equations (20) to (24) at each time $t \geq t_0$ (and in each contingency at t) given initial conditions $\mathbf{w}_{t_0-1}$ and $\{\mathbf{Z}_t^*\}$. Therefore $\{\mathbf{Z}_t^*, \tilde{\mathbf{K}}_t\}$ is a rational expectations equilibrium which also belongs to $\mathcal{G}$ with $\{M_t^*\}$ that can take any value $\tilde{M}_t \geq P_t^* Y_t$ in each contingency in which $i_t^* = 0$. It is clear that the same construction can be repeated for any other appropriately-bounded *balance-sheet policy* and for any other equilibrium belonging to $\mathcal{G}$. The Neutrality Property holds. ■

[70] The requirement that $\tilde{X}_t \geq 0$ for each t and in each contingency imposes appropriate bounds on the *balance-sheet policies* that can be considered.

A.1.7 Proof of Proposition 8

Consider either i) the case in which all the exogenous stochastic disturbances have an absorbing state starting from time τ or ii) the case in which $D_t^C = 0$ for each $t \geq \tau$. Under a passive fiscal policy and a deferred-asset policy of central bank's remittances the Neutrality Property holds if and only if $\tilde{N}_t^C = \bar{N}$ in equilibrium for each $t \geq \tau_1$ with $\tau_1 \geq \tau$.

Proof. Consider first case i). Note that for each $t \geq \tau$ the law of motion of net worth is given by

$$N_t^C = N_{t-1}^C + i_{t-1}(N_{t-1}^C + M_{t-1}) \tag{A.17}$$

since $r_t = i_{t-1}$ for each $t \geq \tau$ as it is implied by conditions (16) and (17) in a deterministic model. Consider case ii), the law of motion of net worth is again given by (A.17) because now $D_t^C = 0$ for each $t \geq \tau$. The following reasonings apply to both cases. We need to show that the condition $\tilde{N}_t^C = \bar{N}$ in equilibrium for each $t \geq \tau_1$ with $\tau_1 \geq \tau$ is equivalent to the necessary and sufficient condition of Proposition 7. Note first that if net worth $\tilde{N}_t^C$ reaches $\bar{N} > 0$ after period τ then $\tilde{N}_t^C$ is going to stay at $\bar{N}$ thereafter, given the deferred-asset regime. Indeed in both cases, profits are never negative and always positive if $i_t > 0$ given that $\Psi_t^C = i_{t-1}(\bar{N} + M_{t-1})$.

Given these observations consider first one of the rational expectations equilibria $\{\mathbf{Z}_t^*, \mathbf{K}_t^*\}$ identified by Proposition 7 for which the Neutrality Property holds and one of the corresponding $\{\mathbf{Z}_t^*, \tilde{\mathbf{K}}_t\}$. Assume by contradiction that $\tilde{N}_t^C < \bar{N}$ for each $t \geq \tau_1$.[71] If $\tilde{N}_t^C < \bar{N}$ for each $t \geq \tau_1$ remittances to the treasury are always zero for each $t \geq \tau_1$ because of the deferred-asset regime. The equilibrium condition (32) implies

$$\frac{\tilde{N}_t^C}{P_t^*} = -E_t \sum_{T=t}^{\infty} R_{t,T} \left(\frac{i_T^*}{1 + i_T^*} \frac{M_T^*}{P_T^*} \right)$$

at each time for each $t \geq \tau_1$ (and contingency only for case ii). Therefore the necessary condition (A.12) of Proposition 7 is violated at each point in time $t \geq \tau_t$. It should be that $\tilde{N}_t = \bar{N}$ in equilibrium $\{\mathbf{Z}_t^*, \tilde{\mathbf{K}}_t\}$ for each $t \geq \tau_1$ with $\tau_1 \geq \tau$.

Consider now a rational expectations equilibrium $\{\mathbf{Z}_t^*, \mathbf{K}_t^*\} \in \mathcal{G}$ characterized by a *conventional monetary policy* $\mathcal{I}(\cdot)$ or $\mathcal{M}(\cdot)$ and a *balance-sheet policy* $\mathcal{B}(\cdot)$ on top of the *transfer policy* identifying the set $\mathcal{G}$ of Proposition 7. Consider now an alternative *balance-sheet policy* $\tilde{\mathcal{B}}(\cdot)$ and assume that given this *balance-sheet policy* $\tilde{N}_t^C = \bar{N}$ in equilibrium $\{\mathbf{Z}_t^*, \tilde{\mathbf{K}}_t\}$ for each $t \geq \tau_1$ with $\tau_1 \geq \tau$. Note that since fiscal policy is passive a *conventional monetary policy* that sets $i_t = 0$ infinitely many times after period τ is not an equilibrium under cases i) and

[71] As discussed above it is not possible that $\tilde{N}_t$ reaches $\bar{N}$ at some point after period τ and then falls below. The only case that contradicts the Proposition is assuming that $\tilde{N}_t < \bar{N}$ at all times after period τ.

ii) for the same reasons discussed in Proposition 4. This implies that when $\tilde{N}_t^C = \bar{N}$ for each $t \geq \tau_1$ profits are strictly positive. Using this result into (32), it implies that the sufficient condition (A.12) is satisfied at each point in time and contingency. ■

A.2 General Model

In this section, we describe the additional features of the general model of Section 4 with respect to the model of Section 2.

We assume that preferences are of the form

$$E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \xi_t \left[\frac{C_t^{1-\rho}}{1-\rho} - \int_0^1 \frac{(L_t(j))^{1+\eta}}{1+\eta} dj \right] \tag{A.18}$$

where C is a consumption bundle of the form

$$C \equiv \left[\int_0^1 C(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}};$$

$C(j)$ is the consumption of a generic good j produced in the economy and θ, with $\theta > 1$, is the intratemporal elasticity of substitution between goods; $L(j)$ is hours worked of variety j which is only used by firm j to produce good j while η is the inverse of the Frisch elasticity of labor supply, with $\eta > 0$. Each household supplies all the varieties of labor used in the production. The asset markets now change to

$$\begin{aligned} M_t + \frac{B_t + X_t}{1+i_t} + Q_t D_t \leq B_{t-1} + X_{t-1} + (1 - \varkappa_t)(1 + \delta Q_t) D_{t-1} + \\ \int_0^1 W_{t-1}(j) L_{t-1}(j) dj - \tilde{T}_t^F + \Phi_{t-1} + (M_{t-1} - P_{t-1} C_{t-1}). \end{aligned} \tag{A.19}$$

In the budget constraint (A.19), $W(j)$ denotes wage specific to labor of quality j. Wage income for each variety of labor j, $W_{t-1}(j)L_{t-1}(j)$, and firms' profits, Φ_{t-1}, of period $t-1$ are deposited in the financial account at the beginning of period t; $\tilde{T}_t^F$ are lump-sum taxes levied by the treasury.

Given that in this general model labor supply is endogenous first-order conditions of the household's problem imply that the marginal rate of substitution between labor and consumption, for each variety j, is given by

$$\frac{(L_t(j))^{\eta}}{C_t^{-\rho}} = \frac{1}{1+i_t} \frac{W_t(j)}{P_t}, \tag{A.20}$$

which is shifted by movements in the nominal interest rate, reflecting the financial friction. Wage income, indeed, can be used to purchase goods only with one-period delay.

We now turn to the supply of goods. We assume that there is a continuum of firms of measure one, each producing one of the goods in the economy. The production function is linear in labor $Y(j) = A_t L(j)$, in which A is a stochastic productivity disturbance which is assumed to follow a Markov process, with transition density $\pi_a(A_{t+1}|A_t)$ and initial distribution f_a. We assume that (π_a, f_a) is such that $A \in [A_{\min}, A_{\max}]$. Given preferences, each firm faces a demand of the form $Y(i) = (P(i)/P)^{-\theta}Y$ where in equilibrium aggregate output is equal to consumption

$$Y_t = C_t. \tag{A.21}$$

Firms are subject to price rigidities as in the Calvo model. A fraction of measure $(1-\alpha)$ of firms with $0 < \alpha < 1$ is allowed to change its price. The remaining fraction α of firms indexes their previously-adjusted prices to the inflation target $\bar{\Pi}$. Adjusting firms choose prices to maximize the presented discounted value of profits under the circumstances that the prices chosen, appropriately indexed to the inflation target, will remain in place until period T with probability α^{T-t}:

$$E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T \left[\bar{\Pi}^{T-t} P_t(j) Y_T(j) - (1-\varrho_T) \frac{W_T(j)}{A_T} Y_T(j) \right],$$

where ϱ_t is a subsidy on firms' labor costs. We assume that ϱ_t is a stochastic disturbance which is assumed to follow a Markov process, with transition density $\pi_\varrho(\varrho_{t+1}|\varrho_t)$ and initial distribution f_ϱ. We assume that (π_ϱ, f_ϱ) is such that $\varrho \in [\varrho_{\min}, \varrho_{\max}]$. The optimality condition implies

$$\frac{P_t^*(j)}{P_t} = \frac{E_t \left\{ \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T \left(\frac{P_T}{P_t} \frac{1}{\bar{\Pi}^{T-t}} \right)^{\theta} \mu_T \frac{W_T(j)}{A_T} Y_T \right\}}{E_t \left\{ \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T P_t \bar{\Pi}^{T-t} \left(\frac{P_T}{P_t} \frac{1}{\bar{\Pi}^{T-t}} \right)^{\theta} Y_T \right\}} \tag{A.22}$$

in which we have used the demand function $Y(i) = (P(i)/P)^{-\theta}Y$ and have defined $\mu_t \equiv \theta(1-\varrho_t)/(\theta-1)$. We can also replace in the previous equation $\lambda_t = C_t^{-\rho}\xi_t/P_t$ and $W_t(j)/P_t$ from (A.20) together with the demand function, $Y(i) = (P(i)/P)^{-\theta}Y$, to obtain

$$\left(\frac{P_t^*}{P_t} \right)^{1+\theta\eta} = \frac{E_t \left\{ \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left(\frac{P_T}{P_t} \frac{1}{\bar{\Pi}^{T-t}} \right)^{\theta(1+\eta)} (1+i_T)\mu_T \left(\frac{Y_T}{A_T} \right)^{1+\eta} \xi_T \right\}}{E_t \left\{ \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left(\frac{P_T}{P_t} \frac{1}{\bar{\Pi}^{T-t}} \right)^{\theta-1} Y_T^{1-\rho} \xi_T \right\}}$$

where P_t^* is the common price chosen by the firms that can adjust it at time t.

Calvo's model further implies the following law of motion of the general price index

$$P_t^{1-\theta} = (1-\alpha)P_t^{*1-\theta} + \alpha P_{t-1}^{1-\theta}\bar{\Pi}^{1-\theta}, \tag{A.23}$$

through which we can write the aggregate supply equation as

$$\left(\frac{1-\alpha\Pi_t^{\theta-1}\bar{\Pi}^{1-\theta}}{1-\alpha}\right)^{\frac{1+\theta\eta}{1-\theta}} = \frac{E_t\left\{\sum_{T=t}^{\infty}(\alpha\beta)^{T-t}\left(\frac{P_T}{P_t}\frac{1}{\bar{\Pi}^{T-t}}\right)^{\theta(1+\eta)}(1+i_T)\mu_T\left(\frac{Y_T}{A_T}\right)^{1+\eta}\xi_T\right\}}{E_t\left\{\sum_{T=t}^{\infty}(\alpha\beta)^{T-t}\left(\frac{P_T}{P_t}\frac{1}{\bar{\Pi}^{T-t}}\right)^{\theta-1}Y_T^{1-\rho}\xi_T\right\}}. \tag{A.24}$$

The additional difference with respect to the model of Section 2 is now in the flow budget constraint of the government which is given by

$$Q_tD_t^G + \frac{B_t^G}{1+i_t} = (1-\varkappa_t)(1+\delta Q_t)D_{t-1}^G + B_{t-1}^G - T_t^F - T_t^C$$

where

$$T_t^F \equiv \tilde{T}_t^F - \varrho_t\int_0^1 W_t(j)L_t(j).$$

A.2.1 Equilibrium

Here, we describe in a compact way the equations that characterize the equilibrium allocation in the general model:

$$\frac{1}{1+i_t} = E_t\left\{\beta\frac{\xi_{t+1}Y_{t+1}^{-\rho}}{\xi_tY_t^{-\rho}}\frac{1}{\Pi_{t+1}}\right\}, \tag{A.25}$$

$$Q_t = E_t\left\{\beta\frac{\xi_{t+1}Y_{t+1}^{-\rho}}{\xi_tY_t^{-\rho}}\frac{(1-\varkappa_{t+1})(1+\delta Q_{t+1})}{\Pi_{t+1}}\right\}, \tag{A.26}$$

$$M_t \geq P_tY_t, \tag{A.27}$$

$$i_t(M_t - P_tY_t) = 0, \tag{A.28}$$

$$\left(\frac{1-\alpha\Pi_t^{\theta-1}\bar{\Pi}^{1-\theta}}{1-\alpha}\right)^{\frac{1+\theta\eta}{1-\theta}} = \frac{F_t}{K_t}, \tag{A.29}$$

$$F_t = \mu_t(1+i_t)\xi_t\left(\frac{Y_t}{A_t}\right)^{1+\eta} + \alpha\beta E_t\left\{\Pi_{t+1}^{\theta(1+\eta)}\bar{\Pi}^{-\theta(1+\eta)}F_{t+1}\right\}, \tag{A.30}$$

$$K_t = \xi_tY_t^{1-\rho} + \alpha\beta E_t\left\{\Pi_{t+1}^{\theta-1}\bar{\Pi}^{1-\theta}K_{t+1}\right\}, \tag{A.31}$$

$$\Delta_t = \Delta\left(\frac{\Pi_t}{\bar{\Pi}}, \Delta_{t-1}\right) \equiv \alpha \left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\theta(1+\eta)} \Delta_{t-1} + (1-\alpha)\left(\frac{1-\alpha\Pi_t^{\theta-1}\bar{\Pi}^{1-\theta}}{1-\alpha}\right)^{\frac{\theta(1+\eta)}{\theta-1}}, \tag{A.32}$$

$$E_t\left\{\sum_{T=t}^{\infty} \beta^{T+1-t}\xi_{T+1}Y_{T+1}^{-\rho}\left[Y_T + \frac{i_T}{1+i_T}\frac{M_T}{P_T}\right]\right\} < \infty, \tag{A.33}$$

$$\lim_{T\longrightarrow\infty} E_t\left[\beta^{T-t}\frac{\xi_T Y_T^{-\rho}}{P_T}\left(M_T + \frac{B_T + X_T}{1+i_T} + Q_T D_T\right)\right] = 0, \tag{A.34}$$

$$Q_t D_t^F + \frac{B_t^F}{1+i_t} = (1-\varkappa_t)(1+\delta Q_t)D_{t-1}^F + B_{t-1}^F - T_t^F - T_t^C, \tag{A.35}$$

$$Q_t D_t^C + \frac{B_t^C}{1+i_t} - M_t - \frac{X_t}{1+i_t} = (1-\varkappa_t)(1+\delta Q_t)D_{t-1}^C + B_{t-1}^C - X_{t-1} - M_{t-1} - T_t^C, \tag{A.36}$$

$$B_t^F = B_t + B_t^C, \tag{A.37}$$

$$D_t^F - D_t = D_t^C. \tag{A.38}$$

A rational expectations equilibrium is a collection of stochastic processes $\{Y_t, \Pi_t, i_t, Q_t, F_t, K_t, M_t, \Delta_t, X_t, B_t, B_t^C, B_t^F, D_t, D_t^C, D_t^F, T_t^F, T_t^C\}$, satisfying each of the conditions in equations (A.25) to (A.38) at each time $t \geq t_0$ (and in each contingency at t) consistently with the specification of a monetary/fiscal policy regime and given the definition $\Pi_t \equiv P_t/P_{t-1}$, the non-negativity constraint on the nominal interest rate $i_t \geq 0$, the stochastic processes for the exogenous disturbances $\{\xi_t, \varkappa_t, A_t, \mu_t\}$ and initial conditions given by the vector $\mathbf{w}_{t_0-1}$ which at least includes $\Delta_{t_0-1}, M_{t_0-1}, X_{t_0-1}, B_{t_0-1}^C, B_{t_0-1}^F, D_{t_0-1}^C, D_{t_0-1}^F$.

A.2.2 Optimal Policy

Optimal policy maximizes the utility of the consumers, the welfare metric can be written as

$$E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\xi_t\left[\frac{Y_t^{1-\rho}}{1-\rho} - \frac{Y_t^{1+\eta}}{1+\eta}\frac{\Delta_t}{A_t^{1+\eta}}\right]. \tag{A.39}$$

We consider the following –partial– specification of the monetary/fiscal policy regime: a *transfer policy* $T_t^F = \mathcal{T}^F(T_t^C, D_{t-1}^F, B_{t-1}^F, P_t, Q_t, \zeta_t)$ and $T_t^C = \mathcal{T}^C(N_{t-1}^C, \Psi_t^C, \zeta_t)$ and a *balance-sheet policy* $B_t^C = \bar{B}_t^C, D_t^C = \bar{D}_t^C, D_t^F = \bar{D}_t^F$ which includes all the cases we are going to consider in our numerical exercises. This specification leaves one degree of freedom along which we choose the optimal policy. The optimal policy is a collection of stochastic processes $\{Y_t, \Pi_t, i_t, Q_t, F_t, K_t, M_t, \Delta_t, X_t, B_t, B_t^C, B_t^F, D_t, D_t^C, D_t^F, T_t^F, T_t^C\}$, satisfying each of the conditions in equations (A.25) to (A.38) at each time $t \geq t_0$ (and in each contingency at t) consistently with $T_t^F = \mathcal{T}^F(T_t^C, D_{t-1}^F, B_{t-1}^F, P_t, Q_t, \zeta_t)$, $T_t^C = \mathcal{T}^C(N_{t-1}^C, \Psi_t^C, \zeta_t)$ and

$B_t^C = \bar{B}_t^C, D_t^C = \bar{D}_t^C, D_t^F = \bar{D}_t^F$ that maximizes (A.39) given the definition $\Pi_t \equiv P_t/P_{t-1}$, the non-negativity constraint on the nominal interest rate $i_t \geq 0$, the stochastic processes for the exogenous disturbances $\{\xi_t, \varkappa_t, A_t, \mu_t\}$ and initial conditions $\mathbf{w}_{t_0-1}$.

To compute the optimal policy, we consider the associated Lagrangian problem maximizing (A.39) and attaching Lagrange multipliers $\lambda_{j,t}$ for $j = 1...15$ to the following constraints (which rewrite those above)

$$\xi_t Y_t^{-\rho} = \beta(1+i_t)E_t\left\{\frac{\xi_{t+1}Y_{t+1}^{-\rho}}{\Pi_{t+1}}\right\}$$

$$F_t = \mu_t(1+i_t)\xi_t\left(\frac{Y_t}{A_t}\right)^{1+\eta} + \alpha\beta E_t\left\{\Pi_{t+1}^{\theta(1+\eta)}\bar{\Pi}^{-\theta(1+\eta)}F_{t+1}\right\}$$

$$K_t = \xi_t Y_t^{1-\rho} + \alpha\beta E_t\left\{\Pi_{t+1}^{\theta-1}\bar{\Pi}^{1-\theta}K_{t+1}\right\}$$

$$\left(\frac{1-\alpha\Pi_t^{\theta-1}\bar{\Pi}^{1-\theta}}{1-\alpha}\right)^{\frac{1+\theta\eta}{1-\theta}} K_t = F_t$$

$$\Delta_t = \alpha\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\theta(1+\eta)}\Delta_{t-1} + (1-\alpha)\left(\frac{1-\alpha\Pi_t^{\theta-1}\bar{\Pi}^{1-\theta}}{1-\alpha}\right)^{\frac{\theta(1+\eta)}{\theta-1}}$$

$$i_t \geq 0$$

$$Q_t\xi_t Y_t^{-\rho} = E_t\left\{\beta\xi_{t+1}Y_{t+1}^{-\rho}\frac{(1-\varkappa_{t+1})(1+\delta Q_{t+1})}{\Pi_{t+1}}\right\}$$

$$m_t = Y_t$$

$$r_t Q_{t-1} = (1-\varkappa_t)(1+\delta Q_t) - Q_{t-1}$$

$$n_t^C = Q_t d_t^C - m_t - \tilde{x}_t$$

$$\xi_t Y_t^{-\rho}(t_t^C - \psi_t^C + n_t^C) = \xi_t Y_t^{-\rho} n_{t-1}^C \Pi_t^{-1}$$

$$\psi_t^C \Pi_t = i_{t-1}(n_{t-1}^C + m_{t-1}) + (r_t - i_{t-1})Q_{t-1}d_{t-1}^C$$

$$t_t^C = \bar{T}^C + \gamma_c\psi_t^C + \phi_c n_{t-1}^C \Pi_t^{-1}$$

$$Q_t d_t^F + \frac{1}{1+i_t}b_t^F = (1+r_t)Q_{t-1}d_{t-1}^F\Pi_t^{-1} + b_{t-1}^F\Pi_t^{-1} - t_t^F - t_t^C$$

$$t_t^F = \bar{T}^F - \gamma_f t_t^C + \phi_f\left[(1+r_t)Q_{t-1}d_{t-1}^F\Pi_t^{-1} + b_{t-1}^F\Pi_t^{-1}\right]$$

where lower-case variables denote the real counterpart of the upper-case variable, while $\tilde{x}_t \equiv (X_t - B_t^C)/(P_t(1+i_t))$.

The first-order conditions with respect to the vector $(Y_t, i_t, \Pi_t, K_t, F_t, \Delta_t, m_t, Q_t, r_t, t_t^C, n_t, \tilde{x},$

ψ_t^C, t_t^F, b_t^F) are respectively:

$$\begin{aligned}
0 &= \xi_t Y_t^{-\rho} - \xi_t Y_t^{\eta}\Delta_t A_t^{-(1+\eta)} - \rho\lambda_{1,t}\xi_t Y_t^{-\rho-1} + \lambda_{1,t-1}\rho(1+i_{t-1})\xi_t Y_t^{-\rho-1}\Pi_t^{-1} \\
&\quad -\lambda_{2,t}(1+\eta)\mu_t(1+i_t)\xi_t \frac{1}{Y_t}\left(\frac{Y_t}{A_t}\right)^{1+\eta} - \lambda_{3,t}(1-\rho)\xi_t Y_t^{-\rho} - \lambda_{7,t}\rho Q_t \xi_t Y_t^{-\rho-1} \\
&\quad +\lambda_{7,t-1}\rho\xi_t Y_t^{-\rho-1}\frac{(1-\varkappa_t)(1+\delta Q_t)}{\Pi_t} - \lambda_{8,t}
\end{aligned}$$

$$0 = \lambda_{1,t}\beta E_t\left\{\xi_{t+1}Y_{t+1}^{-\rho}\Pi_{t+1}^{-1}\right\} + \lambda_{2,t}\mu_t\xi_t\left(\frac{Y_t}{A_t}\right)^{1+\eta} - \lambda_{6,t} + \beta E_t\left\{\lambda_{12,t+1}\right\}(n_t+m_t-Q_t d_t^C) + \frac{\lambda_{14,t}}{(1+i_t)^2}b_t^F$$

$$\begin{aligned}
0 &= \lambda_{1,t-1}(1+i_{t-1})\xi_t Y_t^{-\rho}\Pi_t^{-2} + \lambda_{4,t}K_t\alpha\frac{1+\theta\eta}{1-\alpha}\left(\frac{1-\alpha\Pi_t^{\theta-1}\bar{\Pi}^{1-\theta}}{1-\alpha}\right)^{\frac{\theta(1+\eta)}{1-\theta}}\Pi_t^{\theta-2}\bar{\Pi}^{1-\theta} \\
&\quad -\lambda_{2,t-1}F_t\alpha\theta(1+\eta)\Pi_t^{\theta(1+\eta)-1}\bar{\Pi}^{-\theta(1+\eta)} - \lambda_{3,t-1}K_t\alpha(\theta-1)\Pi_t^{\theta-2}\bar{\Pi}^{1-\theta} \\
&\quad -\lambda_{5,t}\Delta_{t-1}\alpha\theta(1+\eta)\Pi_t^{\theta(1+\eta)-1}\bar{\Pi}^{-\theta(1+\eta)} + \lambda_{5,t}\alpha\theta(1+\eta)\left(\frac{1-\alpha\Pi_t^{\theta-1}\bar{\Pi}^{1-\theta}}{1-\alpha}\right)^{\frac{1+\theta\eta}{\theta-1}}\Pi_t^{\theta-2}\bar{\Pi}^{1-\theta} \\
&\quad +\lambda_{7,t-1}\xi_t Y_t^{-\rho}\frac{(1-\varkappa_t)(1+\delta Q_t)}{\Pi_t^2} + \lambda_{11,t}\xi_t\frac{Y_t^{-\rho}}{\Pi_t^2}n_{t-1}^C + \lambda_{12,t}\psi_t^C + \lambda_{13,t}\phi_c n_{t-1}^C\Pi_t^{-2} \\
&\quad +(\lambda_{14,t}+\phi_f\lambda_{15,t})[(1+r_t)Q_{t-1}d_{t-1}^F + b_{t-1}^F]\Pi_t^{-2}
\end{aligned}$$

$$0 = \lambda_{4,t}\left(\frac{1-\alpha\Pi_t^{\theta-1}\bar{\Pi}^{1-\theta}}{1-\alpha}\right)^{\frac{1+\theta\eta}{1-\theta}} + \lambda_{3,t} - \lambda_{3,t-1}\alpha\Pi_t^{\theta-1}\bar{\Pi}^{1-\theta}$$

$$0 = -\lambda_{4,t} + \lambda_{2,t} - \lambda_{2,t-1}\alpha\Pi_t^{\theta(1+\eta)}\bar{\Pi}^{-\theta(1+\eta)}$$

$$0 = -\xi_t\frac{Y_t^{1+\eta}}{1+\eta}A_t^{-(1+\eta)} + \lambda_{5,t} - \alpha\beta E_t\left\{\lambda_{5,t+1}\Pi_{t+1}^{\theta(1+\eta)}\bar{\Pi}^{-\theta(1+\eta)}\right\}$$

$$0 = \lambda_{8,t} + \lambda_{10,t} - \beta i_t E_t\lambda_{12,t+1}$$

$$\begin{aligned}
0 &= \lambda_{7,t}\xi_t Y_t^{-\rho} - \lambda_{7,t-1}\delta(1-\varkappa_t)\xi_t Y_t^{-\rho}\Pi_t^{-1} + \beta E_t\left\{\lambda_{9,t+1}(1+r_{t+1})\right\} - (1-\varkappa_t)\delta\lambda_{9,t} \\
&\quad -\beta E_t\left\{(\lambda_{14,t+1}+\phi_f\lambda_{15,t+1})(1+r_{t+1})d_t^F\Pi_{t+1}^{-1}\right\} - \lambda_{10,t}d_t^C - \beta d_t^C E_t\left\{\lambda_{12,t+1}(r_{t+1}-i_t)\right\} \\
&\quad +\lambda_{14,t}d_t^F
\end{aligned}$$

$$\lambda_{9,t}Q_{t-1} - \lambda_{12,t}Q_{t-1}d_{t-1}^C - (\lambda_{14,t}+\phi_f\lambda_{15,t})Q_{t-1}d_{t-1}^F\Pi_t^{-1} = 0$$

$$0 = \lambda_{11,t}\xi_t Y_t^{-\rho} + \lambda_{13,t} + \lambda_{14,t} + \gamma_f\lambda_{15,t}$$

$$0 = \lambda_{10,t} - \beta E_t\left\{\lambda_{11,t+1}\xi_{t+1}Y_{t+1}^{-\rho}\Pi_{t+1}^{-1}\right\} + \lambda_{11,t}\xi_t Y_t^{-\rho} - \beta i_t E_t\left\{\lambda_{12,t+1}\right\} - \beta\phi_c E_t\left\{\Pi_{t+1}^{-1}\lambda_{13,t+1}\right\}$$

$$\lambda_{10,t} = 0$$

$$\lambda_{12,t}\Pi_t - \lambda_{11,t}\xi_t Y_t^{-\rho} - \gamma_c \lambda_{13,t} = 0$$

$$\lambda_{14,t} + \lambda_{15,t} = 0$$

$$\frac{\lambda_{14,t}}{1+i_t} - \beta E_t \left\{ (\lambda_{14,t+1} + \phi_f \lambda_{15,t+1}) \Pi_{t+1}^{-1} \right\} = 0.$$

A.2.3 Solution Method

We study the optimal policy problem using linear-quadratic methods. We approximate, around a non-stochastic steady state, the objective welfare function to second order, and the models equilibrium conditions to first order. We solve and simulate the model using the piecewise-linear algorithm developed by Guerrieri and Iacoviello (2015): the approximated system of linear equations is treated as a regime-switching model, where the alternative regimes depend on whether specific constraints are binding or not. In particular, in our model there are two distinct constraints that may occasionally bind. The first one is the familiar zero-lower bound on the nominal interest rate, while the second is a non-negativity constraint that may affect central bank's remittances under some specifications of the transfer policies.

A.2.4 Derivation of equations (35), (36) and (37)

Here, we derive the second-order approximation of the welfare (A.39). The approximation is taken with respect to the efficient steady state. The latter maximizes (A.39) under the resource constraint of the economy. Since $\Delta_t \geq 1$, it is clear from (A.39) that in the efficient allocation $\Delta_t = 1$ and $\bar{Y}^{-\rho} = \bar{Y}^{\eta}$.

In the decentralized equilibrium instead $\mu \bar{Y}^{-\rho} = (1-\bar{\varphi})\bar{Y}^{\eta}$ as it is implied by equations (A.29) to (A.31). To implement the efficient allocation it is sufficient to set a subsidy such that $\mu/(1-\bar{\varphi}) = 1$. This requires setting the employment subsidy $\varrho \equiv 1 - (1-1/\theta)(1-\bar{\varphi})$. A second-order approximation of the utility flows in (A.39) around this steady state can be simply written as

$$\begin{aligned} U_t &= \bar{U} + \bar{Y}^{-\rho}\left[(Y_t - \bar{Y}) - \frac{\rho}{2}\bar{Y}^{-1}(Y_t - \bar{Y})^2\right] - \bar{Y}^{\eta}\left[(Y_t - \bar{Y}) + \frac{\eta}{2}\bar{Y}^{-1}(Y_t - \bar{Y})^2\right] - \\ &\quad - \frac{1}{1+\eta}\bar{Y}^{1+\eta}(\Delta_t - 1) + \mathcal{O}(||\xi||^3) \end{aligned}$$

since Δ_t is already a second-order term. The above expansion can be further simplified to

$$U_t = \bar{U} - \frac{1}{2}(\rho+\eta)\bar{Y}^{1-\rho}\hat{Y}_t^2 - \frac{1}{1+\eta}\bar{Y}^{1-\rho}(\Delta_t - 1) + \mathcal{O}(||\xi||^3) \tag{A.40}$$

in which we have used

$$Y_t = \bar{Y}\left(1 + \hat{Y}_t + \frac{1}{2}\hat{Y}_t^2\right) + \mathcal{O}(||\xi||^3)$$

where $\hat{Y}_t \equiv \ln Y_t/\bar{Y}$. Note that the welfare function (A.39) can be written as

$$W_{t_0} = E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\xi_t U_t$$

whose second-order approximation is simply

$$W_{t_0} = E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}[\bar{U} + (\xi_t - 1)\bar{U} + (U_t - \bar{U}) + (\xi_t - 1)(U_t - \bar{U})] + \mathcal{O}(||\xi||^3)$$

where we have normalized the steady-state of ξ_t to 1. From (A.40), note that $U_t - \bar{U}$ is a second-order term, it follows that the second-order approximation of the welfare can be simply written as

$$W_{t_0} = -E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\bar{Y}^{1-\rho}\left[\frac{1}{2}(\rho+\eta)\hat{Y}_t^2 - \frac{1}{1+\eta}(\Delta_t - 1)\right] + \text{t.i.p.} + \mathcal{O}(||\xi||^3)$$

since $\bar{U}$ and $(\xi_t - 1)\bar{U}$ are terms independent of policy which enters in t.i.p.

Finally, by taking a second-order approximation of (A.32), and integrating appropriately across time, we obtain

$$\sum_{t=t_0}^{\infty}\beta^{t-t_0}\hat{\Delta}_t = \frac{\alpha}{(1-\alpha)(1-\alpha\beta)}\theta(1+\eta)(1+\eta\theta)\sum_{t=t_0}^{\infty}\beta^{t-t_0}\frac{(\pi_t - \bar{\pi})^2}{2} + \text{t.i.p.} + \mathcal{O}(||\xi||^3).$$

We can therefore write

$$W_{t_0} = -(\rho+\eta)\bar{Y}^{1-\rho}\cdot\frac{1}{2}E_t\left\{\sum_{t=t_0}^{\infty}\beta^{t-t_0}[\hat{Y}_t^2 + \lambda_\pi(\pi_t - \bar{\pi})^2]\right\} + \text{t.i.p.} + \mathcal{O}(||\xi||^3)$$

from which the loss function (35) follows, and where we have defined

$$\lambda_\pi \equiv \frac{\theta}{\kappa},$$

$$\kappa \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\frac{(\rho+\eta)}{(1+\eta\theta)}.$$

Log-linear approximation of the equilibrium conditions A first-order approximation of (A.19) delivers

$$\rho E_t(\hat{Y}_{t+1} - \hat{Y}_t) = \hat{\imath}_t - r_t^n - E_t(\pi_{t+1} - \bar{\pi})$$

in which $\hat{\imath}_t \equiv \frac{i_t - \bar{\imath}}{1+\bar{\imath}}$ and

$$r_t^n \equiv \ln \xi_t - E_t \ln \xi_{t+1} + \rho \frac{1+\eta}{\rho+\eta}(E_t \ln A_{t+1} - \ln A_t)$$

while a first-order approximation of (A.29) to (A.31) implies the AS equation

$$\pi_t - \bar{\pi} = \kappa \hat{Y}_t + \zeta \hat{\imath}_t + \beta E_t(\pi_{t+1} - \bar{\pi}) + u_t$$

in which the parameter ζ is defined as $\zeta \equiv \kappa(1+\bar{\imath})/(\rho+\eta)$ where $\bar{\imath}$ is the steady-state level of the nominal interest rate.