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* Views expressed are those of the authors and do not necessarily reflect official positions of De Nederlandsche Bank.

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Global models for a global pandemic: the impact of COVID-19 on small euro area economies

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Abstract

We analyse the COVID-19 pandemic shock on small open economies (SOEs) in the euro area in a unified modelling framework: the Euro Area and the Global Economy model. We find strong negative international spillovers affecting each of the modelled SOEs, stemming not only from the rest of the euro area, but also from the United States and the rest of the world. A lower bound on nominal interest rates in the euro area amplifies these spillovers, especially within the euro area. Furthermore, we find some positive spillovers from the fiscal measures implemented in the Euro area to combat the pandemic, including the new Next Generation EU instrument.

JEL Classification: C53, E32, E52, F45.

Keywords: *DSGE Modelling, International Spillovers, Monetary Union, Euro Area, COVID-19*

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1 Introduction

The COVID-19 pandemic produced an unforeseen global crisis. Faced with this health versus recession trade-off, most governments decided to lock down economies and therefore shut down most non-tradable sectors, but also temporarily tradable sectors. When an economy is small and very open, the immediate impact of such action is worsened by the existing large inter-relations across countries and trade linkages. This mechanism highlights the importance of understanding the role of international spillovers and the transmission of global shocks to domestic economies. The question is even more acute for countries in a monetary union, where a common monetary policy and local fiscal reactions are typical. To alleviate the effects of the shock, governments typically reacted by running large fiscal stimuli that can also spill over to the main trade partners. Of particular interest in this respect is the novelty in the measures taken in response to the COVID-19 pandemic, which was the implementation of an extraordinary fiscal package at the EU level, the so-called the Next Generation EU (NGEU), aiming at supporting countries facing large public deficits.

The existing literature on the transmission and the macroeconomic effects of the pandemic has mainly focused either on closed economy models, abstracting from the analysis of the international spillovers of the shock itself and the policies adopted to counteract its effects (see Eichenbaum et al. (2020), Guerrieri et al. (2020)), or on models for a monetary union with two regions that are symmetric in size (Bartocci et al., 2020). Little is known as regards the impact of pandemic shocks on small open economies (henceforth SOEs) that are part of a monetary union. Their stronger dependence on intra- and extra-union trade makes them more vulnerable to fluctuations in global demand. At the same time, their higher degree of openness, compared to that of larger member states, makes them more vulnerable to fluctuations in the terms of trade and/or exchange rate fluctuations triggered by the first and second order effects of a pandemic shock. For the same reason, such countries can be expected to be more susceptible to the effects of fiscal policy measures taken by their trading partners, which implies that such spillovers carry more weight for policymakers than in large and less open countries. This paper fills the gap in the literature by analysing the effects of the COVID-19 pandemic shock on small open economies in a monetary union, namely the euro area.

We focus on the size and the direction of the international spillovers from the shock itself, and from the ensuing fiscal response. We use the Euro Area and Global Economy (EAGLE) model (Gomes et al., 2012) which is a dynamic general equilibrium model (DSGE) of the euro area within the global economy. Featuring a detailed trade matrix, tradable and non-tradable sectors, the EAGLE provides a rich environment to assess the international spillovers of the COVID-19 pandemic. We calibrate the model for a number of small open economy member states, namely Ireland, Luxembourg, the Netherlands and Slovenia (and, for the purpose of comparison with a larger economy within a monetary union, also to Germany).[1] Note that we use a fully unified modelling framework, which is in contrast to what has typically been the case in cross-country comparisons using DSGE models (e.g., Kilponen et al. (2019), Coenen et al. (2012)). This implies greater comparability between simulations in the paper and is its main advantage compared to similar studies.

We find significant spillover effects in each of the modelled SOEs, either from within the EA or form the outside of the EA. The rest of the EA (REA) represents the largest trading partner of all modelled SOEs. Consequently, the direct international spillover effects mostly stem from this region (intra euro area effects). We find that in the case of the binding effective lower bound (ELB), these intra-EA spillovers become substantially stronger than in normal times. However, we also find large international spillover effects that arise from other two foreign regions, either the rest of the world (RW) or the US, and which transmit to the domestic SOE both directly and via the REA (extra euro area effects).

We also assess fiscal spillovers, as such an environment provides a textbook example of a large-scale countercyclical fiscal policy intervention.[2] To maintain comparability, we run the same set of shocks across models (even though each SOE had its own fiscal response), which is based on

[1] We pick the economies that are small, yet diverse in the structure of their economies (e.g., in terms of trade direction we have economies with more extra-EA trade and more intra-EA trade; in terms of production more manufacturing-oriented and more service-oriented; old members very integrated into the EA and recent joiners). This covers practically all cases and helps us to explain why some channels are more important for some countries than for the others. At the same time, it serves as a robustness check for our findings.

[2] A strand of the literature covers the effects of fiscal shocks on business cycles. We mention only a few studies here. Caggiano et al. (2015) study the state dependent fiscal multipliers of the US economy. Cugnasca and Rother (2015) investigate the impact of fiscal consolidation and multipliers in the EU. Kilponen et al. (2019) estimate output multipliers for alternative fiscal instruments by simulating 15 structural models within the EA. Recently, Arigoni et al. (2023) provide an analysis of different fiscal shocks, while studying the multiplier effects in the case of a large-scale fiscal stimulus package in a larger economic crisis, such as the COVID-19 pandemic.

the EA fiscal response. The results not only indicate significant effects arising from the domestic fiscal stimulus but also highlight spillovers from the fiscal stimulus in REA. Furthermore, these spillovers are magnified when the monetary policy authority in the Euro area faces an ELB on nominal interest rates.

In addition, we explore the robustness of the findings to a number of modelling features, based on country-specific model versions. Among these are the Blanchard-Yaari structure, cross-border workers, search frictions, and import content of government expenditure. Moreover, we also analyse the macroeconomic effects of the recently approved Next Generation EU program. Our simulations show the importance of the instrument used to finance the EU budget.

This paper relates to the growing literature analysing the macroeconomic effects of the pandemic. Angelini et al. (2020) estimate the effects of the COVID-19 pandemic by combining two models: the large-scale semi-structural ECB BASE model (Angelini et al., 2019) and the SIR (Susceptible-Infectious-Recovered) model that incorporates predictive dynamics, based on the Kermack and McKendrick (1927) mathematical epidemiological model. Eichenbaum et al. (2020) and Toda (2020) also adopted the SIR model approach. They find that people's decision to cut back on consumption and work reduces the severity of the epidemic, but negatively contributes to the size of the resulting recession. On the empirical side of the literature, Barro et al. (2020) and Correia et al. (2020) compare the effects of the COVID-19 pandemic with the so-called "Spanish flu" pandemic in the 1918-1920, while Jordà et al. (2020) compare it to the "Black Death" pandemic. Aiyar et al. (2022) examine trade spillovers from national lockdowns, Bergant and Forbes (2023) examine the policy responses, including fiscal policy, and the role of constraints, while Altavilla et al. (2023) focus on monetary policy. From the modelling perspective, few studies attempt to use DSGE models to analyse the COVID-19 pandemic.[3] Eichenbaum et al. (2020) take a simpler DSGE model and combine it with a SIR model, while Mihailov (2020), building on Galí et al. (2012) assesses the macroeconomic effects of the COVID-19 lockdown based on three scenarios in the United States, Germany, France, Italy and Spain. Several papers propose simulating the COVID-19 shock as a combination of existing structural shocks, which is an approach we adopt. These are Primiceri and Tambalotti (2020), Gomme

[3] For empirical estimates of the COVID crisis see Chudik et al. (2020), using a threshold multi-country model, or Kohlscheen et al. (2020), using a GVAR. Both show the importance of spillovers.

(2020), and Bartocci et al. (2020).

Our contribution to the literature is threefold. First, a common model setup calibrated to various SOEs. To the best of our knowledge this is the first paper that analyses the macroeconomic effects of the pandemic on SOEs in a monetary union setting while at the same time disentangling the intra and extra euro area effects, given the multi-region nature of the model at hand. Second, we exploit detailed international and fiscal environments allowing to properly measure international and fiscal spillovers. Third, we provide an original manner to replicate the first wave of the COVID crisis by exploiting the open economy features in the dynamic stochastic general equilibrium models.

The paper is structured as follows. Section 2 presents the scenario of the first wave of the COVID-19 pandemic outbreak. Section 3 presents the common modelling environment used in comparisons and the country specific calibrations of key parameters. Section 4 reports the results from the benchmark calibration as well as the various spillovers, international and fiscal, from within the euro area as well as from the regions outside. Section 5 discusses the robustness of the results to country specific versions of the model and an extension accounting for Next Generation EU (NGEU) funds. Section 6 concludes.

2 Crisis scenario and fiscal measures

Using the approach of Primiceri and Tambalotti (2020), Gomme (2020), and Bartocci et al. (2020), among others, we model the COVID-19 shock as a mixture of structural shocks. Here we provide the rationale for the chosen shocks and align the shocks with the empirical evidence.

2.1 Stylized facts

The global nature of the COVID-19 pandemic and the sharp decrease in international trade suggests important international spillovers. We first look at the global growth dynamics of output and CPI inflation in the main global regions, the EA, the US and the RW in the top panels of Figure 1. Real GDP growth in all regions collapsed in the second quarter of 2020. Changes in CPI inflation were also sudden and large.

The middle panels in Figure 1 show the main macroeconomic variables in the EA. Unlike during the global financial crisis (GFC), it was aggregate consumption that plummeted in the second quarter of 2020. The substantial drop of investment is also worth noting. This lockdown-induced decline is the main cause for the fall in GDP. At the sectoral level, unlike during the GFC, the decline in non-tradable output is larger than its tradable counterpart, since the non-tradable sector is primarily represented by services, which were most affected during the lockdown.

At the country level, similar patterns emerge. The bottom two panels in Figure 1 compare the EA dynamics with those in Ireland, Luxembourg, the Netherlands and Slovenia. CPI inflation and output in all these economies feature similar paths. The negative effects of the COVID-19 pandemic are similar to those in global regions shown in the top panels of the figure.

These observations provide the rationale to use the global model, since its theoretical structure and a large variety of shocks will allow us to model the international dimension of the pandemic.

2.2 Data treatment

The COVID-19 pandemic shock is not a standard shock built in macroeconomic models, which is why we construct the COVID-19 shock as a combination of a set of standard structural shocks that span supply and demand. Specifically, we consider:

1. Negative preference shocks. By reducing the weight households place on current utility from consumption relative to future one, this shock generates an increase in savings. In addition, a negative preference shock can also be viewed as a proxy for the increase in uncertainty caused by the pandemic, which would also reduce households' desire to consume.[4]
2. Reduction in habit formation. Lockdowns strongly and suddenly affected typical consumption, which we model by a temporary reduction in the consumption habit of households. This allows us to generate a fast an strong reduction in consumption, which is unusual in the data, but has been observed at the beginning of the pandemic.
3. Negative investment technology shocks. The economy becomes less efficient at transforming the flow of investment into the stock of capital, triggering a collapse in real investment.[5]
4. Cost-push shocks to tradable and non-tradable goods. These supply side shocks help us in

[4]This is, however, still an imperfect and, strictly speaking, a reduced-form proxy for the effects of uncertainty that one would observe in a model solved using global methods.

[5]The negative investment technology shock can also be viewed as a proxy for financial effects of the pandemic.

two respects. First, they offset the strong downward pressure on inflation from the demand shocks, allowing for a better approximation of observed inflation dynamics. Second, they allow tradable prices to fall more than non-tradable prices, as in the data.

5. Reduction in non-tradable inputs in the consumption basket. This shifts consumption away from non-tradable goods (consistently with the effects of the lockdown) and at the same time lowers tradable output less than non-tradable output, as observed in the data.[6]

The second key factor concerns the propagation of the COVID-19 pandemic over time. Most institutions did not predict a very fast recovery. For example, in 2021, projected 2022 world GDP standed 4% lower than its pre- COVID-19 projection (IMF, 2021). In line with this evidence, we allow our synthetic COVID shock extend over the first four quarters of the simulation.

The third ingredient in the procedure is the size of the synthetic shock, which we calibrate using the following procedure:

- We start with the 2020 economic forecasts prepared in autumn 2019. These projections are not polluted by the pandemic, as they date from before the first coronavirus outbreak.
- We then compare this vintage of forecasts with the ones prepared in spring 2020, once the COVID-19 pandemic had ravaged the world and its economies and assume that the differences between those projections are only due to the pandemic and the fiscal response.[7]

We focus on the key macroeconomic variables for which different forecast vintages are readily available, namely the annual growth rate of GDP, private consumption, private investment, and the GDP deflator. In the RW bloc, we limit our attention to the annual growth rate of GDP. Table 1 presents our empirical targets. It shows that economies across the world experienced steep falls in activity and significant declines in prices. Such co-movement across regions and variables suggests a dual nature for the COVID-19 shock: it affects both supply and demand in an interconnected environment. Note that forecast vintages on sectoral prices and quantities are not readily available and comparable across countries. Therefore, we calibrate the sectoral shocks to be consistent with the following outcomes: (i) the fall of domestic tradable prices is twice as large as that of non-tradable prices; (ii) the decline of domestic tradable output is half

[6]The Netherlands is an exception, as tradable output fell by approximately the same as nontradable output.

[7]This is the same assumption underlying narrative identification strategies and event studies (Zeev (2018), and Antolín-Díaz and Rubio-Ramírez (2018))

as large as that of non-tradable output. These dynamics are common to all economies.[8]

The 2020 Spring Forecast incorporates fiscal packages, which must be taken into account. However, incorporating these measures is not a straightforward task due to the lack of clear data on fiscal measures and the absence of model counterparts for certain policy actions. It is important to note that our analysis focuses on the first wave of the pandemic, during which many fiscal measures were either not yet implemented or not fully used by firms and households, particularly in the case of public guarantees.

To ensure comparability across countries, we have chosen to express the fiscal measures in terms of direct government spending and transfers to households. Specifically, we assume that both the domestic economy and the rest-of-the-euro-area bloc implement identical fiscal measures. These measures include a reduction in labor taxes paid by households amounting to 4.5 percentage points, which translates to transfers to households equivalent to approximately 2% of the ex-ante GDP, and an increase in government consumption amounting to 2.2% of the ex-ante GDP.

Finally, we assume that monetary policy in the euro area is constrained by the ELB during the first three years of the simulation.

3 Modelling environment

3.1 Models setup

All simulations are based on the Euro Area and the Global Economy (EAGLE) model. This is a multi-country dynamic general equilibrium model of the euro area developed by a team from the Bank of Italy, Bank of Portugal and the ECB (Gomes et al. (2010) and Gomes et al. (2012)) and extended with import-content of exports by Brzoza-Brzezina et al. (2014). Similarly to the ECB's New Area Wide model (NAWM, Coenen et al. (2008)) or the IMF's Global Economy Model (GEM, Laxton and Pesenti (2003)), the EAGLE is micro-founded and features nominal price and wage rigidities, capital accumulation, international trade in goods and bonds. The EAGLE is a global extension of the NAWM and shares the same theoretical setup. The introduction of

[8] For example, in Luxembourg, the services component of HICP fell 1% in 2020 relative to its long run average, while the goods component fell 2.1%. In contrast, hours worked in services during the first half of the year declined by 6.5% relative to their long run average, while hours worked in the tradable sector only declined by 3.3%.

tradable and non-tradable sectors, the monetary union and the enhanced fiscal bloc are the main differences with the original NAWM.

The central bank sets the domestic short-term nominal interest rate according to a standard Taylor-type rule, by reacting to increases in consumer price inflation and real activity, both defined at the euro area level. The US and the RW have their own nominal interest rates and nominal exchange rates. On the fiscal dimension, each region in the model has its own fiscal authority responsible for generating revenue through taxes imposed on the private sector and seigniorage from monetary creation. The model incorporates various tax instruments, including a VAT-like tax on consumption, taxes on labor and capital income. The revenue generated from these taxes is used to acquire the final government good and to finance transfer payments to households. Any resulting public debt is financed by government bonds. To maintain fiscal stability, a gradual fiscal rule is in place to ensure convergence towards a desired long-term debt-to-GDP ratio.

3.2 Calibration

Table 2 reports the implied great ratios at the steady state and shows that trade is the main source of heterogeneity across countries. Shares of domestic demand components in nominal GDP are broadly in line with the noticeable exception of LU where the consumption share represents around 35%, almost half the share in other countries. Private investment and public expenditures are around 20% of GDP. Large differences come from import shares: quite low for NL (below 25%), extremely high for LU (160%) while IE and SI are in the middle (75% and 70%, respectively).[9] For all countries, import content is larger for investment goods than for consumption goods. The import content of exports is also significantly larger in LU compared to other blocs (130% against 35% for IE and SI), reflecting the larger degree of openness of the Luxembourgish economy. On the fiscal side, discrepancies are more muted. VAT revenue is between 16% (LU) and 19% (SI) of GDP. The tax burden is generally larger on workers (labour

[9] It is widely known that the Netherlands has a large import sector. However, the biggest bulk of imports is repackaged for re-export and is hence not consumed domestically. The reported percentage refers to imported goods (consumption and investment goods) that are consumed domestically.

income taxes and social contributions) than on firms, except for SI (around 13% against 14%). A row of the table shows region sizes expressed as shares of world GDP.

Table 2 also provides an overview of key structural parameters, which adhere to standard values and are largely based on Gomes et al. (2012), which are based on the estimated New Area-Wide Model (Coenen et al. (2008)) and the Global Economy Model of the IMF (Laxton and Pesenti (2003)). Country calibrations are based on the existing country-specific papers (Bolt et al. (2019), Clancy et al. (2016), Moura and Lambrias (2018)). All countries share the same household preferences specification: log utility function separable in consumption and leisure. The quarterly discount factor is set to imply an annualised steady-state real interest rate about 3%, the Frisch elasticity equal to 0.50 and the habit persistence parameter between 0.6 and 0.7. The share of rule-of-thumb households is 25% in all economies. On the supply side, the production technology for intermediate goods is identical (Cobb-Douglas with capital and labour). In the final-goods production technology (CES), substitutability between domestic and imported tradable goods is much higher than that between tradable and non-tradable goods, in line with the empirical literature. The elasticity of substitution between tradable and non-tradable goods is equal to 0.5 against 2.5 (1.5 for LU) for the elasticity between domestic and imported tradable goods. Mark-ups in the tradable goods sector (proxied by the manufacturing sector, 50%) are always lower than those in the non-tradable goods sector (proxied by the services sector, 20% or 30%), while mark-ups in the labour market are set around 30%. Lastly, real, price and wage rigidities are in the same ballpark, as shown at the bottom of the table.

4 Benchmark simulations

4.1 Baseline scenario

Figures 2 and 3 present the dynamic effects of the synthetic COVID-19 shock in Ireland, Luxembourg, the Netherlands and Slovenia. For comparison, we also include Germany as an example of larger, but still open economy in the euro area. Our baseline scenario provides a suitable setup for measuring international spillovers, as it captures the strong and persistent contraction observed in the data. We first explain the main transmission channels that underlie all dynamic responses, and then we turn to country-specific issues.[10]

Three mechanisms shape these dynamic responses. The leading force is the combination of domestic demand shocks. Specifically, the negative preference shocks cause households to postpone consumption, as the weight they place on current utility falls. This, together with the reduction in habit formation, leads to an almost immediate collapse in aggregate consumption, in line with what has been observed during the first stage of the lockdown. In addition, the negative investment shocks make the economy less productive at transforming investment into capital, thereby penalizing investment, which also falls markedly.[11] These three shocks lead to a steep decline in aggregate demand, labour demand, wages, marginal costs and inflation.[12] The second important mechanism is international trade. As the global economic activity collapses, so do exports from domestic economies to the REA and beyond. Imports fall due to the aforementioned fall in aggregate demand. The net effect of external trade on domestic economy depends on which of the two declines prevails, but for most economies (except Luxembourg), the reduction in exports is not offset by the decline in imports and therefore compounds the damages caused by domestic shocks, as all four domestic economies are very small and open (see Table 2). The third mechanism is the ELB, which interacts with inflation. Two competing forces shape inflation dynamics. On the one hand, the fall in aggregate demand pushes inflation downwards. On the other hand, the cost-push shocks to tradable and non-tradable goods push inflation upwards. Following the calibration of the synthetic COVID shock, the demand effect dominates, so that inflation falls. This raises the real interest rate, because monetary policy is constrained by the ELB in the short run. The rise in the real rate slows spending through an inter-temporal substitution effect (decline in the present discounted value of wealth), depressing activity further.

[10]Note that we analyze the impact of the Covid shock using deterministic simulations (i.e., the households do not expect the shock to occur, but once it has occurred, they know the future path of the shock). We believe this is a more suitable approach than employing a linearized model around a steady state, which is appropriate only for small shocks. Additionally, using a first-order perturbation method would have made it impossible to model the ELB on monetary policy. Moreover, we assume that the pandemic does not permanently alter the underlying structural parameters of the model, except temporarily those that we shock (habit formation, share of nontradables in the consumption bundle).

[11]The decline in investment leads to a decline in the stock of capital, which takes a long time to recover and therefore increases the persistence of the shock. Lower stock of capital and therefore lower aggregate supply is also the main reason why inflation increases and overshoots after the demand shocks cease. Admittedly, the overshooting is not large, but it is persistent and lasts until the capital stock recovers.

[12]In the pandemic shock, hours worked have fallen more than employment. The harmonised model we use features effective labour, whose counterpart in the real world would be the product of the number of workers employed and hours worked.

The nominal exchange rate of the euro area vis-a-vis the US and the RW appreciates due to the binding ELB in the euro area.[13] This reduces the extra-EA exports of all the countries involved. The negative effect of the nominal exchange rate appreciation is reduced by domestic price reduction (which to some extent mitigates the real exchange rate appreciation). This is most obvious if one focuses on the implied real exchange rate appreciation in each country (see figure 3), which is the lowest in Ireland, where inflation falls the most, and the strongest in Slovenia with relatively rigid prices. This strong real exchange rate appreciation, along with very high share of imported investment goods and low investment-adjustment costs that reflect high volatility of investment in the past data, is the reason for the strong decline in investment in Slovenia. Note that flexible prices in Ireland also come at a cost with the binding ELB, which is why the real interest rate in Ireland increases by the most, resulting in a stronger drop in consumption than would be warranted by the decline in GDP alone.

Germany, as an example of a larger economy within the monetary union, fares no better than the four small open economies discussed above. The main reasons are that it is also an open economy that depends on exports, but also that the ECB, which would at least to some extent react to developments in a major European economy, cannot lower interest rates due to the ELB. Germany's size, and therefore its larger weight in the EA aggregate, have thus no impact on monetary-union-wide interest rate setting in the presence of the ELB. Importantly, note that Germany experiences a milder increase in its debt-to-gdp ratio, while German exports overshoot 5 quarters after the initial impact of the Covid shock. Both of these factors speed up its recovery.

All told, our baseline simulation generates plausible paths for quantities and prices based on standard economic mechanisms. To see this point more clearly, the last two columns of Table 1 compare the main empirical targets with the model's ability to match them. Obviously, not all items are exactly on target, but broadly speaking, the baseline simulation is in the right ballpark.

[13]This is in line with the actual behavior of the USD/EURO spot exchange rate. Looking at exchange rate spot data, the euro appreciated vis a vis the USD by 6% in the third quarter of 2020 compared to its level in the last quarter of 2019, and by 8% approximately in the fourth quarter of 2020.

4.2 International spillovers

We now analyse international spillovers from the three foreign blocs to the four domestic economies. Specifically, we assess how domestic output and inflation would have responded to the COVID shock if either the REA, the US or the RW had been immune to the pandemic. To this end, we first define the following statistic:

$$\mu(i,t)^x = X_t^{Bas} - X_t^i. \tag{1}$$

Here X_t^{Bas} is the variable of interest (e.g., Dutch output) at time t in the baseline scenario, and X_t^i is its counterfactual in a hypothetical scenario where bloc $i = \{REA, US, RoW\}$ is not hit by the COVID shock. Both X_t^{Bas} and X_t^i are expressed in either percentage deviations or percentage differences from the deterministic steady state. For each country, we compute and display the spillovers under two cases, namely one with a binding ELB on the interest rate in the euro area and another where the ELB constraint is absent.

Figures 4 - 5 show in the left column that international spillovers matter for both output and inflation. They account for a significant portion of the decline in economic activity in all domestic economies, especially in the first year and in particular in the case where the ELB is binding. There are some differences across the countries, which we discuss below and explain the mechanisms that cause the differences.

Four are three key channels that govern international spillovers in all the economies we analyse. First, there is a direct demand channel. Even if prices remain unchanged, a fall in the economic activity and demand of the main trading partner has a detrimental effect on the economy that exports to this trading partner. This means that the differences in the trade orientation of countries matter. Second, the reaction of prices, and in particular the exchange rate in the case of extra-monetary union trade, can either mitigate or alleviate the consequences of the decrease in foreign demand. Here the binding ELB in the euro area plays an important role, as it causes the euro to appreciate and hence works in a negative direction. Trade direction therefore matters more when the monetary union is bound by the ELB. Finally, when there is a binding ELB in the monetary union, a shock that lowers inflation leads to an increase in the real interest rate, which magnifies the recession, which exacerbates the spillovers within the monetary union. We now turn to illustrate these channels in particular countries.

For **Ireland** (top half of Figure 4), the spillovers reflect the structure of its foreign trade, which is much more open to non-EA regions. Accordingly, in the normal case without the binding ELB, the spillovers from the RW, which includes the UK, are the strongest, amounting to a bit less than 1 p.p. of GDP (i.e., Irish output would drop by 1 p.p. less if there was no shock in the RW), followed by the US (0.5 p.p. of GDP), and the REA (0.2 p.p. of GDP). In the case with the ELB, all spillovers become considerably stronger and this is particularly the case for the spillovers from the rest of the euro area, which become the strongest at about 1.5 p.p. of GDP. Inflation spillovers follow a similar pattern. The structure of foreign trade should be considered together with the movements in relative prices. When there is an ELB for the euro area, but not for the US and the RW, the euro appreciates, because the non-euro blocs are able to lower their interest rates. If the US or the RW are not hit by the shock, the real exchange rate for Ireland appreciates less, which implies that exports to regions not affected by the shock are less affected by the relative price (in addition to more resilience in the quantity demanded). This exchange rate channel is much less powerful in trade with the REA, because the nominal exchange rate is fixed and the real exchange rate only reflects relative changes in goods prices, which are sluggish due to sticky prices. However, with the presence of the ELB, the recession in the REA is stronger, so that quantity demanded drops by more in the case of the shock, which is why in this case spillovers from the REA become stronger despite fixed nominal exchange rate. The appreciation of the nominal exchange rate lowers inflation directly due to lower import prices. It also reduces foreign demand and therefore domestic production and labour demand, resulting in lower wages and lower marginal costs, which further lower inflation.[14] This is why, when there is no shock in the RW (to which Ireland is most exposed) or the US, the drop in inflation in Ireland is lower (by less than 0.5 p.p. without the ELB and about 1 p.p. with the ELB due to the euro appreciation). For trade with the REA, the nominal exchange rate effect is absent. However, the global shock causes an appreciation of the euro even if the shock is not present in the REA in the case of the binding ELB, inducing a recession in the entire monetary union, which magnifies the spillover of inflation.

[14] Wages in Ireland are relatively flexible compared to other countries (even though not for all types of employees, see Lydon and Lozej (2018)), but this is also the case for prices, so that real wages fall only gradually.

In **Luxembourg**, we observe remarkably strong negative spillovers. For example, without any shocks to the rest-of-the-euro-area block, the initial decline in output in Luxembourg is 4.4% instead of 7.1%. These large spillovers can be largely attributed to Luxembourg's high export-to-GDP ratio, which stands close to 190%. It is worth noting that the US contributes the weakest spillovers, as expected, given that US exports account for approximately 4% of Luxembourg's total exports. On the other hand, over the three years following the COVID shock, spillovers from the rest-of-the-euro-area are the strongest. This finding is not surprising since the rest-of-the-euro-area is Luxembourg's largest trading partner. As for inflation, Figure 4 illustrates significant spillover effects and provides an interesting insight. In the absence of substantial output spillovers, the decrease in aggregate demand within Luxembourg is insufficient to counterbalance the inflationary pressures stemming from negative supply shocks, particularly cost-push shocks. As a result, Luxembourg experiences higher inflation in the absence of foreign shocks. Lastly, similar to the other SOEs, the ELB magnifies international spillovers to Luxembourg, as discussed in the preceding paragraph.

For the **Netherlands** (top half of Figure 5), although the differences are small, the paradox of a slightly deeper contraction in output is observed when the REA is not hit by the global shock, in the absence of an ELB constraint. This is triggered by the persistent real effective exchange rate appreciation in the medium-run under this counterfactual scenario, which offsets any potential gains from the short-lived depreciation initially. Given the strong dependence of the Dutch economy on international trade, the persistent real effective exchange rate appreciation harms its exporting sector, and thus output. Therefore, it seems that the REA, when hit by the global shock, acts as an absorber which allows for a milder medium-run real effective exchange rate appreciation in the benchmark scenario. As a result, the exporting sector is hit less in the benchmark economy than in the counterfactual scenario, which leads to a milder contraction. As regards the contribution of the other regions, the US seems to have a non-negligible spillover in the Dutch economy. The spillover from the US economy amounts to approximately 0.6 p.p. of GDP (i.e., Dutch output would drop by 0.6 p.p. less if there was no shock in the US) in the absence of an ELB and to slightly above 1% under ELB. The spillover from the US is almost double the spillover from the RW, in general. The spillovers from the US are stronger in the presence of ELB owing to the appreciation of the Euro vis a vis the dollar. A similar pattern is observed at the spillovers from RW. As regards inflation, the persistent appreciation of the real effective exchange rate in the medium-run, in the absence of ELB, when the global shock is absent in the REA leads to a deeper decline in inflation in the first year. Obviously, the amplified response of inflation is further fuelled by the sharper decline in output in the export sector. Spillovers to inflation from the other two regions are relatively small in the absence of ELB. In the presence of ELB the spillovers from REA and the US to inflation rise and are non-negligible, again due to the appreciation of the euro vis a vis the dollar.[15] As regards the RW, its spillovers to inflation in the Netherlands are negligible.

For **Slovenia** (lower half of Figure 5), as expected, the largest international spillover arises from its biggest trading partner, the REA, amounting up to 1.7 p.p. of GDP if the ELB is binding. This is followed by the 1.5 p.p. of GDP spillover from RW and the 1.3 p.p. of GDP spillover from US, thus reflecting the structure of foreign trade of Slovenia. Removing the ELB leads to significantly lower international spillovers compared to the binding ELB case as the binding ELB leads to the appreciation of the euro vis a vis the dollar, dampening exports from REA and Slovenia to US and RW, thus decreasing output in REA and consequently in Slovenia further. On the other hand, without the ELB the ECB reacts to the shock by lowering the nominal (and real) interest rate, which effectively stimulates private consumption and exports in Slovenia and in the REA. What is noteworthy as well is that in contrast to the binding ELB case, the largest spillover effect on impact now stems from RW and US, while the spillover from the REA stays negligible due to reasons elaborated above.

4.3 Fiscal spillovers

We now explore the role of fiscal policy in shaping dynamic responses to the synthetic COVID shock, and we consider the cases where the ELB is binding and when it is not. This feature could be important, given that it has been argued in the literature that fiscal policy can have asymmetric effects depending on whether the economy is in a recession or not (e.g., Auerbach

[15] Note that the appreciation of the euro vis a vis the dollar dampens exports of REA to the US and thereby its output further, leading ultimately (as a second round effect) to a drop in demand for Dutch goods from REA as well.

and Gorodnichenko (2012) and Ghassibe and Zanetti (2022)). Note that in our case, we only have a recession, but in the presence of the ELB the recession is stronger, so that we only distinguish between more and less severe recessions. As explained below, we mostly find only minor differences in the strength of the fiscal spillovers when the ELB binds. As in the previous subsection, we compare the paths of output and inflation in the baseline scenario with their counterparts in a hypothetical scenario where either the domestic economy or the rest-of-the-euro-area does not implement any fiscal measure.

Figures 4 - 5 also show, on the right-hand side, the effects of fiscal policies during the first three years of the simulation.[16] Although there are significant differences across countries, two key findings emerge. First, domestic fiscal policy generally has mild effects on both quantities and prices. For instance, fiscal support in **Luxembourg** boosts output by 0.4% in the first year, while it leaves inflation unchanged. Moderate effects mainly reflect import leakages in SOEs, as the import content of consumption, investment and exports, and in some cases even government spending tends to be high. This insight is consistent with the empirical literature on fiscal multipliers, which finds that small open economies feature smaller multipliers (Ilzetzki et al., 2013).

The two key finding reveals nontrivial fiscal spillovers from the rest-of-the-euro area block. As mentioned earlier, this bloc lowers labour taxes by 4.5% percentage points and increases government consumption by 2.2% of ex-ante annual GDP. This fiscal package boosts aggregate demand in the REA, especially consumption demand by non-Ricardian households, which in turn fosters domestic exports, and hence domestic output. As expected, the ELB accentuates these fiscal spillovers. The fiscal package implemented in the rest-of-the-euro-area bloc boosts overall expenditure, leading to increased inflationary pressures. Consequently, a standard Taylor rule responds by adopting a less accommodative policy stance, curbing the growth in spending and negatively affecting domestic exports to the rest of the Euro area

Domestic fiscal policy in **Ireland** (top half of Figure 4) has expansionary effects for two reasons. First, government consumption directly stimulates domestic output. Second, a subsidy to households' wages keeps their after-tax income higher, which sustains consumption of non-

[16] Recall that, for the sake of comparability across countries, we simulate the same shock structure across all country versions, as explained in Section 2.2.

Ricardian households. Moreover, wage subsidy also keeps wages lower than they would have been otherwise, which given openness of Ireland and given the relatively more flexible prices in Ireland than elsewhere helps stimulate exports. Without the ELB, this latter effect is important and results in higher GDP through the stimulus from higher external demand. With the ELB, this effect is damaging, because it tends to lower prices and further stimulates the increase in the real rate. However, in the case of the ELB, direct government spending counters this negative inflation effect from lower labour taxes and ameliorates the increase in the real rate (compared to the case with no government consumption spending). In terms of fiscal spillovers, they tend to be small from all regions, except from the REA in the case of the binding ELB, in which case fiscal measures in the REA have mildly positive effects on Irish GDP and inflation, but only in the first year. The effects of fiscal measures on domestic inflation are relatively small, because, as discussed above, both fiscal measures (reduction in taxes and the increase in government consumption) work in the opposite direction in terms of their effect on prices.

In the **Netherlands**, the fiscal stimulus in the first year leads to a milder contraction of output by approximately two percentage points less than in the case of no domestic fiscal support. This result is robust to the existence of an ELB constraint. The fiscal measures are thus quite effective in mitigating the recession. The spillovers of fiscal measures in REA to output in the Netherlands are persistently negligible in the absence of an ELB, but are higher under the ELB owing to the appreciation of the euro. Obviously, not only does the appreciation of the euro vis a vis the dollar contribute but also the even lower REA demand for Dutch goods in the absence of fiscal support measures in REA. As a result, output in the Netherlands would contract slightly more had fiscal measures been absent in REA. As regards inflation, the effects stemming from the domestic fiscal measures are relatively small, mainly due to price stickiness. Looking at the spillovers of fiscal measures in the REA under the ELB, their impact on Dutch inflation builds up gradually until it starts dissipating again. This is rather due to the implications for the euro (i.e. appreciation) and its pass-through to domestic prices.[17]

Similarly to the cases of Ireland and The Netherlands in the case of **Slovenia** (lower half of Figure 5), the domestic fiscal stimulus spillover immediately reaches close to 2 p.p. of GDP and

[17]In the ELB, the euro appreciates more in the absence of fiscal support measures in REA, leading to a deeper contraction in output and to a larger drop in inflation compared to the baseline scenario.

but on the contrary to IE causes only a slight increase of inflation in the first year with binding ELB as well as without the ELB. At the same time, due to the lack of the import content in exports feature of government spending the spillover of the REA fiscal stimulus has persistent and negligible effect on the Slovenian economy throughout the observing period, regardless of the binding condition of the ELB.

5 Robustness to model features

5.1 Overlapping generations in the EAGLE

The Dutch version of the EAGLE introduces an overlapping generations structure that applies to all four regions.[18] This affects the effective planning horizon of households and consequently households have no bequest motive. Borrowing from Blanchard (1985) and Yaari (1965), it is assumed that each period households face a probability of death. Agents discount the far future more heavily, thus placing more weight on current fluctuations in disposable income, as well as medium-term discounted wealth, while individual behaviour is similar to that of the representative agent. As a result, Ricardian equivalence breaks down, implying that changes in lump-sum taxes matter and entail non-negligible wealth effects.

To save space, only the home household's maximization problem is presented. Households die with probability $1-\delta$ each period and every period a newborn generation i represents a fraction $1-\delta$ each of total population, where $0\leq \delta \leq 1$. In other words, δ captures the probability of survival from one period to the next. Therefore, $\sum_{t=0}^{\infty} \delta^t = \frac{1}{1-\delta}$ represents the average household lifetime. As pointed out by Smets and Trabandt (2012), an alternative and empirically more plausible interpretation of $1-\delta$ is that it reflects the effective planning horizon of households. Here, we adopt the planning horizon interpretation as in Smets and Trabandt. Households have no bequest motive and the usual Ricardian equivalence breaks down. Households in all four regions derive utility form consumption and disutility from supplying labour to domestic firms. They are also assumed to have external habits in consumption. In what follows we present the equations for the home country only in order to save space. Similar conditions hold for the rest of

[18] For details, see the online technical appendix.

the three regions. Under the Blanchard-Yaari structure, the utility function of the representative household in each generation i receives the following form:

$$U_t^i = \frac{1-\kappa}{1-\sigma}\left(\frac{C_t^i - \kappa\ C_{t-1}}{1-\kappa}\right)^{1-\sigma} - \frac{\left(N_t^i\right)^{1+\zeta}}{1+\zeta} + \beta\ \delta\ U_{t+1}^i \tag{2}$$

where κ is the degree of habit parameter, σ is the degree of relative risk aversion, ζ is the inverse of the Frisch elasticity of labour supply, C_t^i is generation i's private consumption while C_t is aggregate consumption. N_t^i is generation i's labour supply. Notice that in the utility function above the survival probability, δ, enters the utility function of generation i in discounting the future. Households in the home country trade in two assets, namely a one period home government bond and a one period foreign government bond issued in the US. The latter is subject to a risk premium shock. The aggregated home Euler equations (abstracting from capital for simplicity) in home and foreign bonds receive the following form:

$$\beta\ \frac{R_t}{\Pi_{t+1}}\frac{\Lambda_{t+1}}{\Lambda_t} = \frac{1-\delta}{\delta\ \mu_{t+1}\Pi_{t+1}}\Lambda_{t+1}\ \ (b_t + q_t^*\ b_t^* + m_t) + 1 \tag{3}$$

$$\beta\,(1-\Gamma_{b^*}(\cdot))\,\frac{R_t^{US}}{\Pi_{t+1}}\frac{\Lambda_{t+1}}{\Lambda_t}\frac{S_{t+1}^{H,US}}{S_t^{H,US}} = \frac{1-\delta}{\delta\ \mu_{t+1}^*\Pi_{t+1}}\Lambda_{t+1}\ \ (b_t + q_t^*\ b_t^* + m_t) + 1 \tag{4}$$

In the equations above, Λ_t denotes the marginal utility of consumption in period t. Π_t is gross inflation and R_t is the short-term interest rate on one-period domestic bonds which coincides with the policy rate of the central bank. b_t denote real bond holdings issued at home, b_t^* denote real bond holdings issued in the US which carry price q_t^*. m_t are real money holdings. Note that in this setup money holdings become net wealth. In equation (3), R_t^{US} is the short-term interest rate on one-period bonds issued in the US, which coincides with the policy set by the Fed. $S_t^{H,US}$ denotes the nominal exchange rate defined as the domestic currency (euro) price of one dollar. Note that home households pay a premium in adjusting their holdings of bonds issued in the US. This is captured by the function $\Gamma_b^*(.)$, which is a convex function in foreign bond holdings.[19] Finally, the terms μ_{t+1} and μ_{t+1}^* in both equations are recursive discounting terms.

[19]This function guarantees stationarity.

Observation of the two aggregate Euler equations reveals that bond and money holdings have real effects, through their direct impact on consumption. When households have finite lifetimes, their bond holdings become net wealth. This means that, after aggregation, fluctuations in their holdings affect consumption smoothing, regardless of the fiscal instrument (i.e. distortionary or non-distortionary) used for debt stabilization. This effect is absent in infinite lifetimes which explains why changes in lump-sum taxes leave private consumption unaffected. Clearly, the infinite lifetimes version of the two Euler equations above can be obtained by setting $\delta = 1$. In this case, the asset portfolio terms disappear. In the scenarios presented below, we calibrate the survival probability $\delta = 0.99$, which corresponds to 25 years of effective planning horizon approximately.[20]

We now compare how this overlapping generations structure with the standard version of the model used in section 4 where agents' lifespans are infinite. In Figure 6, we display the responses of output and annualized inflation in the Netherlands and the REA. In each panel, we compare the responses from the version of the model with finite lifetimes (Blanchard-Yaari version) to those from the same model with infinitely lived households. The responses under infinite lifetimes correspond to the baseline scenario presented in the previous subsection, which coincides with the case of $\delta = 1$. The shocks are as in the baseline scenario.

When households have finite lifetimes, the responses of output and inflation are dampened in both regions. In fact, the contraction in output in both regions is now milder and the trough in inflation is also lower. Finitely-lived households discount the future more heavily and care more about the short- to medium-run fluctuations in their disposable income and wealth. The Covid shock and the associated fiscal-support measures lead to an increase in public debt. Given that households' bond holdings are net wealth and looking at the Euler equations (3) and (4), it becomes clear that the rise in debt-to-GDP partially offsets the downward pressures on private consumption due to the Covid shock. Moreover, households with finite lifetimes discount more heavily future increases in taxes that will be brought about in order to stabilize the debt. Hence

[20] Mavromatis (2020) estimates a closed economy DSGE with a Blanchard-Yaari structure using Bayesian techniques and finds 95% posterior interval values for the survival probability between 0.956 and 0.995 approximately. We have implemented additional robustness scenarios by considering lower survival probabilities ($\delta = 0.8702$) as found in Castelnuovo and Nisticò (2010). Our results show that the contraction in output becomes milder as the survival probability declines. A similar pattern is observed in inflation. The qualitative thus conclusions presented in this section are robust to lower values of the survival probability.

their non-Ricardian nature mitigates partially the recessionary effects of the Covid shock while at the same time, it increases the potency of fiscal policy.[21] Instead, infinitely lived households smooth the effects of the adverse shocks, reducing their private consumption by more. As a result of the larger drop in private consumption, infinitely lived households also expect an even larger drop in medium-run inflation, which is reflected in annualized inflation declining more. Given that the economy is at the ELB in the first 12 quarters, a larger drop in inflation leads to higher real interest rates compared to the case with finitely lived households. As a result, the present value of lifetime wealth falls further, adding to the negative effects on private consumption and output.

5.2 Cross-border workers in Luxembourg

Cross-border workers make up nearly half of Luxembourg's workforce. To introduce these agents in the model, we follow Moura and Lambrias (2018), and make a distinction between the labour services provided by resident workers and those provided by cross-border workers. As a result, domestic firms in Luxembourg require both inputs for production. In addition, we adjust certain accounting identities, such as the current account in Luxembourg and the rest of the euro area, to incorporate the wages paid to cross-border workers.[22]

We use this extended framework to confirm that all the insights presented earlier remain valid. For example, the decline in domestic annual output, in the absence of any shocks in the euro area, remains relatively unchanged at around 2.5 percent. Similarly, the decrease in annual inflation continues to be close to 1.4 percentage points. The same applies to spillovers from the United States and the rest of the world, as well as to the effect of domestic and foreign fiscal policy.

[21] Essentially, finite lifetimes imply higher multipliers of the fiscal support measures, which help in mitigating the recessionary effects of the Covid shock. As lifetimes increase this effect becomes weaker.

[22] For more detailed information, please refer to the original paper by Moura and Lambrias (2018).

5.3 Search and matching in the labour market in Ireland

Let us now consider the effects of labour market frictions, which we can analyse using the Irish model featuring search-and-matching frictions in all blocs (Mortensen and Pissarides (1999)).[23] Households supply workers to a continuum of labour firms, each employing one worker. Labour firms hire workers by posting vacancies. Using hired workers, labour firms produce labour services, which they sell to firms in the intermediate tradable and non-tradable sectors. Labour firms also negotiate wages and hours worked with households. The model has sticky wages following Hall (2005), and a labour market where employees can move between tradable and non-tradable sectors without friction, but movements from unemployment to employment are subject to search frictions. Separations are exogenous. Households and labour firms are subject to labour taxes (households pay wage taxes and labour firms pay social security contributions).

Figure 7 shows in the upper two panels the key labour market variables. Employment falls and unemployment increases, while job finding probability (probability that workers find jobs) decreases. Despite the decrease in vacancies, job filling probability (the probability that firms find workers) increases, which is entirely due to the increase in the number of unemployed workers per vacancy.

Importantly, because households care about the after-tax wage income and firms care about total labour cost (including labour taxes paid by firms), changes in taxes are taken into account when wages are negotiated. This channel turns out to be important for the way fiscal measures are transmitted in the model. This is shown in the bottom two panels of Figure 7, which compares inflation and wage responses in the benchmark model discussed above with the country-specific model featuring search frictions (note that we run exactly the same shock in both models and that the calibration of the models is the same, so that the differences are only due to model features). In the model with search frictions labour taxes paid by households are part of the wage negotiations with firms. As a consequence of a large labour tax decrease (recall that wage subsidies amount to 4.5% of GDP in our simulation), households are willing to agree to lower pre-tax wage payments from firms, resulting in the reduction in wages payable by firms (dashed line in the bottom-right panel in Figure 7). This results in a faster and stronger reduction in inflation,

[23] See Jacquinot et al. (2018) for the full description of the model and the online technical appendix for the key details relevant for the discussion here.

but also in a faster recovery of inflation when fiscal measures cease. The same mechanism is at work in the rest of the euro area, where reduction in labour taxes also have negative effects on inflation. Therefore, inflation spillovers in the case of the ELB in the euro area turn negative rather than positive (recall than in the benchmark case in Figure 4, inflation spillovers from the rest of the euro area fiscal measures were positive at 0.4 p.p. in the first year; in contrast, with labour market frictions these spillovers become negative at 0.8 p.p. in the first year).[24]

5.4 Long-term bonds and Next Generation EU

Following the sharp contraction of European economies during the pandemic, the European Council agreed in July 2020 to launch the Next Generation EU (NGEU) instrument. In practice, NGEU allows the European Commission to issue debt to finance grants and loans to EU Member States, with the disbursement of funds focusing on the countries most affected by the crisis.[25]

Against this background, we consider an extended version of the model that features a supranational fiscal authority in the euro Area.[26] The European fiscal authority issues long-term bonds and collects lump-sum or VAT taxes from households in the home country and in the REA.[27] Long-term bonds are modelled as perpetuities following Woodford (2001). The duration of these bonds is 10 years. The fiscal authority sells the bonds to Ricardian households at home and in the REA. For the long-term interest rate to deviate from the union wide short-term rate set by the union central bank, it is assumed that long-term bonds are subject to transaction costs. We model transaction costs as a function that is increasing in the GDP share of outstanding long-term debt issued by the supranational fiscal. Lump sum or VAT taxes imposed on home and REA households increase with the share of outstanding long-term debt but are weighted by the size of the region that the household belongs to. This ensures that tax incidence falls more on households that reside in the bigger region.

[24] The effect on non-fiscal spillovers is only affected in terms of the magnitude and tends to be somewhat stronger in case of the ELB than in the benchmark case without labour frictions. The spillover effects on inflation without the ELB are almost nil. To save space, we do not report the full set of spillovers.

[25] See also Bańkowski et al. (2021) for more details on the implementation.

[26] For details, see the online technical appendix.

[27] An alternative and more realistic assumption would be local governments to impose lump-sum taxes of households the proceeds of which are then rebated to the fiscal authority, as in Bartocci et al. (2020). Given that households are infinitely lived, and local government debt is stabilized through lump-sum taxes, the two approaches have exactly the same wealth effects.

The supranational authority uses the resources raised by issuing long-term debt either to provide support to the non-tradable goods sector or to finance part of member states' outstanding debt. The two scenarios are considered separately. In both cases, it is assumed that the supranational authority implements additional spending for three quarters. As in Bartocci et al. (2020), taxes used to finance debt issued by the supranational fiscal authority are not active in the first three quarters when resources are distributed to member states. In both scenarios, we assume that additional union wide spending is set exogenously by the supranational authority and amounts to five percent of euro area GDP.

In all the scenarios considered in this section, we take the Netherlands as the home country. We report the impulse responses of output and inflation in the Netherlands in Figure 8. The top panels display three impulse responses corresponding to three scenarios: no support from the EU (black solid lines), EU transfers to the non-tradable goods sector financed by EU lump-sum taxes (red dashed lines) and EU transfers to the non-tradable goods sector financed by EU VAT taxes (blue dotted lines). The bottom panels report the same set of impulse responses but for EU transfers to finance national governments' outstanding debt. In the scenario of no EU support, no other fiscal backing is assumed. That is, fiscal support measures from national governments are turned off. This isolates the effects of fiscal transfers from the EU by excluding additional effects stemming from national government support.

Let us first focus on EU lump-sum taxes. The impulse responses reveal that the Netherlands benefits only marginally from EU transfers towards the non-tradable goods sector. The trough in the contraction of output comes at an earlier stage and the recovery starts faster. However, in the years that follow the path of output overlaps with that under no EU support. Turning to lump-sum transfers to finance national government debt (bottom panels), the effects are similar. Obviously, this is largely driven by the fact that both national governments and the supranational authority use lump-sum taxes to stabilize debt. Note that, in the case of EU lump-sum taxes to stabilize EU debt, the trough in output is not milder and this is due to the fact that non-Ricardian household consumption is hit to a large extent, offsetting the short-run positive effects of EU transfers. A similar picture is observed when looking at inflation. Given the ELB, the drop in inflation leads to a rise in the real rate which in turn acts as an additional offsetting anchor of EU transfers.

When the supranational authority uses VAT taxes to finance its debt, both forms of fiscal support measures from the supranational authority are beneficial. Specifically, transfers to the non-tradable goods sector and transfers to finance national debt both reduce the contraction in NL output. This result is evidently driven by the fact that the supranational authority stabilizes its debt using a distortionary tax. The intuition is as follows. Households are aware that the supranational authority will not raise VAT taxes immediately, but only in the future.[28] From the Euler equation, private consumption rises before the increase in EU VAT taxes takes place. Households frontload the effects of higher future VAT and, as a result, decide to raise their consumption (or limit the decline in their consumption) before the rise in taxes is actually implemented. This puts upward pressure on prices in both regions. This channel is absent when the supranational authority stabilizes its debt via lump-sum taxes. Moreover, given that monetary policy is constrained at the ELB for the first three years, the real rate declines, mitigating thus the recession. Therefore, the key difference between lump-sum EU taxes and VAT EU taxes is that the latter lead to frontloading in private consumption and to lower real interest rates.

5.5 Fiscal extension of the EAGLE model in Slovenia

For small and very open economies, demand from foreign governments or government-related entities for goods produced in their economy may be an important feature, in particular if the foreign economy is large. An example are pharmaceuticals and private consumption subsidized by the government (for instance, the cash-for-clunkers scheme in Germany, where car parts of cars bought with the subsidy are produced in Slovenia). This issue has been explored in Clancy et al. (2016), but here we apply this model feature to fiscal spillovers in the presence of the ELB. This allows us to illustrate a difference in the magnitude of fiscal spillovers, akin to the discussion of the different strength of fiscal multipliers found in the empirical and theoretical literature (for the first, see Auerbach and Gorodnichenko (2012), and for the latter, see e.g. Ghassibe and Zanetti (2022)).

To see the difference, compare spillovers in the bottom half of Figure 5, where there is no

[28]EU VAT taxes react to lagged debt to gdp.

import-content of government spending, with those in Figure 9, where governments use part of their expenditure on imported goods. Note that when government spending has an import component, it is impossible to distinguish international and fiscal spillovers, since the imported part of fiscal spending is also an international spillover. In Figure 9), the largest spillover comes from the REA with ELB, and it has the opposite sign from that in the bottom half of Figure 5, but only in the presence of the ELB. The reason for this strong asymmetry is the following. When the ELB binds, a negative shock causes deflation and with it an increase in the real interest rate, which further reduces domestic consumption, which in turn reduces inflation, which again reduces the real rate, etc. In this situation, even a small effect of foreign demand that pulls the economy from this deflationary spiral can have very strong effects, and this is what the model shows. When the REA shock is switched off, government spending in the REA is also switched off, and with it some of the positive effect it has on the Slovene economy. Given the size of the REA and the size of government spending shock (2.2% of ex-ante GDP of the REA), this can, thorough government imports, affect the small economy, and just enough to alleviate the strong deflationary spiral that can occur int he presence of the ELB. This asymmetry is not present when there is no ELB, which can be seen in both figures.

6 Conclusion

We investigate the role of international spillovers during the COVID-19 pandemic for small open economies that are members of the monetary union. We use a unified framework, based on versions of the Euro Area and the Global Economy (EAGLE) model. Despite particular features of the country-specific versions, they share the same basic framework, ensuring a much higher degree of comparability than most cross-country studies. We find that a substantial part of the decline in 2020 economic activity in Ireland, Luxembourg, the Netherlands and Slovenia resulted from foreign factors, mostly related to trade and less so to fiscal policies. This finding is not surprising for small open economies. However, the effective lower bound on interest rates, the specific design of fiscal measures, and the monetary union framework can significantly affect the results. We explain the transmission mechanisms behind these findings. In particular, the interaction of the effective lower bound with the effective lower bound in the euro area is important, mainly for for international spillovers within the euro area. However, the binding ELB in the euro area also leads to a magnification of spillovers from the outside of the monetary union. The unprecedented fiscal measures taken by euro area countries boosted aggregate demand in the whole union, which, according to our models, also had some non-trivial effects in the small economies studied here.

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Table 1. Empirical targets

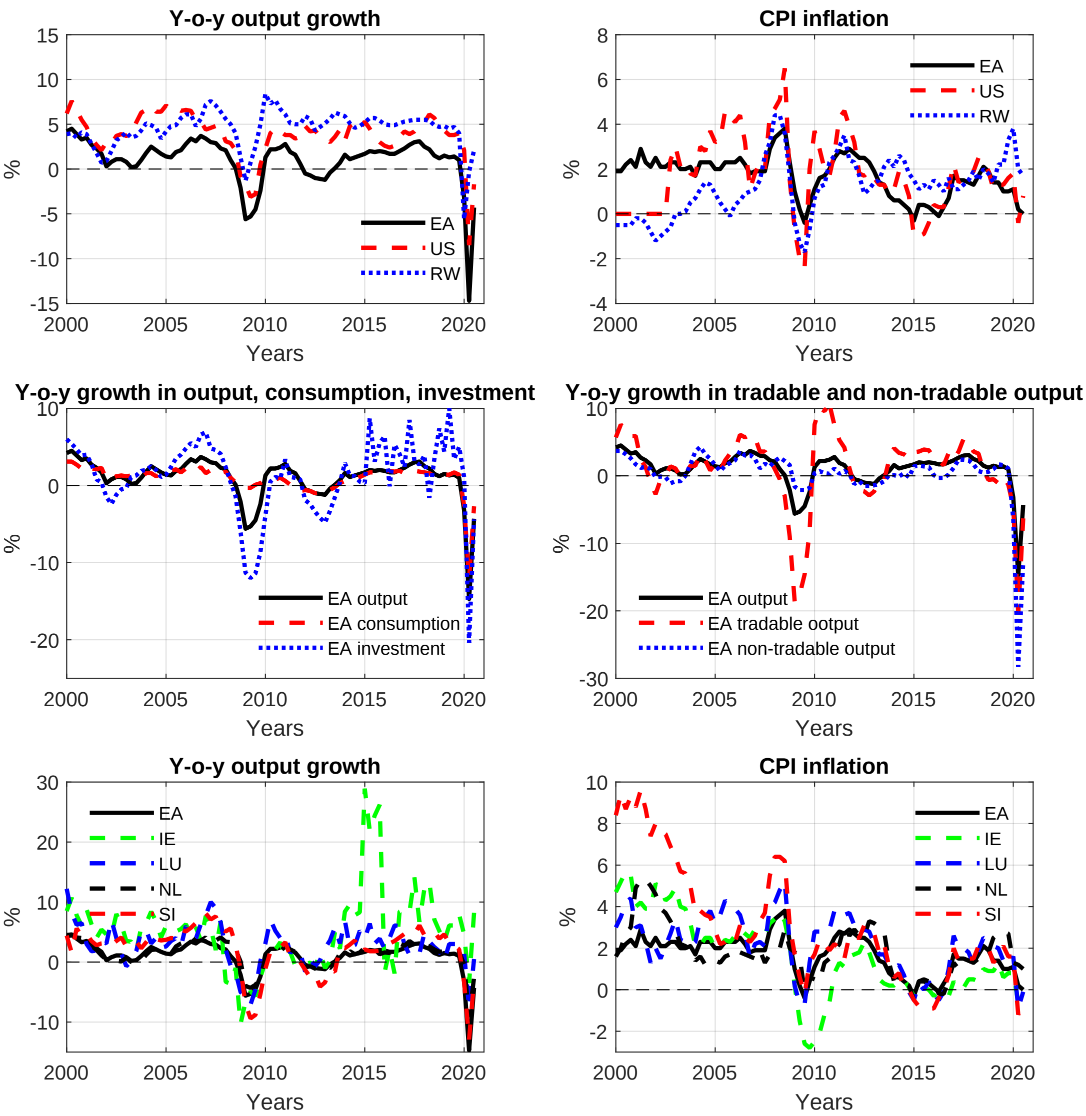
	E.C 2020 Forecasts Autumn 2019	Spring 2020	Targets	Model simulations
Ireland				
GDP	3.5	-7.9	-11.4	-7.1
Consumption	2.5	-8.8	-11.3	-9.5
Investment	4.5	-41.6	-46.1	-12.4
Inflation	1.5	1.3	-0.2	-3.5
Luxembourg				
GDP	2.6	-5.4	-8.0	-7.1
Consumption	2.7	-4.1	-6.8	-6.3
Investment	2.9	-12.0	-14.9	-19.6
Inflation	1.9	0.4	-1.5	-4.3
Netherlands				
GDP	1.3	-6.8	-8.1	-6.0
Consumption	1.7	-9.5	-11.2	-3.95
Investment	1.8	-11.2	-13.0	-13.7
Inflation	1.5	1.1	-0.4	-1.93
Slovenia				
GDP	2.7	-7.0	-9.7	-10.0
Consumption	2.9	-6.1	-9.0	-9.3
Investment	6.0	-13.0	-19.0	-37.8
Inflation	2.4	2.1	-0.3	-2.9
Euro area				
GDP	1.2	-7.7	-8.9	-8.6
Consumption	1.2	-9.0	-10.2	-8.7
Investment	2.0	-13.3	-15.3	-19.2
Inflation	1.5	1.3	-0.2	-2.0
United States				
GDP	1.8	-6.5	-8.3	-7.5
Consumption	2.2	-7.2	-9.4	-9.1
Investment	1.1	-12.2	-13.3	-8.9
Inflation	1.8	-0.6	-2.4	-2.4
Rest of the world				
GDP	4.3	-1.3	-5.6	-7.2

Note: The Autumn 2019 and the Spring 2020 European Economic Forecasts by the European Commission provides the data for all European economies, the Euro Area, and the United States. The October 2019 and April 2020 World Economic Database by the International Monetary Fund provides the data for the Rest of the World. The Rest of the World refers to an average of China, Japan and the United Kingdom, where each country is weighted according to its 2018 Gross Domestic Product.

Table 2. Great ratios and calibration

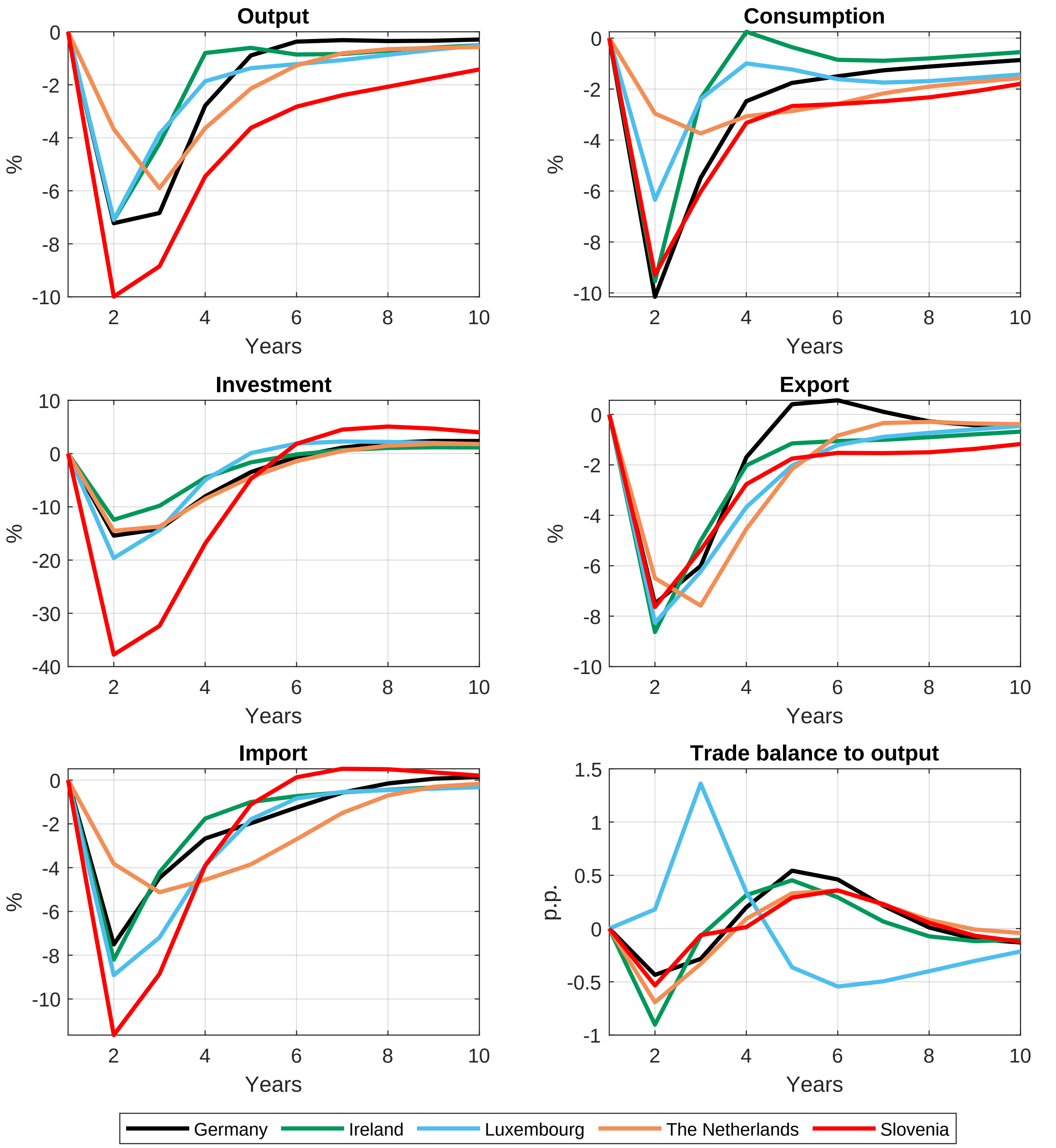
	IE	LU	NL	SI
Share in percentage of GDP				
Private consumption	59.79	32.00	56.56	62.87
Private investment	17.00	19.00	20.56	19.00
Public expenditure	18.80	21.00	22.55	19.00
Imports total	77.78	160	24.40	61.29
Imports consumption	29.20	15.00	14.40	15.71
Imports investment	11.60	15.00	10.00	8.86
Imports exports	36.97	130		36.72
Imports public expenditures				
Share non-tradable sector	35.94	9.55	60.00	44.69
Tax rates				
VAT	18.3	16.00	16.80	19.70
Labour income tax rate	14.90	13.94	18.30	12.89
SSC by firms	9.90	10.89	13.00	11.10
Share of world GDP (%)	0.3	0.05	1.1	0.2
Mark-up				
Wages – households	1.30	1.30	1.30	1.30
Prices – domestic tradable goods	1.20	1.30	1.20	1.20
Prices – exports	1.30	1.30	1.20	1.30
Prices – domestic non-tradable goods	1.50	1.50	1.50	1.50
Real rigidities				
Investment	6.00	5.00	5.00	1.80
Imports – consumption	2.00	2.00	2.00	2.00
Imports – investment	1.00	1.00	2.00	1.00
Nominal rigidities				
Households				
Wage stickiness	0.75	0.75	0.80	0.81
Wage indexation	0.75	0.75	0.75	0.75
Tradable goods sector				
Price stickiness (domestic goods)	0.75	0.90	0.90	0.90
Price indexation (domestic goods)	0.50	0.50	0.50	0.50
Price stickiness (exported goods)	0.75	0.75	0.90	0.75
Price indexation (exported goods)	0.50	0.50	0.50	0.50
Non-tradable goods sector				
Price stickiness (domestic goods)	0.75	0.75	0.90	0.90
Price indexation (domestic goods)	0.50	0.50	0.50	0.50

Figure 1. Stylised facts



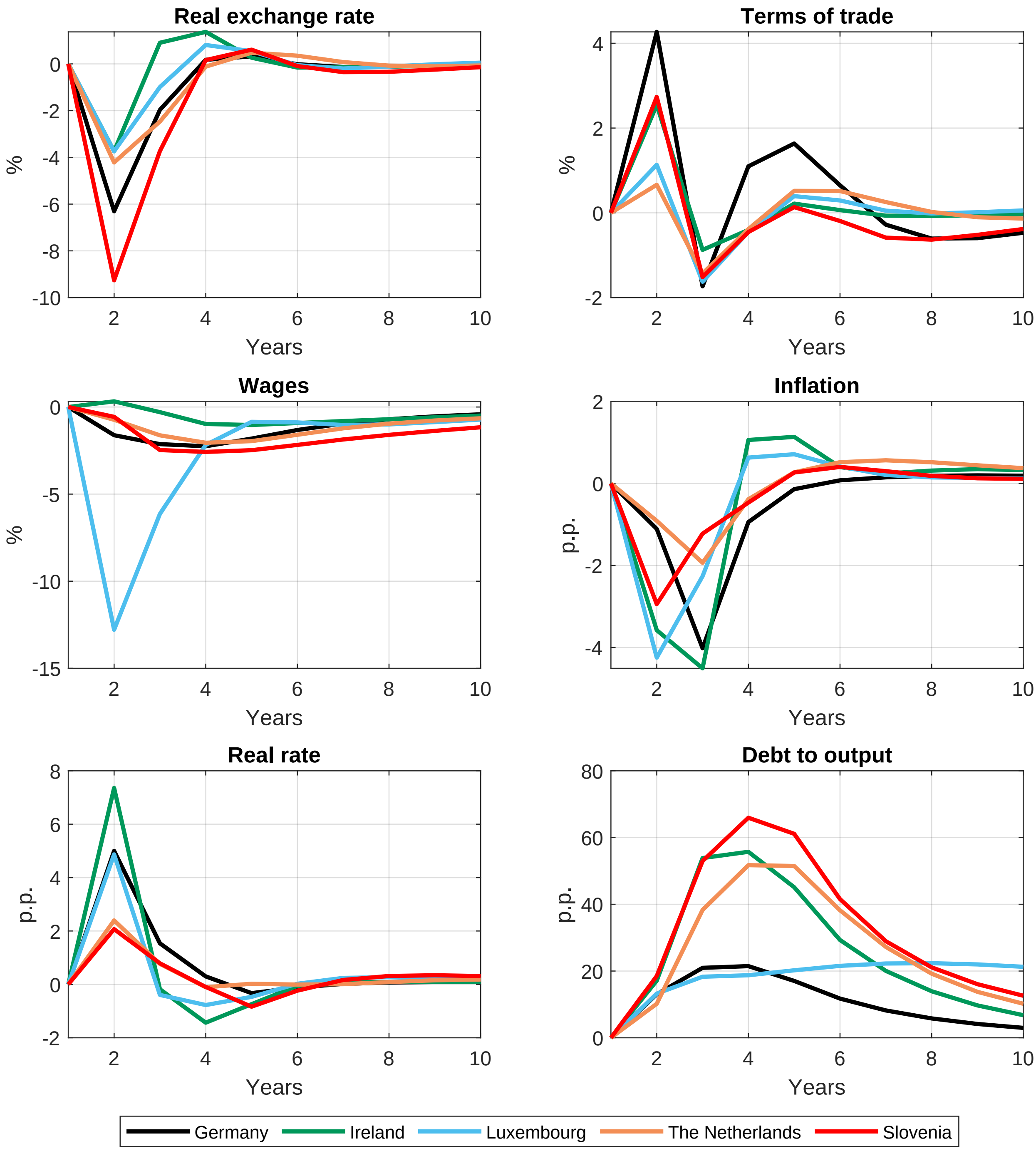
Source: Years. Eurostat, OECD, St. Louis FED (FRED).

FIGURE 2. Benchmark scenario, responses of quantities



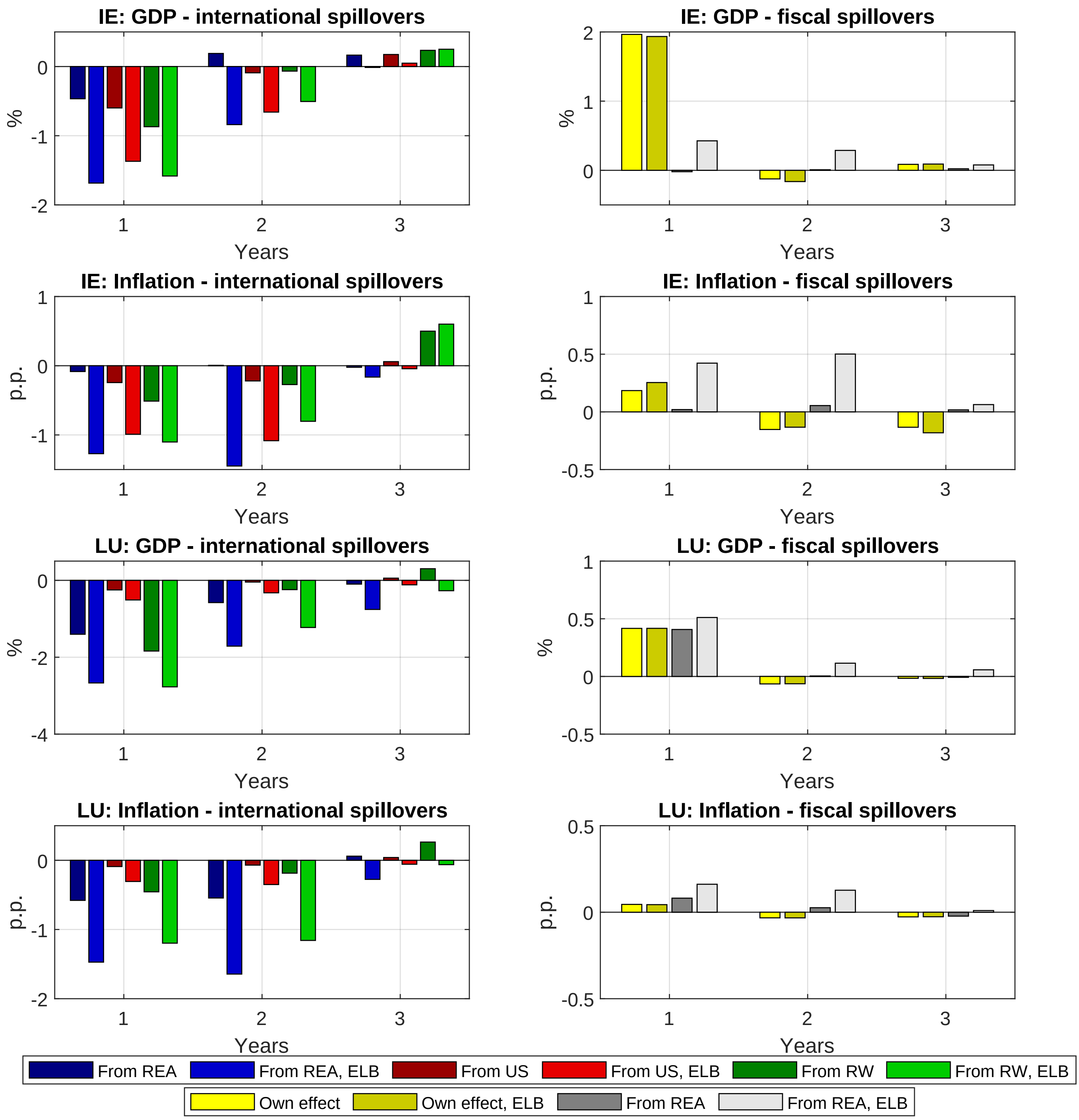
Horizontal axes: Years. Vertical axes: Percent deviations from the initial value, except for the trade balance (trade-balance-to-GDP ratio, in p.p. deviations). GDP and its components are reported in real terms.

Figure 3. Benchmark scenario, responses of prices and interest rates



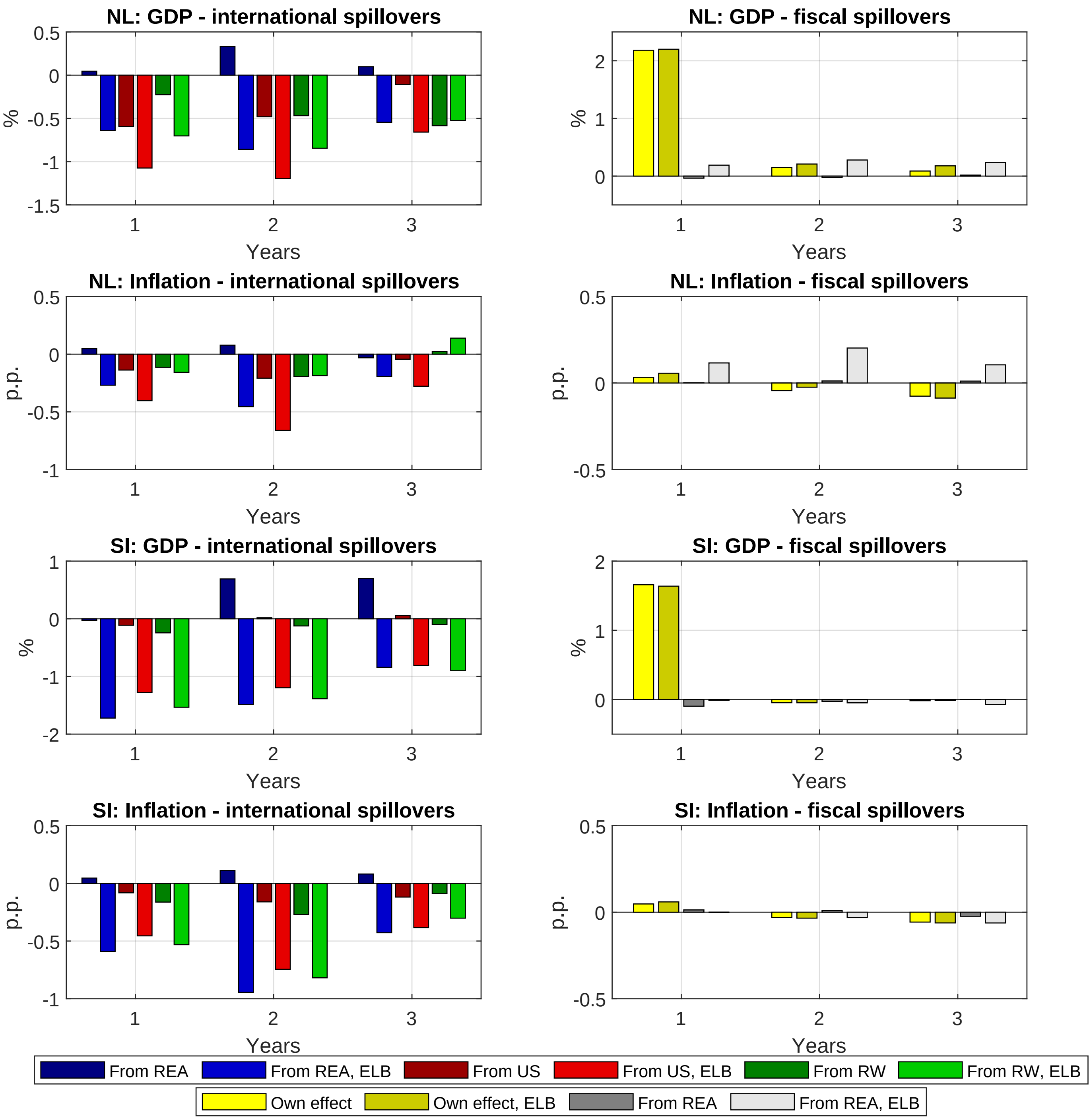
Horizontal axes: Years. Vertical axes: Percent deviations from the initial value, except for inflation and interest rates (annualised percentage point deviations) and the debt ratio (in p.p. deviations).

FIGURE 4. Spillovers in Ireland and in Luxembourg



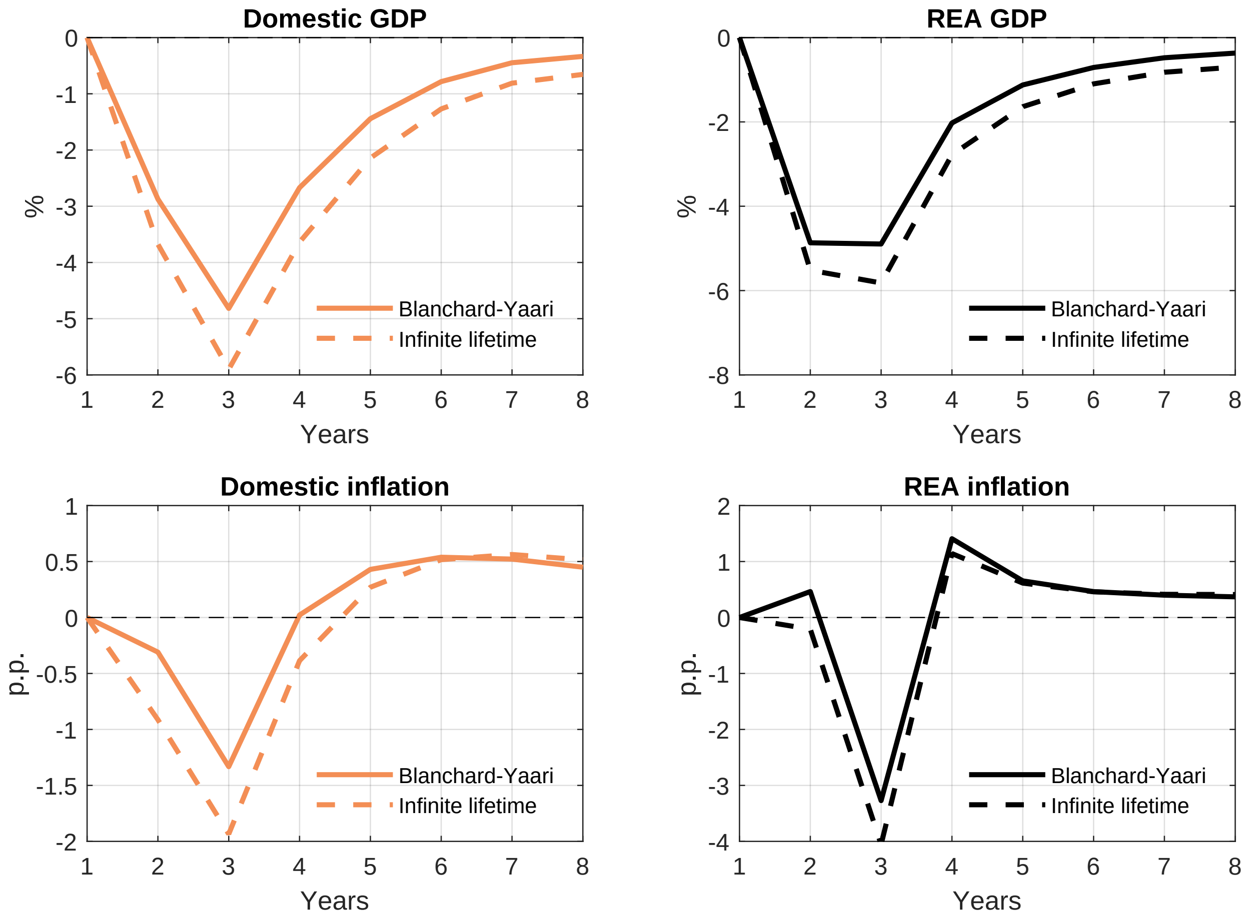
Horizontal axes: Years. Vertical axes: Percentage point deviations from the initial value.

Figure 5. Spillovers in The Netherlands and in Slovenia



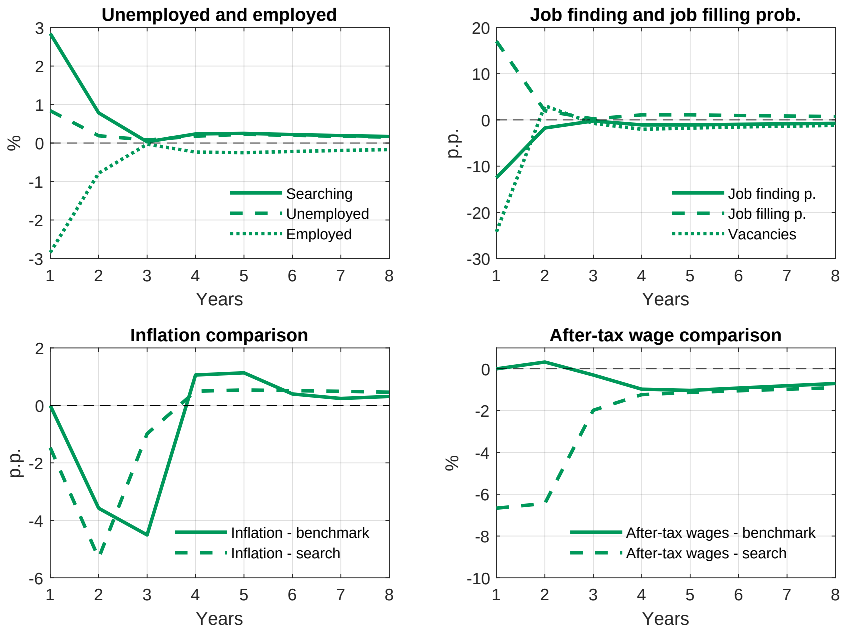
Horizontal axes: Years. Vertical axes: Percentage point deviations from the initial value.

Figure 6. Blanchard-Yaari vs. infinite-lifetime utility (NL)



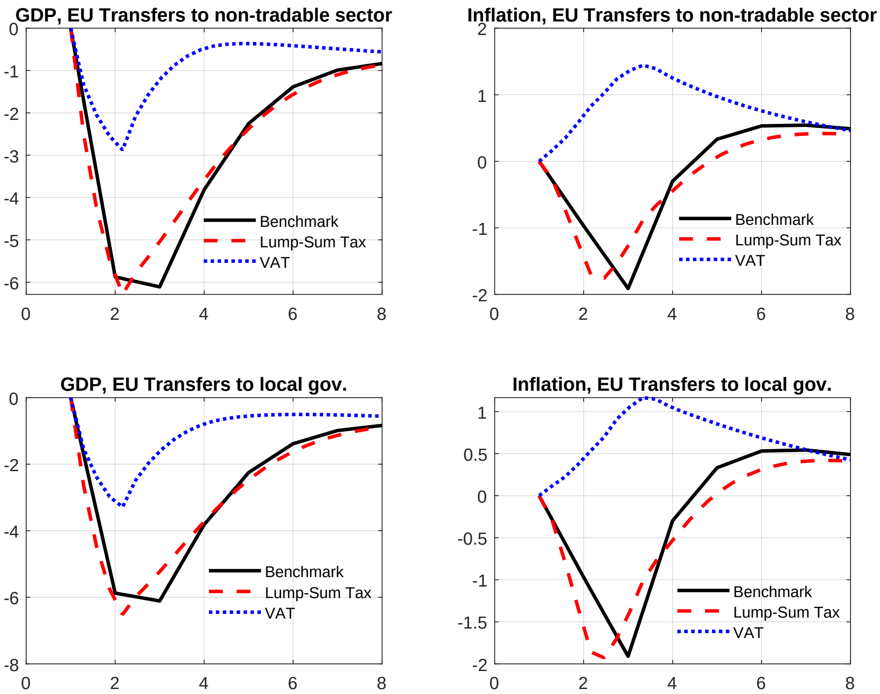
Horizontal axes: Years. Vertical axes: Percent deviations from the initial value, except for inflation (percentage point deviations).

Figure 7. Responses of the labour market specific variables



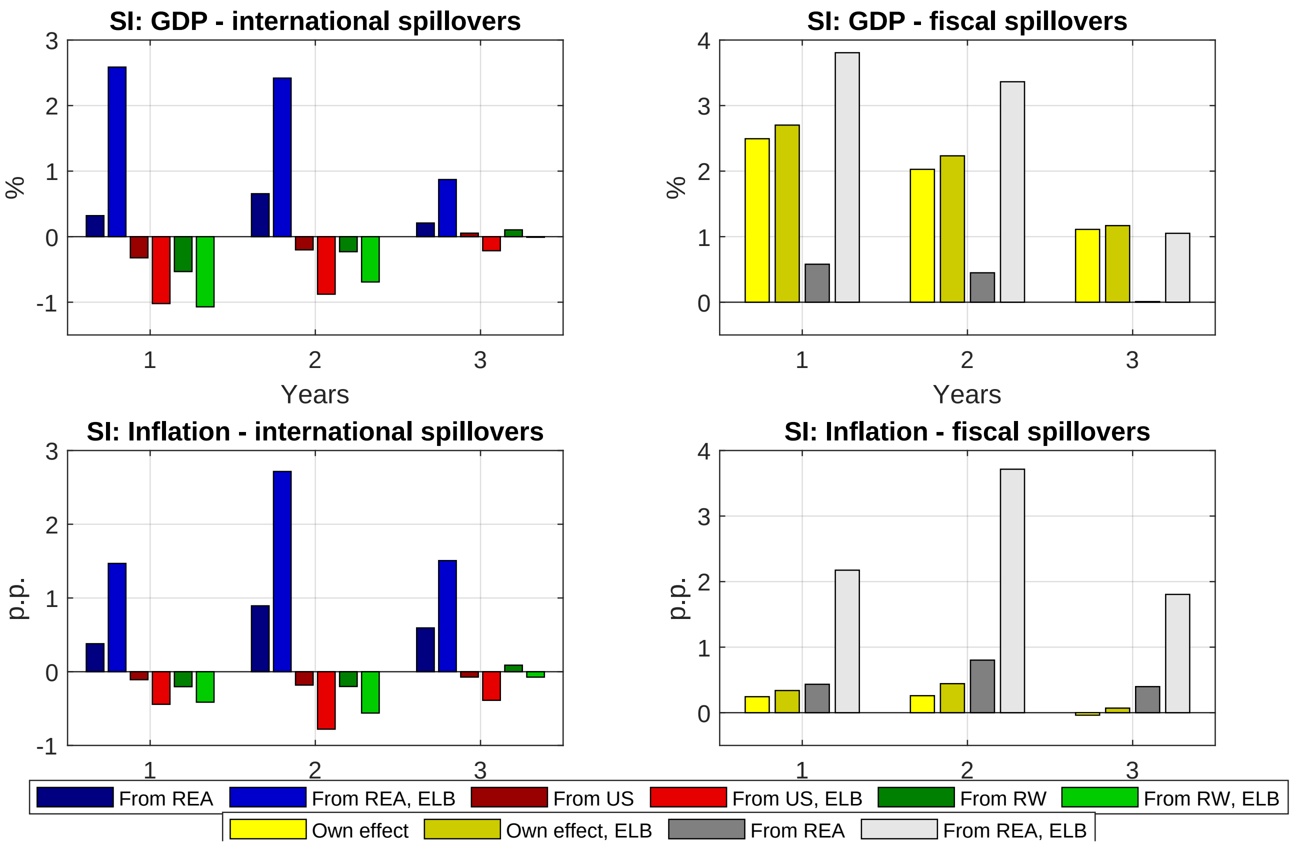
Horizontal axes: Years. Vertical axes: Percent deviations from the initial value, except for probabilities (percentage point deviations).

Figure 8. EU transfers



Horizontal axes: Years. Vertical axes: Percent deviations from the initial value, except for inflation (percentage point deviations). The black solid lines are the responses from the baseline version of the model without NGEU support measures and no fiscal support from national governments. The red-dashed lines are the responses in the case where the supranational fiscal authority finances its debt through lump-sum taxes, while the blue-dotted lines are the responses when the supranational authority finances its debt through a value-added tax. The top two panels are the responses following NGEU transfers to the non-tradable goods sector while the bottom two panels display the responses following NGEU transfers to national governments.

Figure 9. Spillovers in Slovenia with fiscal extension EAGLE model



Horizontal axes: Years. Vertical axes: Percentage point deviations from the initial value.

Appendix

A Labour market

The model with labour market frictions follows the approach pioneered by Mortensen and Pissarides (1999), and its extension in Christoffel et al. (2009). We assume there is a continuum of labour firms, each employing one worker. Firms enter the market by posting a vacancy and, if they find a worker, sell homogeneous labour services intermediate goods producers. Labour firms pay labour taxes and bargain with households over wages.

The flows on the labour market are as follows. The number of workers that are employed after the matching process has been completed, nde_t, evolves as follows:

$$nde_t = (1 - \delta_x)\ nde_{t-1} + M_t,$$

where M_t is the number of new matches formed in a period, and δ_x is the fraction of existing employment relationships that have (exogenously) separated in each period. The number of matches is defined as

$$M_t = \varphi_M\ un_t^{\mu}\ vac_t^{1-\mu} = p_t^W\ un_t = p_t^F\ vac_t,$$

where φ_M is matching productivity, un_t is the number of searching workers, vac_t is the number of vacancies, $p_t^W W$ is the matching probability for workers, p_t^F is the matching probability for firms, and μ is the elasticity of the matching function with respect to unemployment.

The number of searching workers is not identical to the number of workers who end up being unemployed at the end of the period, denoted by une_t These workers are the ones who receive unemployment benefits. Their number evolves as follows

$$une_t = une_{t-1} + \delta_x\ nde_{t-1},$$

The population of each bloc in the model is standardised to 1, so that the number of unemployed at the end of the period is also defined as $une_t = 1 - nde_t$.

The above laws of motion lead to associated value functions of employed and unemployed workers, and those of a firm. The value function for an employed household member

$$
\begin{aligned}
E_t \;=\; & (1-\tau_t^{WH})\; w_t\; h_t - \frac{\chi}{\lambda_t}\,\frac{h_t^{1+\zeta}}{1+\zeta} + \beta\,\frac{\lambda_{t+1}}{\lambda_t}\,\left(\delta_x\left[1-p_{t+1}^{W}\right]\; U_{t+1}\right. \\
& \left. +(1-\delta_x\left[1-p_{t+1}^{W}\right])\; E_{t+1}\right),
\end{aligned}
$$

where E_t is the value of being employed, U_t is the value of being unemployed, τ_t^{WH} is the labour tax rate, β is the discount factor, w_t is real wage per hour worked, $h_t$is the number of hours worked, λ_t is the marginal utility of consumption, and $\frac{\chi}{\lambda_t}\,\frac{h_t^{1+\zeta}}{1+\zeta}$ is the marginal disutility of an additional hour worked, measured in terms of consumption goods. The value of being employed is therefore the net wages received by the worker, minus the disutility of having to work, plus the discounted value of the future state for the worker. This is a weighted average of the values of employment and unemployment, weighted by the probabilities that each state occurs. The value of being unemployed is

$$
U_t = b_t + \beta\,\frac{\lambda_{t+1}}{\lambda_t}\,\left(\left[1-p_{t+1}^{W}\right]\; U_{t+1} + p_{t+1}^{W}\; E_{t+1}\right),
$$

where b_t stands for unemployment benefits. These are determined as $b_t = rr\; w_t$, where rr is the replacement ratio. The above equation states that the value of being unemployed is the value of unemployment benefits, plus the discounted value of the future state, which is a weighted average of the value of remaining unemployed and the value of getting a job in the next period.

Analogously to households, labour firms also have value functions. They pay w_t to households for labour and transform this labour into effective labour y, using the technology $y_t = h_t^{\alpha_H}$, which they sell to intermediate goods producers at a rate x_t. The value to such labour firm of having a worker, J_t, is therefore

$$
J_t = x_t\; h_t^{\alpha_H} - (1+\tau_t^{WF})w_t\; h_t + \beta\,\frac{\lambda_{t+1}}{\lambda_t}\,(1-\delta_x)\;\; J_{t+1},
$$

where τ_t^{WF} is the labour tax rate paid by the labour firm. The value of a worker for a firm consists of the revenue that a firm obtains from labour services produced by the worker, minus the wage (gross of labour taxes) that it needs to pay to the worker, plus the discounted continuation value of the match, if the match does not separate. Labour firms also post vacancies, with per-period cost of having a vacancy open denoted by Ψ. The value of having a vacancy, V_t, is

$$V_t = -\Psi + p_t^F \; J_t + \beta \, \frac{\lambda_{t+1}}{\lambda_t} \left(1 - p_{t+1}^F\right) \; V_{t+1}.$$

We assume that vacancies expire every period, and that the entry to the market is free, so that firms post vacancies until the value of having a vacancy open is zero. This gives us the free-entry condition,

$$\Psi = p_t^F \; J_t,$$

which states that the cost of having a vacancy open must equal the expected gain from posting a vacancy, where the latter consists of the probability of finding a worker, times the value of this worker to a firm.

Assuming standard (efficient) Nash bargaining, wages and hours worked are determined by the following equations. Wages are split according to the Nash sharing rule,

$$\eta(1 - \tau_t^{WH}) \; J_t = (1 - \eta)(1 + \tau_t^{WF})(E_t - U_t),$$

where η is the bargaining power of households. The intuition for the above rule is that households and labour firms bargain over each other's matching surpluses, where the surplus of the firm is the value of the worker (because in equilibrium the value of the vacancy is zero), and for the household the value of the match is the difference between the values of being employed and unemployed. Hours worked are determined by

$$\alpha_H \; x_t \; h_t^{\alpha_{H-1}} = \frac{\chi}{\lambda_t} \, \frac{(1 + \tau_t^{WF})}{(1 - \tau_t^{WH})} \; h_t^{\zeta}.$$

The term on the left is the marginal revenue from labour services brought about by an additional hour worked and the term on the right is the marginal disutility of having to work an additional hour, in after-tax consumption terms.

B The Blanchard-Yaari structure in EAGLE

In this section, we present the extension of the standard EAGLE model of Gomes et al (2012). We adopt the specification of the Blanchard (1985) and Yaari (1965) model of perpetual youth in discrete time similar to Devereux (2011) and Smets (2018). Households die with probability $1-\lambda$ each period and every period a newborn generation i represents a fraction $1-\lambda$ of total population, where $0 \leq \lambda \leq 1$. In other words, λ captures the probability of survival from one period to the next. Therefore, $\sum_{t=0}^{\infty} \lambda^t = \frac{1}{1-\lambda}$ represents the average household lifetime. As pointed out by Trabandt and Smets (2012), an alternative and empirically more plausible interpretation of $1/1-\lambda$ is that it reflects the effective planning horizon of households. In this paper, we adopt the planning horizon interpretation. Households have no bequest motive and the usual Ricardian equivalence breaks down. The same structure applies in all four regions. Here, we present only the case of the home country to save space. First, we look at how household type I and type J's maximization problem changes under the Blanchard-Yaari structure. Subsequently, we derive the aggregate budget constraint for both types of households as well as the respective aggregate Euler equations.

I-type households

As in the standard version of EAGLE, household i gains utility from consumption $C_t(i)$ and disutility from working $N_t(i)$. In particular, there is external habit formation in consumption, which means that its utility depends positively on the difference between the current level of individual consumption, $C_t(i)$, and the lagged average consumption level of households of type I, $C_{I,t-1}$. Household i lifetime utility function is then:

$$E_t \left[\sum_{k=0}^{\infty} (\beta\lambda)^k \left(\frac{1-\kappa}{1-\sigma} \left(\frac{C_{I,t+k}(i) - \kappa C_{I,t+k-1}}{1-\kappa} \right)^{1-\sigma} - \frac{1}{1+\zeta} N_{I,t+k}(i)^{1+\zeta} \right) \right] \tag{5}$$

where $(0 < \beta, \lambda < 1)$ is the discount rate, σ $(\sigma > 0)$ denotes the inverse of the intertemporal elasticity of substitution and ζ (> 0) is the inverse of the elasticity of work effort with respect to the real wage (Frisch elasticity). The parameter κ $(0 < \kappa < 1)$ measures the degree of external habit formation in consumption. The individual budget constraint for household i is:

$$
\begin{aligned}
&\left(1+\tau_t^C+\Gamma_v(.)\right) P_{C,t}C_t(i)+P_{I,t}I_t(i)+R_t^{-1}B_{t+1}(i)+\left(\left(1-\Gamma_B^*(.)\right)R_t^*\right)^{-1}S_t^{H,US}B_{t+1}^*(i)+M_t(i)+\Phi_t(i)+\Xi_t \\
= &\quad \left(1-\tau_t^N-\tau_t^{W_h}\right) W_t(i)N_t(i)+\left(1-\tau_t^D\right) D_t(i)+TR_t(i)-T_t(i) \\
&+\frac{1}{\lambda}\left[\left(\left(1-\tau_t^K\right)\left(R_{K,t}u_t(i)-\Gamma_u(.)P_{I,t}\right)+\tau_t^K\delta P_{I,t}\right)K_t(i)+B_t(i)+S_t^{H,US}B_t^*(i)+M_{t-1}\right].
\end{aligned} \tag{6}
$$

Notice that stock variables on the RHS of the budget constraint (6) are multiplied by the term $1/\lambda$. This implies that the first order conditions stay the same as in Gomes et al (2012). However, as we show below this is not the case after aggregation. Therefore, in what follows and in order to save space, we do not present the first order conditions of household i since they are exactly the same as in the standard version of EAGLE. We therefore refer the reader to the appendix of the paper by Gomes et al (2012). In the next two subsections, we derive the aggregate budget constraint, first, and, second, the aggregate Euler equation.

J-type households

The lifetime utility of J-type household j changes in the same way as that of I-type households above. Its budget constraint reads as follows:

$$
\begin{aligned}
&\left(1+\tau_t^C+\Gamma_v(.)\right) P_{C,t}C_t(j)+M_t(j)+\Phi_t(j) \\
= &\quad \left(1-\tau_t^N-\tau_t^{W_h}\right) W_t(j)N_t(j)+TR_t(j)-T_t(j)+\frac{1}{\lambda}M_{t-1}
\end{aligned} \tag{7}
$$

B.1 Aggregate Budget Constraint

I-type households

The budget constraints of the different generations of type-I living in the economy at a given time t are given as follows:

$$
\begin{aligned}
&(1-\lambda)\Bigg[\left(1+\tau_t^C+\Gamma_v(.)\right)P_{C,t}C_t(i)+P_{I,t}I_t(i)+R_t^{-1}B_{t+1}(i)+\left((1-\Gamma_B^*(.))\,R_t^*\right)^{-1}S_t^{H,US}B_{t+1}^*(i)\\
&+M_t(i)+\Phi_t(i)+\Xi_t\Bigg]\\
&=(1-\lambda)\left[\left(1-\tau_t^N-\tau_t^{W_h}\right)W_t(i)N_t(i)+\left(1-\tau_t^D\right)D_t(i)+TR_t(i)-T_t(i)\right]
\end{aligned}
\tag{8}
$$

$$
\begin{aligned}
&(1-\lambda)\,\lambda\Bigg[\left(1+\tau_t^C+\Gamma_v(.)\right)P_{C,t}C_t(i)+P_{I,t}I_t(i)+R_t^{-1}B_{t+1}(i)+\left((1-\Gamma_B^*(.))\,R_t^*\right)^{-1}S_t^{H,US}B_{t+1}^*(i)\\
&+M_t(i)+\Phi_t(i)+\Xi_t\Bigg]\\
&=(1-\lambda)\,\lambda\left[\left(1-\tau_t^N-\tau_t^{W_h}\right)W_t(i)N_t(i)+\left(1-\tau_t^D\right)D_t(i)+TR_t(i)-T_t(i)\right]\\
&\quad+\frac{(1-\lambda)\,\lambda}{\lambda}\left[\left(\left(1-\tau_t^K\right)\left(R_{K,t}u_t(i)-\Gamma_u(.)P_{I,t}\right)+\tau_t^K\delta P_{I,t}\right)K_t(i)+B_t(i)+S_t^{H,US}B_t^*(i)+M_{t-1}\right].
\end{aligned}
\tag{9}
$$

$$
\begin{aligned}
&(1-\lambda)\,\lambda^2\Bigg[\left(1+\tau_t^C+\Gamma_v(.)\right)P_{C,t}C_t(i)+P_{I,t}I_t(i)+R_t^{-1}B_{t+1}(i)+\left((1-\Gamma_B^*(.))\,R_t^*\right)^{-1}S_t^{H,US}B_{t+1}^*(i)\\
&+M_t(i)+\Phi_t(i)+\Xi_t\Bigg]\\
&=(1-\lambda)\,\lambda^2\left[\left(1-\tau_t^N-\tau_t^{W_h}\right)W_t(i)N_t(i)+\left(1-\tau_t^D\right)D_t(i)+TR_t(i)-T_t(i)\right]\\
&\quad+\frac{(1-\lambda)\,\lambda^2}{\lambda}\left[\left(\left(1-\tau_t^K\right)\left(R_{K,t}u_t(i)-\Gamma_u(.)P_{I,t}\right)+\tau_t^K\delta P_{I,t}\right)K_t(i)+B_t(i)+S_t^{H,US}B_t^*(i)+M_{t-1}\right].
\end{aligned}
\tag{10}
$$

...

Let us denote the relation between generation specific variable x_t^i and aggregate variable x_t as follows:

$$x_t = \sum_{i=-\infty}^{t} (1-\delta)\,\delta^{t-i} x_t^i$$

$$x_{t-1} = \sum_{i=-\infty}^{t} (1-\delta)\,\delta^{t-1-i} x_{t-1}^i$$

So that the aggregate budget constraint reads as follows:

$$\begin{aligned}
& \left(1+\tau_t^C+\Gamma_\upsilon(.)\right) P_{C,t}C_t + P_{I,t}I_t(i) + R_t^{-1}B_{t+1} + \left(\left(1-\Gamma_B^*(.)\right)R_t^*\right)^{-1} S_t^{H,US} B_{t+1}^*(i) + M_t + \Phi_t + \Xi_t \\
= \quad & \left(1-\tau_t^N-\tau_t^{W_h}\right) W_t N_t + \left(1-\tau_t^D\right) D_t + TR_t(i) - T_t \\
& + \left(\left(1-\tau_t^K\right)\left(R_{K,t}u_t(i) - \Gamma_u(.)P_{I,t}\right) + \tau_t^K \delta P_{I,t}\right) K_t + B_t + S_t^{H,US} B_t^*(i) + M_{t-1}. \qquad (11)
\end{aligned}$$

which coincides exactly with the aggregate budget constraint when households have infinite lifetimes.

J-type households

Following the same steps, we receive the aggregate budget constraint for I-type households:

$$\begin{aligned}
& \left(1+\tau_t^C+\Gamma_\upsilon(.)\right) P_{C,t}C_t + M_t + \Phi_t \\
= \quad & \left(1-\tau_t^N-\tau_t^{W_h}\right) W_t N_t + TR_t - T_t + M_{t-1} \qquad (12)
\end{aligned}$$

B.2 Aggregate Euler Equation

In this section, we derive the aggregate Euler equations for both types of households. To save space we derive the aggregate relationships for type I households only. The same steps are applied in the derivation of the aggregate euler equation of type J households.

I-type households

As discussed above, the flow budget constraint of household i in a generic period t is specified as in 6. Let us derive the aggregate Euler equation for home bonds B_t. Define variable $\Delta_t(i)$ as follows:

$$\Delta_t(i) = \left(1 - \tau_t^N - \tau_t^{W_h}\right) W_t(i)N_t(i) + \left(1 - \tau_t^D\right) D_t(i) + TR_t(i) - T_t(i) - \Phi_t(i) - \Xi_t(i) \quad (13)$$

Writing the budget constraint for periods $t+1$, $t+2$, ..., we receive:

$$\Bigg[\left(1 + \tau_{t+1}^C + \Gamma_v(.)\right) P_{C,t+1} C_{t+1}(i) + \frac{P_{I,t+1}}{1 - \Gamma_{I,t+1}(.)} K_{t+2}(i) + R_{t+1}^{-1} B_{t+2}(i) + \left(\left(1 - \Gamma_B^*(.)\right) R_{t+1}^*\right)^{-1} S_{t+1}^{H,US} B_{t+2}^*(i)$$
$$+ M_{t+1}(i) - \Delta_{t+1}(i) \Bigg] \lambda$$
$$= \Bigg[\left(\left(1 - \tau_{t+1}^K\right) \left(R_{K,t+1} u_{t+1}(i) - \Gamma_u(.) P_{I,t+1}\right) + P_{I,t+1} \left(\tau_{t+1}^K \delta + \frac{\lambda(1-\delta)}{1 - \Gamma_{I,t+1}(.)} \right) \right) K_{t+1}(i)$$
$$+ S_{t+1}^{H,US} B_{t+1}^*(i) + M_t \Bigg] + B_{t+1}(i). \quad (14)$$

$$\Bigg[\left(1 + \tau_{t+2}^C + \Gamma_v(.)\right) P_{C,t+2} C_{t+2}(i) + \frac{P_{I,t+2}}{1 - \Gamma_{I,t+2}(.)} K_{t+3}(i) + R_{t+2}^{-1} B_{t+3}(i) + \left(\left(1 - \Gamma_B^*(.)\right) R_{t+2}^*\right)^{-1} S_{t+2}^{H,US} B_{t+3}^*(i)$$
$$+ M_{t+2}(i) - \Delta_{t+2}(i) \Bigg] \lambda$$
$$= \Bigg[\left(\left(1 - \tau_{t+2}^K\right) \left(R_{K,t+2} u_{t+2}(i) - \Gamma_u(.) P_{I,t+2}\right) + P_{I,t+2} \left(\tau_{t+2}^K \delta + \frac{\lambda(1-\delta)}{1 - \Gamma_{I,t+2}(.)} \right) \right) K_{t+2}(i)$$
$$+ S_{t+2}^{H,US} B_{t+2}^*(i) + M_{t+1} \Bigg] + B_{t+2}(i). \quad (15)$$

...

Notice that in (14) and (15) above we have isolated at the very end of each expression B_{t+1} and B_{t+2}, respectively. This is because we will work with these terms as our focus in this subsection is to derive the aggregate euler equation for home bonds B_t. Using (14) to substitute for $B_{t+1}(i)$ in (6) we receive:

$$
\begin{aligned}
&\left(1+\tau_t^C+\Gamma_\upsilon(.)\right)P_{C,t}C_t(i)+\frac{P_{I,t}}{1-\Gamma_{I,t}(.)}K_{t+1}(i)+\left(\left(1-\Gamma_B^*(.)\right)R_t^*\right)^{-1}S_t^{H,US}B_{t+1}^*(i)\\
&\qquad+M_t(i)+\Phi_t(i)+\Xi_t-\Delta_t(i)\\
&\frac{\lambda}{R_t}\Bigg[\left(1+\tau_{t+1}^C+\Gamma_\upsilon(.)\right)P_{C,t+1}C_{t+1}(i)+\frac{P_{I,t+1}}{1-\Gamma_{I,t+1}(.)}K_{t+2}(i)+\left(\left(1-\Gamma_B^*(.)\right)R_{t+1}^*\right)^{-1}S_{t+1}^{H,US}B_{t+2}^*(i)\\
&+M_{t+1}(i)-\Delta_{t+1}(i)\Bigg]+\lambda\frac{B_{t+2}(i)}{R_tR_{t+1}}\\
=&\qquad\frac{1}{\lambda}\Bigg[\left(\left(1-\tau_t^K\right)\left(R_{K,t}u_t(i)-\Gamma_u(.)P_{I,t}\right)+P_{I,t}\left(\tau_t^K\delta+\frac{\lambda\left(1-\delta\right)}{1-\Gamma_{I,t}(.)}\right)\right)K_t(i)\\
&\qquad+B_t(i)+S_t^{H,US}B_t^*(i)+M_{t-1}\Bigg]\\
&\qquad+\frac{1}{R_t}\Bigg[\left(\left(1-\tau_{t+1}^K\right)\left(R_{K,t+1}u_{t+1}(i)-\Gamma_u(.)P_{I,t+1}\right)+P_{I,t+1}\left(\tau_{t+1}^K\delta+\frac{\lambda\left(1-\delta\right)}{1-\Gamma_{I,t+1}(.)}\right)\right)K_{t+1}(i)\\
&\qquad+S_{t+1}^{H,US}B_{t+1}^*(i)+M_t\Bigg] \qquad (16)
\end{aligned}
$$

Using (15) to substitute for $B_{t+2}(i)$ in (16), we get:

$$
\begin{aligned}
&\left(1+\tau_t^C+\Gamma_v(.)\right) P_{C,t}C_t(i) + \frac{P_{I,t}}{1-\Gamma_{I,t}(.)}K_{t+1}(i) + \left(\left(1-\Gamma_B^*(.)\right)R_t^*\right)^{-1} S_t^{H,US} B_{t+1}^*(i) \\
&\quad + M_t(i) + \Phi_t(i) + \Xi_t - \Delta_t(i) \\
&\frac{\lambda}{R_t}\Bigg[\left(1+\tau_{t+1}^C+\Gamma_v(.)\right) P_{C,t+1}C_{t+1}(i) + \frac{P_{I,t+1}}{1-\Gamma_{I,t+1}(.)}K_{t+2}(i) + \left(\left(1-\Gamma_B^*(.)\right)R_{t+1}^*\right)^{-1} S_{t+1}^{H,US} B_{t+2}^*(i) \\
&+ M_{t+1}(i) - \Delta_{t+1}(i)\Bigg] \\
&\frac{\lambda^2}{R_tR_{t+1}}\Bigg[\left(1+\tau_{t+2}^C+\Gamma_v(.)\right) P_{C,t+2}C_{t+2}(i) + \frac{P_{I,t+2}}{1-\Gamma_{I,t+2}(.)}K_{t+3}(i) + \left(\left(1-\Gamma_B^*(.)\right)R_{t+2}^*\right)^{-1} S_{t+2}^{H,US} B_{t+3}^*(i) \\
&+ M_{t+2}(i) - \Delta_{t+2}(i)\Bigg] + \lambda^2 \frac{B_{t+3}(i)}{R_tR_{t+1}R_{t+2}} \\
= \quad & \frac{1}{\lambda}\Bigg[\left(\left(1-\tau_t^K\right)\left(R_{K,t}u_t(i) - \Gamma_u(.)P_{I,t}\right) + P_{I,t}\left(\tau_t^K\delta + \frac{\lambda\left(1-\delta\right)}{1-\Gamma_{I,t}(.)}\right)\right) K_t(i) \\
&\quad + B_t(i) + S_t^{H,US}B_t^*(i) + M_{t-1}\Bigg] \\
&\quad + \frac{1}{R_t}\Bigg[\left(\left(1-\tau_{t+1}^K\right)\left(R_{K,t+1}u_{t+1}(i) - \Gamma_u(.)P_{I,t+1}\right) + P_{I,t+1}\left(\tau_{t+1}^K\delta + \frac{\lambda\left(1-\delta\right)}{1-\Gamma_{I,t+1}(.)}\right)\right) K_{t+1}(i) \\
&\quad + S_{t+1}^{H,US}B_{t+1}^*(i) + M_t\Bigg] \\
&\quad + \frac{\lambda}{R_tR_{t+1}}\Bigg[\left(\left(1-\tau_{t+2}^K\right)\left(R_{K,t+2}u_{t+2}(i) - \Gamma_u(.)P_{I,t+2}\right) + P_{I,t+2}\left(\tau_{t+2}^K\delta + \frac{\lambda\left(1-\delta\right)}{1-\Gamma_{I,t+2}(.)}\right)\right) K_{t+2}(i) \\
&\quad + S_{t+2}^{H,US}B_{t+2}^*(i) + M_{t+1}\Bigg] \qquad (17)
\end{aligned}
$$

Iterating forward we receive:

$$
\begin{aligned}
&\sum_{s=0}^{\infty} \frac{\lambda^s}{\prod_{s=0}^{s-1} R_{t+s}} \left[\left(1 + \tau^C_{t+s} + \Gamma_\upsilon(.)\right) P_{C,t+s} C_{t+s}(i) \right] \\
&+ \sum_{s=0}^{\infty} \frac{\lambda^s}{\prod_{s=0}^{s-1} R_{t+s}} \Bigg\{ \left(\frac{R_{t+s}-1}{R_{t+s}} \right) M_{t+s}(i) - \Delta_{t+s}(i) + \left(\frac{S^{H,US}_{t+s}}{\left(1 - \Gamma^*_{t+s,B}(.)\right) R^*_{t+s}} - \frac{S^{H,US}_{t+s+1}}{R_{t+s}} \right) B^*_{t+s+1}(i) \\
&\left[\frac{P_{I,t+s}}{1 - \Gamma_{I,t+s}(.)} - \frac{1}{R_{t+s}} \left(\left(1 - \tau^K_{t+s}\right) \left(R_{K,t+s} u_{t+s}(i) - \Gamma_u(.) P_{I,t+s}\right) + P_{I,t+s} \left(\tau^K_{t+1} \delta + \frac{\lambda(1-\delta)}{1 - \Gamma_{I,t+s}(.)} \right) \right) \right] K_{t+s}(i) \Bigg\} \\
&= \frac{1}{\lambda} \Bigg[\left(\left(1 - \tau^K_t\right) \left(R_{K,t} u_t(i) - \Gamma_u(.) P_{I,t}\right) + P_{I,t} \left(\tau^K_t \delta + \frac{\lambda(1-\delta)}{1 - \Gamma_{I,t}(.)} \right) \right) K_t(i) \\
&\qquad + B_t(i) + S^{H,US}_t B^*_t(i) + M_{t-1} \Bigg] \qquad (18)
\end{aligned}
$$

where we have imposed the transversality condition:

$$
\lim_{s \to \infty} \frac{\lambda^s}{\Pi_{s=0}^{s-1} R_{t+s}} \left[\frac{B_{t+s+1}(i)}{R_{t+s}} + \frac{S^{H,US}_{t+s} B^*_{t+s+1}(i)}{\left(\left(1 - \Gamma^*_B(.)\right) R^*_{t+s}\right)} + \frac{P_{I,t+s}}{1 - \Gamma_{I,t+s}} K_{t+s+1} + M_{t+s} \right] = 0 \qquad (19)
$$

Let us now work with the first term on the LHS (18). For simplicity, let us assume log-preferences in consumption (i.e. σ). The Euler equation for home bonds for household I writes as follows:

$$
\beta E_t \frac{\left(1 + \tau^C_t + \Gamma_{\upsilon,t}(.) + \Gamma'_{\upsilon,t}(.)\upsilon_t\right) \left(C_t(i) - \kappa C_{t-1}\right)}{\left(1 + \tau^C_{t+1} + \Gamma_{\upsilon,t+1}(.) + \Gamma'_{\upsilon,t+1}(.)\upsilon_{t+1}\right) \left(C_{t+1}(i) - \kappa C_t\right)} \Pi^{-1}_{t+1} = 1 \qquad (20)
$$

Iterating forward, we receive:

$$
\begin{aligned}
&\frac{\lambda^s}{\Pi_{s=0}^{s-1} R_{t+s}} P_{C,t+s} \left(1 + \tau^C_{t+s} + \Gamma_{\upsilon,t+s}(.) + \Gamma'_{\upsilon,t+s}(.)\upsilon_{t+s}\right) \left(C_{t+s}(i) - \kappa C_{t+s-1}\right) \\
&\qquad = (\lambda\beta)^s P_{C,t} \left(1 + \tau^C_t + \Gamma_{\upsilon,t}(.) + \Gamma'_{\upsilon,t}(.)\upsilon_t\right) \left(C_t(i) - \kappa C_{t-1}\right) \qquad (21)
\end{aligned}
$$

Adding and subtracting $\kappa \sum_{s=0}^{\infty} \frac{\lambda^s}{\Pi_{s=0}^{s-1} R_{t+s}} P_{C,t+s} \left(1 + \tau^C_{t+s} + \Gamma_{\upsilon,t+s}(.)\right) C_{t+s-1}$ and $\sum_{s=0}^{\infty} \frac{\lambda^s}{\Pi_{s=0}^{s-1} R_{t+s}} \left(\Gamma'_{\upsilon,t+s}(.)\upsilon_{t+s}\right) P_{C,t+s} \left(C_{t+s}(i) - \kappa C_{t+s-1}\right)$ in (18), we receive:

$$\sum_{s=0}^{\infty}\frac{\lambda^s}{\prod_{s=0}^{s-1}R_{t+s}}\left[\left(1+\tau^C_{t+s}+\Gamma_\upsilon(.)+\Gamma'_{\upsilon,t+s}(.)\upsilon_{t+s}\right)P_{C,t+s}\left(C_{t+s}(i)-\kappa C_{t+s-1}\right)\right]$$

$$-\sum_{s=0}^{\infty}\frac{\lambda^s}{\Pi_{s=0}^{s-1}R_{t+s}}\left(\Gamma'_{\upsilon,t+s}(.)\upsilon_{t+s}\right)P_{C,t+s}\left(C_{t+s}(i)-\kappa C_{t+s-1}\right)+\kappa\sum_{s=0}^{\infty}\frac{\lambda^s}{\Pi_{s=0}^{s-1}R_{t+s}}P_{C,t+s}\left(1+\tau^C_{t+s}+\Gamma_{\upsilon,t+s}(.)\right)C_{t+s-}$$

$$+\sum_{s=0}^{\infty}\frac{\lambda^s}{\prod_{s=0}^{s-1}R_{t+s}}\Bigg\{\left(\frac{R_{t+s}-1}{R_{t+s}}\right)M_{t+s}(i)-\Delta_{t+s}(i)+\left(\frac{S^{H,US}_{t+s}}{\left(1-\Gamma^*_{t+s,B}(.)\right)R^*_{t+s}}-\frac{S^{H,US}_{t+s+1}}{R_{t+s}}\right)B^*_{t+s+1}(i)$$

$$\left[\frac{P_{I,t+s}}{1-\Gamma_{I,t+s}(.)}-\frac{1}{R_{t+s}}\left(\left(1-\tau^K_{t+s}\right)\left(R_{K,t+s}u_{t+s}(i)-\Gamma_u(.)P_{I,t+s}\right)+P_{I,t+s}\left(\tau^K_{t+1}\delta+\frac{\lambda(1-\delta)}{1-\Gamma_{I,t+s}(.)}\right)\right)\right]K_{t+s}(i)\Bigg\}$$

$$=\frac{1}{\lambda}\Bigg[\left(\left(1-\tau^K_t\right)\left(R_{K,t}u_t(i)-\Gamma_u(.)P_{I,t}\right)+P_{I,t}\left(\tau^K_t\delta+\frac{\lambda(1-\delta)}{1-\Gamma_{I,t}(.)}\right)\right)K_t(i)$$

$$+B_t(i)+S^{H,US}_tB^*_t(i)+M_{t-1}\Bigg] \tag{22}$$

Substituting (21) in (22) above yields:

$$P_{C,t}\left(1+\tau^C_t+\Gamma_{\upsilon,t}(.)+\Gamma'_{\upsilon,t}(.)\upsilon_t\right)\left(C_t(i)-\kappa C_{t-1}\right)\sum_{s=0}^{\infty}(\lambda\beta)^s$$

$$-P_{C,t}\left(1+\tau^C_t+\Gamma_{\upsilon,t}(.)+\Gamma'_{\upsilon,t}(.)\upsilon_t\right)\left(C_t(i)-\kappa C_{t-1}\right)\sum_{s=0}^{\infty}(\lambda\beta)^s\frac{\Gamma'_{\upsilon,t+s}(.)\upsilon_{t+s}}{1+\tau^C_{t+s}+\Gamma_{\upsilon,t+s}(.)+\Gamma'_{\upsilon,t+s}(.)\upsilon_{t+S}}$$

$$+\kappa\sum_{s=0}^{\infty}\frac{\lambda^s}{\Pi_{s=0}^{s-1}R_{t+s}}P_{C,t+s}\left(1+\tau^C_{t+s}+\Gamma_{\upsilon,t+s}(.)\right)C_{t+s-1}$$

$$+\sum_{s=0}^{\infty}\frac{\lambda^s}{\prod_{s=0}^{s-1}R_{t+s}}\Bigg\{\left(\frac{R_{t+s}-1}{R_{t+s}}\right)M_{t+s}(i)-\Delta_{t+s}(i)+\left(\frac{S^{H,US}_{t+s}}{\left(1-\Gamma^*_{t+s,B}(.)\right)R^*_{t+s}}-\frac{S^{H,US}_{t+s+1}}{R_{t+s}}\right)B^*_{t+s+1}(i)$$

$$\left[\frac{P_{I,t+s}}{1-\Gamma_{I,t+s}(.)}-\frac{1}{R_{t+s}}\left(\left(1-\tau^K_{t+s}\right)\left(R_{K,t+s}u_{t+s}(i)-\Gamma_u(.)P_{I,t+s}\right)+P_{I,t+s}\left(\tau^K_{t+1}\delta+\frac{\lambda(1-\delta)}{1-\Gamma_{I,t+s}(.)}\right)\right)\right]K_{t+s}(i)\Bigg\}$$

$$=\frac{1}{\lambda}\Bigg[\left(\left(1-\tau^K_t\right)\left(R_{K,t}u_t(i)-\Gamma_u(.)P_{I,t}\right)+P_{I,t}\left(\tau^K_t\delta+\frac{\lambda(1-\delta)}{1-\Gamma_{I,t}(.)}\right)\right)K_t(i)$$

$$+B_t(i)+S^{H,US}_tB^*_t(i)+M_{t-1}\Bigg] \tag{23}$$

Gathering the first two terms on the LHS:

$$
\begin{aligned}
&P_{C,t}\left(1+\tau_t^C+\Gamma_{\upsilon,t}(.)+\Gamma'_{\upsilon,t}(.)\upsilon_t\right)\left(C_t(i)-\kappa C_{t-1}\right)\left[\sum_{s=0}^{\infty}(\lambda\beta)^s-\sum_{s=0}^{\infty}(\lambda\beta)^s\frac{\Gamma'_{\upsilon,t+s}(.)\upsilon_{t+s}}{1+\tau_{t+s}^C+\Gamma_{\upsilon,t+s}(.)+\Gamma'_{\upsilon,t+s}(.)\upsilon_{t+s}}\right]\\
&+\kappa\sum_{s=0}^{\infty}\frac{\lambda^s}{\Pi_{s=0}^{s-1}R_{t+s}}P_{C,t+s}\left(1+\tau_{t+s}^C+\Gamma_{\upsilon,t+s}(.)\right)C_{t+s-1}\\
&+\sum_{s=0}^{\infty}\frac{\lambda^s}{\prod_{s=0}^{s-1}R_{t+s}}\Bigg\{\left(\frac{R_{t+s}-1}{R_{t+s}}\right)M_{t+s}(i)-\Delta_{t+s}(i)+\left(\frac{S_{t+s}^{H,US}}{\left(1-\Gamma_{t+s,B}^*(.)\right)R_{t+s}^*}-\frac{S_{t+s+1}^{H,US}}{R_{t+s}}\right)B_{t+s+1}^*(i)\\
&\left[\frac{P_{I,t+s}}{1-\Gamma_{I,t+s}(.)}-\frac{1}{R_{t+s}}\left(\left(1-\tau_{t+s}^K\right)\left(R_{K,t+s}u_{t+s}(i)-\Gamma_u(.)P_{I,t+s}\right)+P_{I,t+s}\left(\tau_{t+1}^K\delta+\frac{\lambda(1-\delta)}{1-\Gamma_{I,t+s}(.)}\right)\right)\right]K_{t+s}(i)\Bigg\}\\
&=\frac{1}{\lambda}\Bigg[\left(\left(1-\tau_t^K\right)\left(R_{K,t}u_t(i)-\Gamma_u(.)P_{I,t}\right)+P_{I,t}\left(\tau_t^K\delta+\frac{\lambda(1-\delta)}{1-\Gamma_{I,t}(.)}\right)\right)K_t(i)\\
&\qquad+B_t(i)+S_t^{H,US}B_t^*(i)+M_{t-1}\Bigg]
\end{aligned}
\tag{24}
$$

Rearranging terms yields:

$$
\begin{aligned}
&P_{C,t}\left(1+\tau_t^C+\Gamma_{\upsilon,t}(.)+\Gamma'_{\upsilon,t}(.)\upsilon_t\right)\left(C_t(i)-\kappa C_{t-1}\right)\\
&=\frac{1}{\lambda\Psi_t}\Bigg[\left(\left(1-\tau_t^K\right)\left(R_{K,t}u_t(i)-\Gamma_u(.)P_{I,t}\right)+P_{I,t}\left(\tau_t^K\delta+\frac{\lambda(1-\delta)}{1-\Gamma_{I,t}(.)}\right)\right)K_t(i)\\
&\qquad+B_t(i)+S_t^{H,US}B_t^*(i)+M_{t-1}(i)\Bigg]\\
&-\frac{\kappa}{\Psi_t}\sum_{s=0}^{\infty}\frac{\lambda^s}{\Pi_{s=0}^{s-1}R_{t+s}}P_{C,t+s}\left(1+\tau_{t+s}^C+\Gamma_{\upsilon,t+s}(.)\right)C_{t+s-1}\\
&-\frac{1}{\Psi_t}\sum_{s=0}^{\infty}\frac{\lambda^s}{\prod_{s=0}^{s-1}R_{t+s}}\Bigg\{\left(\frac{R_{t+s}-1}{R_{t+s}}\right)M_{t+s}(i)-\Delta_{t+s}(i)+\left(\frac{S_{t+s}^{H,US}}{\left(1-\Gamma_{t+s,B}^*(.)\right)R_{t+s}^*}-\frac{S_{t+s+1}^{H,US}}{R_{t+s}}\right)B_{t+s+1}^*(i)\\
&-\left[\frac{P_{I,t+s}}{1-\Gamma_{I,t+s}(.)}-\frac{1}{R_{t+s}}\left(\left(1-\tau_{t+s}^K\right)\left(R_{K,t+s}u_{t+s}(i)-\Gamma_u(.)P_{I,t+s}\right)+P_{I,t+s}\left(\tau_{t+1}^K\delta+\frac{\lambda(1-\delta)}{1-\Gamma_{I,t+s}(.)}\right)\right)\right]K_{t+s}(i)
\end{aligned}
\tag{25}
$$

where Ψ_t is:

$$\Psi_t = \sum_{s=0}^{\infty} (\lambda\beta)^s - \sum_{s=0}^{\infty} (\lambda\beta)^s \frac{\Gamma'_{\upsilon,t+s}(.)\upsilon_{t+s}}{1+\tau^C_{t+s}+\Gamma_{\upsilon,t+s}(.)+\Gamma'_{\upsilon,t+s}(.)\upsilon_{t+s}} \tag{26}$$

Aggregate:

$$\begin{aligned}
&P_{C,t}\left(1+\tau_t^C+\Gamma_{\upsilon,t}(.)+\Gamma'_{\upsilon,t}(.)\upsilon_t\right)\left(C_{I,t}-\kappa C_{I,t-1}\right)\\
&=\frac{1}{\Psi_t}\Bigg[\left(\left(1-\tau_t^K\right)\left(R_{K,t}u_t-\Gamma_u(.)P_{I,t}\right)+P_{I,t}\left(\tau_t^K\delta+\frac{\lambda\left(1-\delta\right)}{1-\Gamma_{I,t}(.)}\right)\right)K_t\\
&\qquad+B_t+S_t^{H,US}B_t^*+M_{I,t-1}\Bigg]\\
&-\frac{\kappa}{\Psi_t}\sum_{s=0}^{\infty}\frac{\lambda^s}{\Pi_{s=0}^{s-1}R_{t+s}}P_{C,t+s}\left(1+\tau_{t+s}^C+\Gamma_{\upsilon,t+s}(.)\right)C_{I,t+s-1}\\
&-\frac{1}{\Psi_t}\sum_{s=0}^{\infty}\frac{\lambda^s}{\prod_{s=0}^{s-1}R_{t+s}}\Bigg\{\left(\frac{R_{t+s}-1}{R_{t+s}}\right)M_{I,t+s}-\Delta_{t+s}+\left(\frac{S_{t+s}^{H,US}}{\left(1-\Gamma^*_{t+s,B}(.)\right)R^*_{t+s}}-\frac{S_{t+s+1}^{H,US}}{R_{t+s}}\right)B^*_{t+s+1}\\
&-\left[\frac{P_{I,t+s}}{1-\Gamma_{I,t+s}(.)}-\frac{1}{R_{t+s}}\left(\left(1-\tau_{t+s}^K\right)\left(R_{K,t+s}u_{t+s}-\Gamma_u(.)P_{I,t+s}\right)+P_{I,t+s}\left(\tau_{t+1}^K\delta+\frac{\lambda\left(1-\delta\right)}{1-\Gamma_{I,t+s}(.)}\right)\right)\right]K_{t+s+1}\Bigg\}
\end{aligned} \tag{27}$$

Adding and subtracting $\frac{\lambda}{R_t}\frac{1}{\Psi_t}\Bigg[\left(\left(1-\tau_{t+1}^K\right)\left(R_{K,t+1}u_{t+1}(i)-\Gamma_u(.)P_{I,t+1}\right)+P_{I,t+1}\left(\tau_{t+1}^K\delta+\frac{\lambda(1-\delta)}{1-\Gamma_{I,t+1}(.)}\right)\right)K_{t+1}(i)$ $B_{t+1}+S_{t+1}^{H,US}B^*_{t+1}+M_t\Bigg]$, we receive:

$$
\begin{aligned}
&P_{C,t}\left(1+\tau_t^C+\Gamma_{v,t}(.)+\Gamma'_{v,t}(.)v_t\right)\left(C_{I,t}-\kappa C_{I,t-1}\right)\\
&=\frac{1}{\Psi_t}\Bigg\{\left(\left(1-\tau_t^K\right)\left(R_{K,t}u_t-\Gamma_u(.)P_{I,t}\right)+P_{I,t}\left(\tau_t^K\delta+\frac{\lambda\left(1-\delta\right)}{1-\Gamma_{I,t}(.)}\right)\right)K_t\\
&\quad+B_t+S_t^{H,US}B_t^*+M_{I,t-1}+\Delta_t\\
&-\left(\frac{R_t-1}{R_t}\right)M_{I,t}-\left(\frac{S_t^{H,US}}{\left(1-\Gamma^*_{t,B}(.)\right)R_t^*}-\frac{S_{t+1}^{H,US}}{R_t}\right)B_{t+1}^*\\
&+\left[\frac{P_{I,t}}{1-\Gamma_{I,t}(.)}-\frac{1}{R_t}\left(\left(1-\tau_{t+1}^K\right)\left(R_{K,t+1}u_{t+1}-\Gamma_u(.)P_{I,t+1}\right)+P_{I,t+1}\left(\tau_{t+1}^K\delta+\frac{\lambda\left(1-\delta\right)}{1-\Gamma_{I,t+1}(.)}\right)\right)\right]K_{t+1}\\
&-\kappa P_{C,t}\left(1+\tau_t^C+\Gamma_{v,t}(.)\right)C_{I,t-1}\\
&-\frac{\lambda}{R_t}\Bigg[\left(\left(1-\tau_{t+1}^K\right)\left(R_{K,t+1}u_{t+1}(i)-\Gamma_u(.)P_{I,t+1}\right)+P_{I,t+1}\left(\tau_{t+1}^K\delta+\frac{\lambda\left(1-\delta\right)}{1-\Gamma_{I,t+1}(.)}\right)\right)K_{t+1}(i)\\
&\quad+B_{t+1}+S_{t+1}^{H,US}B_{t+1}^*+M_{I,t}\Bigg]\\
&+\frac{\lambda}{R_t}\Psi_{t+1}P_{C,t+1}\left(1+\tau_{t+1}^C+\Gamma_{v,t+1}(.)+\Gamma'_{v,t+1}(.)v_{t+1}\right)\left(C_{I,t+1}-\kappa C_{I,t}\right)\Bigg\}
\end{aligned}
\tag{28}
$$

Using the aggregate budget constraint for type I households, (11), to substitute out for the terms in the first line of RHS in (28), we receive:

$$
\begin{aligned}
&P_{C,t}\left(1+\tau_t^C+\Gamma_{\upsilon,t}(.)+\Gamma'_{\upsilon,t}(.)\upsilon_t\right)\left(C_{I,t}-\kappa C_{I,t-1}\right)\\
&=\frac{1}{\Psi_t}\Bigg\{\left(1+\tau_t^C+\Gamma_\upsilon(.)\right)P_{C,t}\left(C_t-\kappa C_{I,t-1}\right)+\frac{P_{I,t}}{1-\Gamma_{I,t+1}(.)}K_{t+1}+R_t^{-1}B_{t+1}+\frac{S_t^{H,US}}{\left(1-\Gamma_B^*(.)\right)R_t^*}B_{t+1}^*\\
&+M_{I,t}-\left(\frac{R_t-1}{R_t}\right)M_{I,t}-\left(\frac{S_t^{H,US}}{\left(1-\Gamma_{t,B}^*(.)\right)R_t^*}-\frac{S_{t+1}^{H,US}}{R_t}\right)B_{t+1}^*\\
&-\left[\frac{P_{I,t}}{1-\Gamma_{I,t}(.)}-\frac{1}{R_t}\left(\left(1-\tau_{t+1}^K\right)\left(R_{K,t+1}u_{t+1}-\Gamma_u(.)P_{I,t+1}\right)+P_{I,t+1}\left(\tau_{t+1}^K\delta+\frac{\lambda\left(1-\delta\right)}{1-\Gamma_{I,t+1}(.)}\right)\right)\right]K_{t+1}\\
&-\frac{\lambda}{R_t}\Bigg[\left(\left(1-\tau_{t+1}^K\right)\left(R_{K,t+1}u_{t+1}(i)-\Gamma_u(.)P_{I,t+1}\right)+P_{I,t+1}\left(\tau_{t+1}^K\delta+\frac{\lambda\left(1-\delta\right)}{1-\Gamma_{I,t+1}(.)}\right)\right)K_{t+1}(i)\\
&\qquad+B_{t+1}+S_{t+1}^{H,US}B_{t+1}^*+M_{I,t}\Bigg]\\
&+\frac{\lambda}{R_t}\Psi_{t+1}P_{C,t+1}\left(1+\tau_{t+1}^C+\Gamma_{\upsilon,t+1}(.)+\Gamma'_{\upsilon,t+1}(.)\upsilon_{t+1}\right)\left(C_{I,t+1}-\kappa C_{I,t}\right)\Bigg\} \qquad (29)
\end{aligned}
$$

or

$$\left[\left(1+\tau_t^C+\Gamma_{\upsilon,t}(.)+\Gamma'_{\upsilon,t}(.)\upsilon_t\right)-\frac{1+\tau_t^C+\Gamma_\upsilon(.)}{\Psi_t}\right]P_{C,t}\left(C_{I,t}-\kappa C_{I,t-1}\right)$$
$$=\frac{1}{\Psi_t}\Bigg\{\frac{P_{I,t}}{1-\Gamma_{I,t+1}(.)}K_{t+1}+R_t^{-1}B_{t+1}+\frac{S_t^{H,US}}{\left(1-\Gamma_B^*(.)\right)R_t^*}B_{t+1}^*$$

$$+M_{I,t}-\left(\frac{R_t-1}{R_t}\right)M_{I,t}-\left(\frac{S_t^{H,US}}{\left(1-\Gamma_{t,B}^*(.)\right)R_t^*}-\frac{S_{t+1}^{H,US}}{R_t}\right)B_{t+1}^*$$
$$-\left[\frac{P_{I,t}}{1-\Gamma_{I,t}(.)}-\frac{1}{R_t}\left(\left(1-\tau_{t+1}^K\right)\left(R_{K,t+1}u_{t+1}-\Gamma_u(.)P_{I,t+1}\right)+P_{I,t+1}\left(\tau_{t+1}^K\delta+\frac{\lambda\left(1-\delta\right)}{1-\Gamma_{I,t+1}(.)}\right)\right)\right]K_{t+1}$$
$$-\frac{\lambda}{R_t}\Bigg[\left(\left(1-\tau_{t+1}^K\right)\left(R_{K,t+1}u_{t+1}(i)-\Gamma_u(.)P_{I,t+1}\right)+P_{I,t+1}\left(\tau_{t+1}^K\delta+\frac{\lambda\left(1-\delta\right)}{1-\Gamma_{I,t+1}(.)}\right)\right)K_{t+1}$$
$$+B_{t+1}+S_{t+1}^{H,US}B_{t+1}^*+M_{I,t}\Bigg]$$
$$+\frac{\lambda}{R_t}\Psi_{t+1}P_{C,t+1}\left(1+\tau_{t+1}^C+\Gamma_{\upsilon,t+1}(.)+\Gamma'_{\upsilon,t+1}(.)\upsilon_{t+1}\right)\left(C_{I,t+1}-\kappa C_{I,t}\right)\Bigg\} \tag{30}$$

Gathering terms yields:

$$\left[\left(1+\tau_t^C+\Gamma_{\upsilon,t}(.)+\Gamma'_{\upsilon,t}(.)\upsilon_t\right)\Psi_t-1-\tau_t^C-\Gamma_\upsilon(.)\right]\frac{R_t}{\lambda}P_{C,t}\left(C_{I,t}-\kappa C_{I,t-1}\right)$$
$$=\frac{1-\lambda}{\lambda}\Bigg[B_{t+1}+\frac{S_t^{H,US}}{\left(1-\Gamma_B^*(.)\right)R_t^*}B_{t+1}^*+M_{I,t}$$
$$+\left(\left(1-\tau_{t+1}^K\right)\left(R_{K,t+1}u_{t+1}(i)-\Gamma_u(.)P_{I,t+1}\right)+P_{I,t+1}\left(\tau_{t+1}^K\delta+\frac{\lambda\left(1-\delta\right)}{1-\Gamma_{I,t+1}(.)}\right)\right)K_{t+1}\Bigg]$$
$$+\Psi_{t+1}P_{C,t+1}\left(1+\tau_{t+1}^C+\Gamma_{\upsilon,t+1}(.)+\Gamma'_{\upsilon,t+1}(.)\upsilon_{t+1}\right)\left(C_{I,t+1}-\kappa C_{I,t}\right) \tag{31}$$

where we may write Ψ_t recursively as follows:

$$\Psi_t=1-\frac{\Gamma'_{\upsilon,t}(.)\upsilon_t}{\left(1+\tau_t^C+\Gamma_{\upsilon,t}(.)+\Gamma'_{\upsilon,t}(.)\upsilon_t\right)}+\lambda\beta\Psi_{t+1} \tag{32}$$

Substituting (32) into (33 for Ψ_t and dividing both sides by $P_{C,t+1}$, we end up to the final form of the Euler equation for home bonds B_t:

$$\begin{aligned}
&\beta\left(1+\tau_t^C+\Gamma_{\upsilon,t}(.)+\Gamma'_{\upsilon,t}(.)\upsilon_t\right)\frac{R_t}{\Pi_{C,t+1}}\left(C_{I,t}-\kappa C_{I,t-1}\right)\\
&=\frac{1-\lambda}{\lambda\Psi_{t+1}\Pi_{C,t+1}}\Bigg[B_{t+1}+S_{t+1}^{H,US}B_{t+1}^*+M_{I,t}\\
&+\left(\left(1-\tau_{t+1}^K\right)\left(R_{K,t+1}u_{t+1}(i)-\Gamma_u(.)P_{I,t+1}\right)+P_{I,t+1}\left(\tau_{t+1}^K\delta+\frac{\lambda\left(1-\delta\right)}{1-\Gamma_{I,t+1}(.)}\right)\right)K_{t+1}\Bigg]\\
&+\left(1+\tau_{t+1}^C+\Gamma_{\upsilon,t+1}(.)+\Gamma'_{\upsilon,t+1}(.)\upsilon_{t+1}\right)\left(C_{I,t+1}-\kappa C_{I,t}\right)
\end{aligned}\tag{33}$$

Clearly, setting $\lambda=1$, the above expression collapses to the Euler equation under infinite lifetimes. Following similar steps for foreign bond holdings, we end up to the Euler equation for foreign bonds, B_t^*.

$$\begin{aligned}
&\beta\left(1+\tau_t^C+\Gamma_{\upsilon,t}(.)+\Gamma'_{\upsilon,t}(.)\upsilon_t\right)\frac{\left(1-\Gamma_B^*(.)\right)R_t^*S_{t+1}^{H,US}}{S_t^{H,US}\Pi_{C,t+1}}\left(C_{I,t}-\kappa C_{I,t-1}\right)\\
&=\frac{1-\lambda}{\lambda\Psi_{t+1}\Pi_{C,t+1}}\Bigg[B_{t+1}+S_{t+1}^{H,US}B_{t+1}^*+M_{I,t}\\
&+\left(\left(1-\tau_{t+1}^K\right)\left(R_{K,t+1}u_{t+1}(i)-\Gamma_u(.)P_{I,t+1}\right)+P_{I,t+1}\left(\tau_{t+1}^K\delta+\frac{\lambda\left(1-\delta\right)}{1-\Gamma_{I,t+1}(.)}\right)\right)K_{t+1}\Bigg]\\
&+\left(1+\tau_{t+1}^C+\Gamma_{\upsilon,t+1}(.)+\Gamma'_{\upsilon,t+1}(.)\upsilon_{t+1}\right)\left(C_{I,t+1}-\kappa C_{I,t}\right)
\end{aligned}\tag{34}$$

J-type households

We now derive the aggregate Euler equation for money holdings of type-J households. It is convenient to define:

$$V_t(j) = \left(1 - \tau_t^N - \tau_t^{W_h}\right) W_t(j) N_t(j) + TR_t(j) - T_t(j) - \Phi_t(j) \tag{35}$$

Writing the budget constraint for periods $t+1$, $t+2$, ..., we receive:

$$\left[\left(1 + \tau_{t+1}^C + \Gamma_{\upsilon,t+1}(.)\right) P_{C,t+1} C_{t+1}(j) + M_{t+1}(j) - V_{t+1}\right] \lambda = M_t \tag{36}$$

$$\left[\left(1 + \tau_{t+2}^C + \Gamma_{\upsilon,t+2}(.)\right) P_{C,t+2} C_{t+2}(j) + M_{t+2}(j) - V_{t+2}\right] \lambda = M_{t+1} \tag{37}$$

...

Using (36) to substitute for $M_t(j)$ in (7), we get:

$$\left(1 + \tau_t^C + \Gamma_\upsilon(.)\right) P_{C,t} C_t(j) + \lambda \left(1 + \tau_{t+1}^C + \Gamma_{\upsilon,t+1}(.)\right) P_{C,t+1} C_{t+1}(j) + \lambda M_{t+1}(j) + \lambda V_{t+1}(j) - V_t(j)$$
$$= \frac{1}{\lambda} M_{t-1}(j) \tag{38}$$

Plugging expression (37) in (38), we receive:

$$\left(1 + \tau_t^C + \Gamma_{\upsilon,t}(.)\right) P_{C,t} C_t(j) + \lambda \left(1 + \tau_{t+1}^C + \Gamma_{\upsilon,t+1}(.)\right) P_{C,t+1} C_{t+1}(j) + \lambda^2 \left(1 + \tau_{t+2}^C + \Gamma_{\upsilon,t+2}(.)\right) P_{C,t+2} C_{t+2}(j)$$
$$- \lambda^2 V_{t+2}(j) - \lambda V_{t+1}(j) - V_t(j) + \lambda^2 M_{t+2}(j)$$
$$= \frac{1}{\lambda} M_{t-1} \tag{39}$$

Iterating, we may write:

$$\sum_{s=0}^{\infty}\lambda^s\left(1+\tau_{t+s}^C+\Gamma_{\upsilon,t+s}(.)\right)P_{C,t+s}C_{t+s}(j)-\sum_{s=0}^{\infty}\lambda^s V_{t+s}(j)=\frac{1}{\lambda}M_{t-1}(j) \tag{40}$$

where we have imposed the transversality:

$$\lim_{s\to\infty}\lambda^s M_{t+s}=0 \tag{41}$$

Let us now work with the first order on the LHS of expression (40). Iterating the euler equation forward, we receive:

$$\begin{aligned}&\lambda^s\left(1+\tau_{t+s}^C+\Gamma_{\upsilon,t+s}(.)+\Gamma'_{\upsilon,t+s}(.)\upsilon_{t+s}\right)P_{C,t+s}\left(C_{t+s}(j)-\kappa C_{J,t+s-1}\right)\\&\quad=(\lambda\beta)^s\frac{\left(1+\tau_t^C+\Gamma_{\upsilon,t}(.)+\Gamma'_{\upsilon,t}(.)\upsilon_t\right)}{\Pi_{s=0}^{s-1}\left(1-\Gamma'_{\upsilon,t+s}(.)\upsilon_{t+s}^2\right)}P_{C,t}\left(C_t(j)-\kappa C_{J,t-1}\right)\end{aligned} \tag{42}$$

Adding and subtracting $\sum_{s=0}^{\infty}\lambda^s\left(\Gamma'_{\upsilon,t+s}(.)\upsilon_{t+s}\right)P_{C,t+s}\left(C_{t+s}(j)-\kappa C_{J,t+s-1}\right)$ and

$\kappa\sum_{s=0}^{\infty}\lambda^s\left(1+\tau_{t+s}^C+\Gamma_{\upsilon,t+s}(.)\right)P_{C,t+s}C_{J,t+s-1}$ in (40):

$$\begin{aligned}&\sum_{s=0}^{\infty}\lambda^s\left(1+\tau_{t+s}^C+\Gamma_{\upsilon,t+s}(.)+\Gamma'_{\upsilon,t+s}(.)\upsilon_{t+s}\right)P_{C,t+s}\left(C_{t+s}(j)-\kappa C_{J,t+s-1}\right)\\&+\kappa\sum_{s=0}^{\infty}\lambda^s\left(1+\tau_{t+s}^C+\Gamma_{\upsilon,t+s}(.)\right)P_{C,t+s}C_{J,t+s-1}-\sum_{s=0}^{\infty}\lambda^s\left(\Gamma'_{\upsilon,t+s}(.)\upsilon_{t+s}^2\right)P_{C,t+s}\left(C_{t+s}(j)-\kappa C_{J,t+s-1}\right)\\&-\sum_{s=0}^{\infty}\lambda^s V_{t+s}(j)=\frac{1}{\lambda}M_{t-1}(j)\end{aligned} \tag{43}$$

Using the Euler equation (42):

$$\left(1+\tau_t^C+\Gamma_{\upsilon,t}(.)+\Gamma'_{\upsilon,t}(.)\upsilon_t\right)P_{C,t}\left(C_t(j)-\kappa C_{J,t-1}\right)\sum_{s=0}^{\infty}\frac{(\lambda\beta)^s}{\Pi_{s=0}^{s-1}\left(1-\Gamma'_{\upsilon,t+s}(.)\upsilon_{t+s}^2\right)}$$
$$=\sum_{s=0}^{\infty}\lambda^s\left(\Gamma'_{\upsilon,t+s}(.)\upsilon_{t+s}\right)P_{C,t+s}\left(C_{t+s}(j)-\kappa C_{J,t+s-1}\right)-\kappa\sum_{s=0}^{\infty}\lambda^s\left(1+\tau_{t+s}^C+\Gamma_{\upsilon,t+s}(.)\right)P_{C,t+s}C_{J,t+s-1}$$
$$-\sum_{s=0}^{\infty}\lambda^s V_{t+s}(j)=\frac{1}{\lambda}M_{t-1}(j) \tag{44}$$

Gathering terms:

$$\left(1+\tau_t^C+\Gamma_{\upsilon,t}(.)+\Gamma'_{\upsilon,t}(.)\upsilon_t\right)P_{C,t}\left(C_t(j)-\kappa C_{J,t-1}\right)\Upsilon_t$$
$$=-\kappa\sum_{s=0}^{\infty}\lambda^s\left(1+\tau_{t+s}^C+\Gamma_{\upsilon,t+s}(.)\right)P_{C,t+s}C_{J,t+s-1}+\sum_{s=0}^{\infty}\lambda^s V_{t+s}(j)+\frac{1}{\lambda}M_{t-1}(j) \tag{45}$$

where Υ_t is:

$$\Upsilon_t=\left[\sum_{s=0}^{\infty}\frac{(\lambda\beta)^s}{\Pi_{s=0}^{s-1}\left(1-\Gamma'_{\upsilon,t+s}(.)\upsilon_{t+s}^2\right)}-\sum_{s=0}^{\infty}\frac{(\lambda\beta)^s}{\Pi_{s=0}^{s-1}\left(1-\Gamma'_{\upsilon,t+s}(.)\upsilon_{t+s}^2\right)}\frac{\Gamma'_{\upsilon,t+s}(.)\upsilon_{t+s}}{\left(1+\tau_{t+s}^C+\Gamma_{\upsilon,t+s}(.)+\Gamma'_{\upsilon,t+s}(.)\upsilon_{t+s}\right)}\right] \tag{46}$$

Adding and subtracting $\frac{\lambda}{\Upsilon_t}M_{J,t}$ in (45) and then aggregating:

$$\left(1+\tau_t^C+\Gamma_{\upsilon,t}(.)+\Gamma'_{\upsilon,t}(.)\upsilon_{J,t}\right)P_{C,t}\left(C_{J,t}-\kappa C_{J,t-1}\right)$$
$$=\frac{1}{\Upsilon_t}\left[M_{t-1}-\kappa\left(1+\tau_t^C+\Gamma_{\upsilon,t}(.)\right)C_{J,t-1}+V_t\right]-\frac{\lambda}{\Upsilon_t}M_{J,t}$$
$$\frac{\lambda}{\Upsilon_t}\left[M_{J,t}-\kappa\sum_{s=0}^{\infty}\lambda^s\left(1+\tau_{t+s+1}^C+\Gamma_{\upsilon,t+s+1}(.)\right)P_{C,t+s+1}C_{J,t+s}+\sum_{s=0}^{\infty}\lambda^s V_{t+s+1}\right] \tag{47}$$

or

$$
\begin{aligned}
&\left(1+\tau_t^C+\Gamma_{\upsilon,t}(.)+\Gamma'_{\upsilon,t}(.)\upsilon_{J,t}\right)P_{C,t}\left(C_{J,t}-\kappa C_{J,t-1}\right)\\
&=\frac{1}{\Upsilon_t}\Bigg\{\left[M_{J,t-1}-\kappa\left(1+\tau_t^C+\Gamma_{\upsilon,t}(.)\right)C_{J,t-1}+V_t\right]-\lambda M_{J,t}\\
&\quad+\lambda\Upsilon_{t+1}\left(1+\tau_{t+1}^C+\Gamma_{\upsilon,t+1}(.)+\Gamma'_{\upsilon,t+1}(.)\upsilon_{J,t+1}\right)P_{C,t+1}\left(C_{J,t+1}-\kappa C_{J,t}\right)\Bigg\} \qquad (48)
\end{aligned}
$$

Using the aggregate budget constraint (12) to substitute for $M_{J,t-1}$ in (48), we receive:

$$
\begin{aligned}
&\left(1+\tau_t^C+\Gamma_{\upsilon,t}(.)+\Gamma'_{\upsilon,t}(.)\upsilon_{J,t}\right)P_{C,t}\left(C_{J,t}-\kappa C_{J,t-1}\right)\\
&=\frac{1}{\Upsilon_t}\Bigg\{\left(1+\tau_t^C+\Gamma_{\upsilon,t}(.)\right)\left(C_{J,t}-\kappa C_{J,t-1}\right)+\left(1-\lambda\right)M_{J,t}\\
&\quad+\lambda\Upsilon_{t+1}\left(1+\tau_{t+1}^C+\Gamma_{\upsilon,t+1}(.)+\Gamma'_{\upsilon,t+1}(.)\upsilon_{J,t+1}\right)P_{C,t+1}\left(C_{J,t+1}-\kappa C_{J,t}\right)\Bigg\} \qquad (49)
\end{aligned}
$$

Rearranging:

$$
\begin{aligned}
&\left[1+\tau_t^C+\Gamma_{\upsilon,t}(.)+\Gamma'_{\upsilon,t}(.)\upsilon_{J,t}-\frac{1+\tau_t^C+\Gamma_{\upsilon,t}(.)}{\Upsilon_t}\right]P_{C,t}\left(C_{J,t}-\kappa C_{J,t-1}\right)\\
&=\frac{1}{\Upsilon_t}\Bigg\{\left(1-\lambda\right)M_{J,t}\\
&\quad+\lambda\Upsilon_{t+1}\left(1+\tau_{t+1}^C+\Gamma_{\upsilon,t+1}(.)+\Gamma'_{\upsilon,t+1}(.)\upsilon_{J,t+1}\right)P_{C,t+1}\left(C_{J,t+1}-\kappa C_{J,t}\right)\Bigg\} \qquad (50)
\end{aligned}
$$

or

$$
\begin{aligned}
&\left[\left(1+\tau_t^C+\Gamma_{\upsilon,t}(.)+\Gamma'_{\upsilon,t}(.)\upsilon_{J,t}\right)\Upsilon_t-1-\tau_t^C-\Gamma_{\upsilon,t}(.)\right]P_{C,t}\left(C_{J,t}-\kappa C_{J,t-1}\right)\\
&=\left(1-\lambda\right)M_{J,t}\\
&\quad+\lambda\Upsilon_{t+1}\left(1+\tau_{t+1}^C+\Gamma_{\upsilon,t+1}(.)+\Gamma'_{\upsilon,t+1}(.)\upsilon_{t+1}\right)P_{C,t+1}\left(C_{t+1}-\kappa C_{J,t}\right) \qquad (51)
\end{aligned}
$$

We may rewrite (46) recursively:

$$\Upsilon_t = 1 - \frac{\Gamma'_{\upsilon,t}(.)\upsilon_{J,t}}{1 + \tau_t^C + \Gamma_{\upsilon,t}(.) + \Gamma'_{\upsilon,t}(.)\upsilon_{J,t}} + \frac{\lambda\beta}{1 - \Gamma'_{\upsilon,t}(.)\upsilon_{J,t}^2}\Upsilon_{t+1} \tag{52}$$

Using (52) to substitute out for Υ_t in (51) and dividing both sides by $P_{C,t+!}$:

$$\begin{aligned} &\beta\frac{1 + \tau_t^C + \Gamma_{\upsilon,t}(.) + \Gamma'_{\upsilon,t}(.)\upsilon_{J,t}}{\left(1 - \Gamma'_{\upsilon,t}(.)\upsilon_{J,t}^2\right)\Pi_{C,t+1}}\left(C_{J,t} - \kappa C_{J,t-1}\right) \\ &= \frac{1-\lambda}{\lambda\Upsilon_{t+1}\Pi_{C,t+1}}M_{J,t} \\ &\quad + \left(1 + \tau_{t+1}^C + \Gamma_{\upsilon,t+1}(.) + \Gamma'_{\upsilon,t+1}(.)\upsilon_{t+1}\right)\left(C_{J,t+1} - \kappa C_{J,t}\right) \end{aligned} \tag{53}$$

C Supranational Fiscal Authority

In this section, we present the Euro Area wide supranational fiscal authority. The fiscal authority issues long-term bonds which are sold only to domestic (Euro Area) households. It uses the proceeds from those bonds to finances its expenditures which are then directed to either financing part of local governments' debt or to boost demand in the non-tradables sector. The distribution of those expenditures across the union accounts for the weight of each region. Namely, the resources that each country receives is weighed by its size. Accordingly, the tax burden on each country is weighed by the corresponding country size. To finance its debt, the supranational authority collects either lump-sum or VAT taxes from euro area households. In order to keep the analysis simple we do not assume that these taxes are imposed by local governments and then rebated lump-sum to the supranational authority. We assume instead that the supranational authority imposes and collects taxes directly.

Long-term bonds are modeled as perpetuities following Woodford (2001). Specifically, a long-term bond has a payment structure ρ^{T-t-1} for $T > t$ and $0 \leq \rho \leq 1$. Hence, the value of a long-term bond issued in period t, in any future period $t+j$, is given by $q_{L,t+j}^{-j} = \rho^j q_{L,t+j}^{EU}$, where ρ captures the maturity.[29] The nominal yield on long-term bonds can thus be expressed as $R_{L,t}^{EU} = \frac{1}{q_{L,t}^{EU}} + \rho$. Note that these bonds are denominated in the currency of the home country.

[29]When $\rho = 0$ this asset collapses to a one-period bond, while for $\rho = 1$ this asset resembles a console.

Even though the model guarantees that the bilateral exchange rate between the home country and the rest of the Euro Area is fixed and equal to one, this detail is important when considering these bonds in real terms where the bilateral real exchange rate needs to be taken into account. We distinguish the case of lump-sum taxes and VAT taxes separately. In the case of lump-sum EU taxes, the budget constraint of the supranational authority writes as follows:

$$\begin{aligned} &P_{G,t} * G_t^{EU} + q_{L,t}^{EU} B_{L,t}^{EU,H} + q_{L,t}^{EU} S_t^{H,REA} B_{L,t}^{EU,REA} \\ &\quad = T_t^{EU,H} + T_t^{EU,REA} + (1 + \rho q_{L,t}) B_{L,t-1}^{EU,H} + \left(1 + \rho q_{L,t}^{EU}\right) S_t^{H,REA} B_{L,t-1}^{EU,REA} \end{aligned} \tag{54}$$

where $B_{L,t}^{EU,H}$ and $B_{L,t}^{EU,REA}$ are the home and REA long-term bond holdings issued by the supranational authority. $T_t^{EU,H}$ and $T_t^{EU,REA}$ are revenues from lump-sum taxes imposed on home and REA households. G_t^{EU} is spending by the supranational authority. The long-term rate is specified as follows:

$$R_{L,t}^{EU} = \frac{1}{q_{L,t,}^{EU}} + \rho \tag{55}$$

and the lump sum taxes follow the following rule:

$$\begin{aligned} \tau_t^{EU,H} &= n^{H,EA} \phi_b^{EU} \left(\frac{q_{L,t}^{EU} B_{L,t}^{EU,H} + q_{L,t}^{EU} S_t^{H,REA} B_{L,t}^{EU,REA}}{\overline{P_Y^{EA} Y^{EA}}} - \overline{BY^{EU}} \right) \\ \tau_t^{EU,REA} &= n^{REA,EA} \phi_b^{EU} \left(\frac{q_{L,t}^{EU} B_{L,t}^{EU,H} + q_{L,t}^{EU} S_t^{H,REA} B_{L,t}^{EU,REA}}{\overline{P_Y^{EA} Y^{EA}}} - \overline{BY^{EU}} \right) \end{aligned} \tag{56}$$

where $\tau_t^{EU,H} = T_t^{EU,H} / \overline{P_Y^{EA} Y^{EA}}$ and $\tau_t^{EU,REA} = T_t^{EU,REA} / \overline{P_Y^{EA} Y^{EA}}$. $n^{H,EA}$ and $n^{REA,EA}$ is the size of the home country and the REA in the Euro Area, respectively. We calibrate $\phi_b^{EU} = 0.3$. When the supranational authority finances its debt via VAT (or consumption) taxes, the budget constraint summarizes to:

$$P_{G,t} * G_t^{EU} + q_{L,t}^{EU} B_{L,t}^{EU,H} + q_{L,t}^{EU} S_t^{H,REA} B_{L,t}^{EU,REA}$$
$$= \tau_{C,t}^{EU,H} C_t^H + \tau_{C,t}^{EU,H} * C_t^{REA} + (1 + \rho q_{L,t}) B_{L,t-1}^{EU,H} + \left(1 + \rho q_{L,t}^{EU}\right) S_t^{H,REA} B_{L,t-1}^{EU,REA} \quad (57)$$

and the VAT taxes follow the following rule:

$$\tau_{C,t}^{EU,H} = n^{H,EA} \left(\frac{\phi_b^{EU}}{\bar{C^H}}\right) \left(\frac{q_{L,t}^{EU} B_{L,t}^{EU,H} + q_{L,t}^{EU} S_t^{H,REA} B_{L,t}^{EU,REA}}{\overline{P_Y^{EA} Y^{EA}}} - \overline{BY^{EU}}\right)$$

$$\tau_{C,t}^{EU,REA} = n^{REA,EA} \left(\frac{\phi_b^{EU}}{\bar{C^{REA}}}\right) \left(\frac{q_{L,t}^{EU} B_{L,t}^{EU,H} + q_{L,t}^{EU} S_t^{H,REA} B_{L,t}^{EU,REA}}{\overline{P_Y^{EA} Y^{EA}}} - \overline{BY^{EU}}\right)$$

(58)

Note that we divide ϕ_b^{EU} by steady state consumption in both rules. This is to guarantee that the VAT and lump-sum taxes have exactly the same impact on debt. Households in the home country and the REA pay a financial intermediation premium, Γ_t^{EU} which is common in both regions and is specified as:

$$\Gamma_t^{EU} = \gamma_{B^{EU}} \left(exp\left(\frac{q_{L,t}^{EU} B_{L,t}^{EU,H} + q_{L,t}^{EU} S_t^{H,REA} B_{L,t}^{EU,REA}}{\overline{P_Y^{EA} Y^{EA}}} - \overline{BY^{EU}}\right) - 1\right) \quad (59)$$

We calibrate $\overline{BY^{EU}} = 2.40$ which is equivalent to the 60% debt-to-GDP target and $\gamma_{B^{EU}} = 0.01$. The presence of the intermediation premium gives rise to a wedge between the common long-term rate and the Euro Area policy rate away from the steady state. The first order condition and the respective Euler equations at home and in the REA are straightforward.

D The fiscal extension in EAGLE

The Clancy, Jacquinot and Lozej (2016) extension of the fiscal version of the EAGLE model introduces an additional sector that results in the final goods production for the government. The assumption for the firms producing in this sector is symmetricity and operating under perfect competition. These firms use intermediate tradable and non-tradable goods as inputs while they produce final government consumption and investment bundles, $Q_t^{G_C}$ and $Q_t^{G_I}$ respectively. The final government consumption goods are assembled following the constant elasticity of substitution (CES) technology process

$$Q_t^{G_C} = \left[\nu_{G_C}^{\frac{1}{\mu_{G_C}}} \left(TT_t^{G_C} \right)^{\frac{\mu_{G_C}-1}{\mu_{G_C}}} + (1 - \nu_{G_C})^{\frac{1}{\mu_{G_C}}} \left(NT_t^{G_C} \right)^{\frac{\mu_{G_C}-1}{\mu_{G_C}}} \right]^{\frac{\mu_{G_C}}{\mu_{G_C}-1}} \tag{60}$$

where the government demand for non-tradable goods $NT_t^{G_C}$ is defined as

$$NT_t^{G_C} = (1 - \nu_{G_C}) \left(\frac{P_{NT,t}}{P_{G_C,t}} \right)^{-\mu_{G_C}} Q_t^{G_C} \tag{61}$$

If the value of the parameter ν_{G_C} takes the value of 0, then all the government consumption is spent on non-tradable goods, consequently downsizing the extended fiscal EAGLE model back to the original EAGLE model of Gomes, Jacquinot and Pisani (2012). On the other side of the final consumption , the tradable good $TT_t^{G_C}$, which is consumed by the government, is defined as a bundle of home-produced tradable goods and imported goods, so that

$$TT_t^{G_C} = \left[\nu_{TG_C}^{\frac{1}{\mu_{TG_C}}} \left(HT_t^{G_C} \right)^{\frac{\mu_{TG_C}-1}{\mu_{TG_C}}} + (1 - \nu_{TG_C})^{\frac{1}{\mu_{TG_C}}} \left(IM_t^{G_C} \right)^{\frac{\mu_{TG_C}-1}{\mu_{TG_C}}} \right]^{\frac{\mu_{TG_C}}{\mu_{TG_C}-1}} \tag{62}$$

Demands of home-produced tradable goods $HT_t^{G_C}$ and imported goods $IM_t^{G_C}$ are defined as

$$HT_t^{G_C} = \nu_{TG_C} \left(\frac{P_{HT,t}}{P_{TTG_C,t}} \right)^{-\mu_{TG_C}} TT_t^{G_C} \tag{63}$$

and

$$IM_t^{G_C} = \left[\sum_{CO \neq H} \left(\nu_{MG_C}^{H,CO}\right)^{\frac{1}{\mu_{MG_C}}} \left(IM_t^{G_C,CO}\right)^{\frac{\mu_{MG_C}-1}{\mu_{MG_C}}} \right]^{\frac{\mu_{MG_C}}{\mu_{MG_C}-1}} \tag{64}$$

where $\sum \nu_{MG_C}^{H,CO} = 1$ and the demand for imported goods of each foreign region is specified as

$$IM_t^{G_C,CO} = \nu_{MG_C}^{H,CO} \left(\frac{P_{IM,t}}{P_{IMG_C,t}} \right)^{-\mu_{MG_C}} IM_t^{G_C} \tag{65}$$

We have to define the government consumption goods prices as well. They correspond to the CES-aggregated bundles, so that

$$P_{G_C,t} = \left[\nu_{G_C} \left(P_{TTG_C,t}\right)^{1-\mu_{G_C}} + (1-\nu_{G_C}) \left(P_{NT,t}\right)^{1-\mu_{G_C}} \right]^{\frac{1}{1-\mu_{G_C}}} \tag{66}$$

where

$$P_{TTG_C,t} = \left[\nu_{TG_C} \left(P_{HT,t}\right)^{1-\mu_{TG_C}} + (1-\nu_{TG_C}) \left(P_{IMG_C,t}\right)^{1-\mu_{TG_C}} \right]^{\frac{1}{1-\mu_{TG_C}}} \tag{67}$$

and

$$P_{IMG_C,t} = \left[\sum_{CO \neq H} \nu_{MG_C}^{H,CO} \left(P_{IM,t}^{CO}\right)^{1-\nu_{MG_C}} \right]^{\frac{1}{1-\nu_{MG_C}}} \tag{68}$$

The complementarity assumption of Coenen, Straub and Trabandt (2012) between the private and government consumption requires us to introduce the government consumption also in the utility function in a non-separable manner. This means that the utility depends on consumption $\tilde{C}$ and implies that the government consumption affects optimal private consumption decisions directly. It is defined as

$$\tilde{C} = \left[\nu_{CCES}^{\frac{1}{\mu_{CCES}}} \left(C_{I,t}\right)^{\frac{\mu_{CCES}-1}{\mu_{CCES}}} + \left(\nu_{CCES}^{\frac{1}{\mu_{CCES}}} \right) \left(G_{C,t}\right)^{\frac{\mu_{CCES}-1}{\mu_{CCES}}} \right]^{\frac{\mu_{CCES}}{\mu_{CCES}-1}} \tag{69}$$

The final government investment goods are assembled analogous to the final government consumption goods. What is the key difference between the final government consumption and government investment is the assumption that the government investment is not wasteful. It adds to public capital by following its law of motion

$$K_{G,t+1} = (1 - \delta_G) K_{G,t} + G_{I,t} \tag{70}$$

The government investment nevertheless enters the production function of the private tradable sector (and analogous for the non-tradable sector)

$$Y_{T,t}^S = z_{T,t} K_{G,t}^{\alpha_G} \left(K_{T,t}^D\right)^{\alpha_T} \left(N_{T,t}^D\right)^{1-\alpha_T} - \psi_T \tag{71}$$

As a consequence, the government capital enhances the productivity of private capital as its role is similar to the technological progress. The increase in government capital implies lower marginal costs of the intermediate tradable goods' sector (and analogous for the non-tradable sector)

$$MC_{T,t} = \frac{1}{z_{T,t} K_{G,t}^{\alpha_G} (\alpha_T)^{\alpha_T} (1-\alpha_T)^{1-\alpha_T}} \left(R_t^K\right)^{\alpha_T} \left(\left(1 + \tau_t^{W_f}\right) W_t\right)^{1-\alpha_T} \tag{72}$$

In the end, compared to the baseline EAGLE model of Gomes, Jacquinot and Pisani (2012), some modifications are needed in the market clearing section as well. These are as follows

$$Q_t^{G_C} = G_{C_t} \tag{73}$$

$$Q_t^{G_I} = G_{I_t} \tag{74}$$

$$NT_t = NT_t^C + NT_t^I + NT_t^{G_C} + NT_t^{G_I} \tag{75}$$

$$HT_t = HT_t^C + HT_t^I + HT_t^{G_C} + HT_t^{G_I} \tag{76}$$

The total imports equation is also modified as we add government imports of consumption and investments

$$IM^{H,CO} = \sum_{j=C,I,X} IM_t^{j,H,CO} \frac{1-\Gamma_{IM^j}}{\Gamma_{IM^j}^{H,CO,j}} + IM_t^{G_C} + IM_t^{G_I} \tag{77}$$

We assume no adjustment costs that are associated with these import goods. We also modify the government budget constraint in order to reflect the government spending in both forms, i.e. consumption and investment

$$\begin{aligned} P_{G_C,t}G_{C,t} + P_{G_I,t}G_{I,t} + TR_t + B_t + M_{t-1} \\ = \tau_t^C P_{C,t}C_t + (\tau_t^N + \tau_t^{W_h}\frac{1}{s^H}\left(\int_0^{s^H(1-\omega)} W_t(i)N_t(i)di + \int_{s^H(1-\omega)}^{s^H} W_t(j)N_t(j)dj\right) \\ + \tau_t^{W_f}W_tN_t + \tau_t^K(R_{k,t}u_t - (\Gamma_u(u_t)+\delta)P_{I,t})K_t \\ + \tau_t^D D_t + T_t + R_t^{-1}B_{t+1} + M_t \end{aligned} \tag{78}$$

The aggregate resource constraint is modified to

$$\begin{aligned} P_{Y,t}Y_t = P_{C,t}Q_t^C + P_{I,t}Q_t^I + P_{NT,t}Q_t^{G_C} + P_{HT,t}Q_t^{G_I} \\ + \sum_{CO\neq H} S_t^{H,CO}P_{X,t}^{H,CO}X_t^{H,CO} - \sum_{CO\neq H} P_{IM,t}^{H,CO}IM_t^{H,CO} \end{aligned} \tag{79}$$

The autoregressive shocks to government consumption and investment are also added, where $i = C, I$

$$g_t^i = (1-\rho_{g^i})\overline{g}^i + \rho_{g^i}g_{t-1}^i + \epsilon_{g^i,t} \tag{80}$$

E Spillovers when US monetary policy is at the ELB

In the main text, we do not incorporate a lower bound on the nominal interest rate in the US. Consequently, the monetary authority in that region responds to the Covid shock by reducing the nominal interest rate, thus supporting aggregate demand. While these dynamics are not unrealistic, as evidenced by the Federal Reserve's 1.5 percentage point cut in the federal funds rate in March 2020, it is important to note that they overlook the fact that US nominal rates did reach the zero lower bound during the height of the Covid crisis.

To address this concern, this appendix explores the impact of our Covid shock when nominal interest rates are constrained by the ELB in both the US and the Euro area. It is worth noting that incorporating the ELB in two blocks simultaneously introduces numerical challenges, making the simulation more complex.

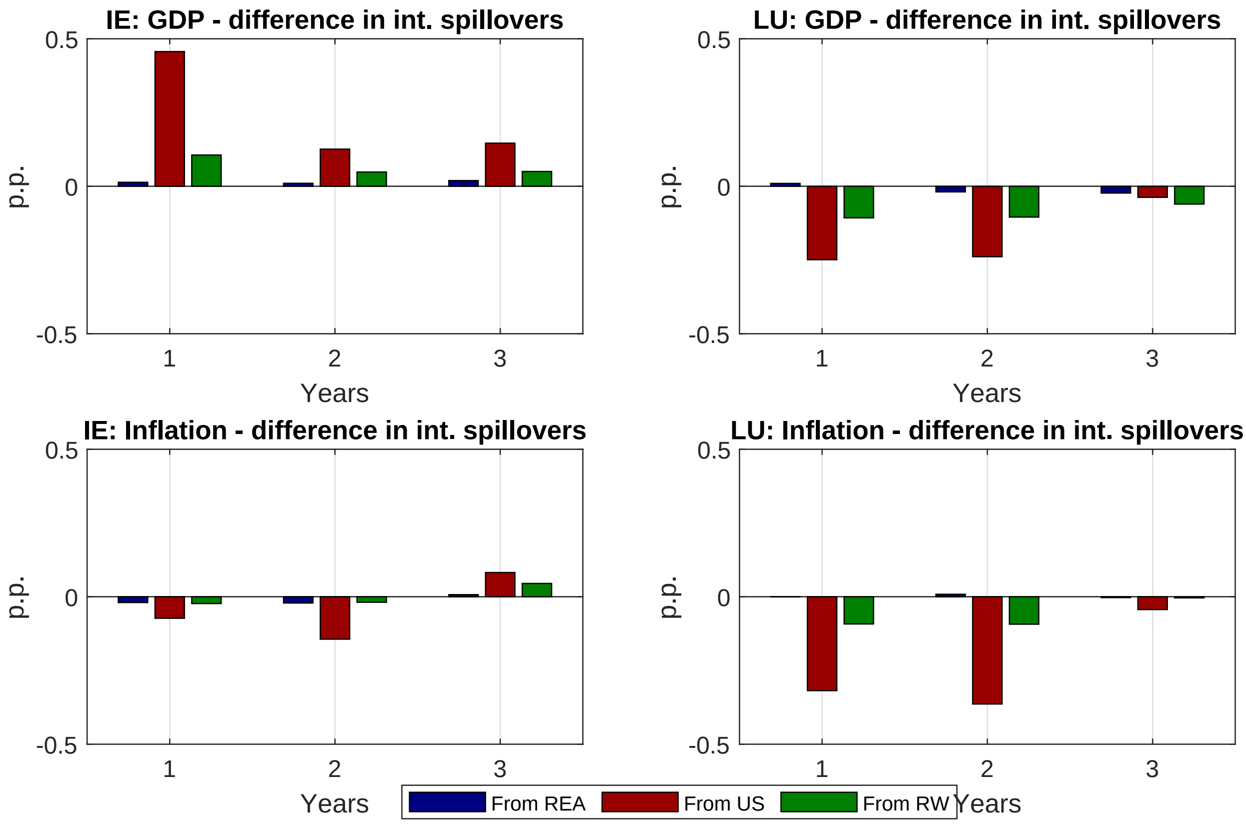
We examine the impact of a binding constraint on US nominal rates on international spillovers as follows. First, we use equation 1 in subsection 4.2 to calculate international spillovers when both the US and the Euro area face an ELB on nominal rates. Next, we repeat the same analysis, but with the ELB binding only in the Euro area. Finally, we compare the results of these two scenarios by computing the difference between them. Figure 10 plots the results for Ireland and Luxembourg. As discussed in the main text and illustrated in Figure 4, Ireland has the highest level of exposure to the US, while Luxembourg has the lowest spillovers from the US. Consequently, we focus on these two extreme cases.

The main finding is that introducing an ELB in the US has a weak impact on the magnitude of international spillovers. This is evident from Figure 10, where all bars, especially those representing spillovers from the rest-of-the-euro-area and the rest-of-the-world, are relatively close to zero.

The reason for the limited effects of the ELB in the US on international spillovers is nuanced. When the ELB only constrains the Euro area, the euro appreciates against the dollar due to the widening gap in nominal interest rates between the two regions. This appreciation of the euro leads to reduced exports from our domestic economies to the US. Conversely, when the ELB is present in both the Euro area and the US, nominal interest rates in both regions move together, resulting in a relatively stable real exchange rate.[30] However, the US experiences a more severe economic downturn with a binding ELB, leading to decreased US aggregate demand and, hence, lower domestic exports to that region. Therefore, although the underlying forces differ, both scenarios result in similar consequences for international spillovers.

[30] Note that there is some nominal effective exchange rate appreciation for the EA also if we explicitly consider the binding ELB in the US. The reason is that the euro still appreciates in nominal terms against the RW, which is not constrained by the ELB.

Figure 10. Spillovers in Ireland and Luxembourg with and without an US ELB



Horizontal axes: Years. Vertical axes: Percent differences from the baseline scenario where only the Euro Area face an ELB.

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