A Demand Theory of the Price Level

Marcus Hagedorn*
University of Oslo and CEPR

This Version: August 24, 2017

Abstract

In this paper I show that the price level is determinate in a large class of incomplete market models. Monetary policy works through setting nominal interest rates, e.g. an interest rate peg or an interest rate rule, while fiscal policy is committed to satisfying the present value budget constraint at all times (in contrast to the Fiscal Theory of the Price Level). Jointly, these policies determine the unique price level, as well as consumption and employment.

In this new approach to price level determinacy several puzzles disappear which arise in New Keyensian models during a liquidity trap. Forward guidance has negligible output effects, the size of the fiscal multiplier decreases if prices are less sticky, and technological regress reduces output. Determinacy also enables a richer analysis of monetary and fiscal policy coordination. In particular, interest rates need not be raised aggressively whenever fiscal policy succeeds in stimulating inflation and demand.

In contrast to the conventional view the long-run inflation rate here is, in the absence of output growth and even when monetary policy operates an interest rate rule with a different inflation target, equal to the growth rate of nominal government spending, which is controlled by fiscal policy. This novel theory, where nominal government spending anchors aggregate demand and therefore current and future prices, offers a different perspective on a range of important issues. These include the fiscal and monetary transmission mechanism, policies at the zero-lower bound, U.S. inflation history, and recent attempts to stimulate inflation in the Euro area.

JEL Classification: D52, E31, E43, E52, E62, E63
Keywords: Price level Determinacy, Incomplete Markets, Inflation, Monetary Policy, Fiscal Policy, Zero Lower Bound, Fiscal Multiplier, Policy Coordination

*University of Oslo, Department of Economics, Box 1095 Blindern, 0317 Oslo, Norway.
Email: marcus.hagedorn07@gmail.com

1 Introduction

This paper shows that the price level is globally determinate in a large class of incomplete market models, with monetary and fiscal policies resembling those actually implemented in practice. The idea is quite simple. Monetary policy works through setting an arbitrary sequence of nominal interest rates, for example through an interest rate peg or an interest rate rule. Fiscal policy sets sequences of nominal government spending, taxes, and government debt, for example through a fiscal rule, and these sequences satisfy the present value government budget constraint at all times and for all prices. The price level is then determined such that demand equals supply in the goods market or equivalently in the asset market. Both monetary and fiscal policies that aim at increasing or decreasing the price level are effective in this endeavor only if they can stimulate or contract nominal aggregate demand.

Price level determinacy in incomplete market models offers a new perspective on various issues in monetary economics. I show that, because of determinacy, several puzzles disappear, which are otherwise observed during a liquidity trap in New Keynesian models. There is no forward guidance puzzle, as commitment to future monetary policy has only negligible effects here. Improvements in technology increase output when the price level is determinate, in contrast to New Keynesian models. The size of the fiscal multiplier becomes smaller if prices are less sticky, whereas it gets arbitrarily large in New Keynesian models.

Furthermore, a determinate price level allows for policy analyses not only when the zero lower bound is binding, but also more generally when not using an interest rate rule satisfying the Taylor principle. In standard complete market models the nominal interest rate has to respond to inflation aggressively enough to guarantee a locally determinate inflation rate and to overcome the indeterminacy pointed out in Sargent and Wallace (1975).[1] To avoid indeterminacy, policy analysis is then restricted to this subset of aggressive monetary policies, and hence also during periods when actual policy did not follow the Taylor principle. Indeed, Clarida et al. (2000) find that US monetary policy pre-1979 increased nominal interests rates by less than expected inflation. This case violated the assumptions needed for determinacy and implied that self-fulfilling inflation bursts cannot be ruled out, and that further analysis either is precluded or would have to invoke further assumptions to overcome indeterminacy. Incomplete market models offer an alternative theory of price level determination, where monetary policy can be represented by any arbitrary sequence of nominal interest rates or an interest rate rule (not) satisfying the Taylor principle. They can do so simply because for each sequence and each rule, determinacy of the equilibrium

[1] Without determinacy agents long-run price expectations are not anchored and beliefs can coordinate on a high price level, unchanged prices or even lower prices, with potentially quite different short-run implications. Clearly a predetermined price, for example if set in the previous period, does not overcome this indeterminacy issue.

is ensured. This allows study of fiscal and monetary policy coordination without restrictions on policy rules and for both time periods, pre- and post-1979. For example, fiscal policy could be expansionary to stimulate the economy, while at the same time monetary policy keeps nominal interest rates constant so as not to offset the stimulus. It is also conceivable that the pre-1979 US experience may just reflect a lower level of concern about inflation among both monetary and fiscal policy makers than was the case during the Volcker-Greenspan era, without inducing indeterminacy.[2]

Heterogenous agents incomplete markets models not only offer a new way to think about price level determinacy but also are an attractive model class for studying questions in macroeconomics, public finance and consumption theory. With good reason: the model can match the joint distribution of earnings, consumption and wealth; generate a realistic distribution of marginal propensities to consume; generate realistic consumption responses to income shocks and transfers; thereby allowing meaningful study of redistributive policies. This class of models, however, neither allows output to be demand-determined as prices are fully flexible, nor does it take the zero lower bound (ZLB) into account. These factors limit its applicability to many questions raised by the Great Recession. Adding a nominal side to the model, allowing for price rigidities and taking the ZLB into account on the other hand forces us to address the same questions we confront in complete market models: Is the price level determinate? What type of monetary and fiscal policies guarantee determinacy? Must the researcher select among different equilibria? Without answers to these question, for example, one of the most important policy questions, the magnitude of the fiscal multiplier, cannot be addressed satisfactorily.[3]

This paper shows that using the workhorse incomplete markets model not only provides an empirically superior model of consumption - a key part of the monetary transmission mechanism - but also entails a windfall gain: the price level is determinate. I show determinacy for all monetary policy rules, those not responding, weakly responding or strongly responding to prices. Interestingly, if responding too strongly to price increases fiscal policy may induce (not remove)

[2]Price level indeterminacy also implies that adding a small amount of nominal rigidities renders purely real models unstable (Kocherlakota (2016)) implying that "purely real models are too fragile to be of practical value". Price level determinacy overcomes this unpleasant conclusion by removing this instability.

[3]The literature has responded recently and has incorporated price rigidities into incomplete market models. Kaplan et al. (2016), Auclert (2016) and Lütticke (2015) study monetary policy in a model with incomplete markets and pricing frictions, however with a different focus. Whereas these authors emphasize and quantify several redistributive channels of the transmission mechanism of monetary policy which are absent in standard complete market models, the price level is endogenously determinate in equilibrium only in my paper. Earlier contributions are Oh and Reis (2012) and Guerrieri and Lorenzoni (2015), who were among the first to add nominal rigidities to a Bewley-Imrohoroglu-Huggett-Aiyagari model and Gornemann et al. (2012) who were the first to study monetary policy in the same environment. More recent contributions include McKay and Reis (2016) (impact of automatic stabilizers), McKay et al. (2015) (forward guidance), Bayer et al. (2015) (impact of time-varying income risk), Ravn and Sterk (2013) (increase in uncertainty causes a recession) and Den Haan et al. (2015) (increase in precautionary savings magnifies deflationary recessions).

indeterminacy, which in turn requires more active monetary policy to reestablish determinacy. I characterize these determinacy conditions in section 4.6.

One of the striking empirical findings in the consumption literature (Campbell and Deaton (1989), Attanasio and Davis (1996), Blundell et al. (2008), Attanasio and Pavoni (2011)) is that a permanent income gain - like a permanent tax rebate - increases household consumption less than one-for-one and thus increases savings too. This simple fact which holds in standard incomplete market models, but violates the permanent income hypothesis, is the key to the determinacy result. It implies that taking prices as given, a permanent decrease in government spending by one dollar and a simultaneous permanent tax rebate of the same amount to private households lowers real total aggregate demand - the sum of private and government demand. The same logic also establishes why in a steady state, real aggregate demand depends on the price level. Given monetary and fiscal policy, a higher steady-state price level lowers real government consumption since government spending is (partially) fixed in nominal terms. At the same time, it lowers the tax burden for the private sector by the same amount. As private sector demand does not substitute one-for-one for the drop in government consumption but instead saves a fraction of the tax reduction, aggregate real demand falls, establishing a downward sloping aggregate demand-price curve. The unique equilibrium steady-state price level is then such that aggregate real demand equals real supply.[4] Moving beyond steady-state results, and establishing price level determinacy globally, require us to assume that the above empirical finding holds also outside the steady state; that is, that a precautionary demand never vanishes.[5]

It is important to emphasize, however, that first, it is the presence of precautionary savings that delivers the result and second, prices are not determinate in every model where Ricardian equivalence fails. For example, the price level is not determinate in an economy where a fraction of households are hand-to-mouth consumers while the remaining households act according to the permanent income hypothesis (PIH). The reason for the indeterminacy is the absence of precautionary savings. The PIH consumers increase their consumption one-for-one in response to a permanent tax rebate, since this what PIH households do. The hand-to-mouth consumers do the same, not as a result of optimization but by assumption. In such a model an increase in the price level also lowers real government consumption (since it is fixed in nominal terms) and increases private consumption. The higher price level does not affect total consumption though, because the drop in government consumption is offset exactly by the increase in private demand.

[4]Werning (2015) (using incomplete markets) and Angeletos and Lian (2016) (using incomplete information) also propose theories of aggregate demand, with the important difference that those theories are real. I propose a nominal theory, a prerequisite to obtain a determinate price level.

[5]In a standard incomplete markets model (Bewley-Imrohoroglu-Huggett-Aiyagari), the precautionary savings motive arises due to a potentially binding credit constraint and associated low consumption and utility levels. It is therefore easy to satisfy the assumption, as I explain at the end of Section A.I.

As a result aggregate demand equals supply for an infinite number of price levels, each of these corresponding to a different size of government. Similar arguments apply to the perpetual youth model and variants of it (Yaari (1965), Blanchard (1985), Bénassy (2005, 2008)), where Ricardian equivalence does not hold but the price level is indeterminate, as I explain in Appendix A.I.

To keep the theoretical model tractable, I use the simplest setup that delivers the empirical finding on individual consumption and precautionary saving behavior discussed above. The key simplifying assumption is that households are members of a family that provides insurance, such that the distribution of asset holdings across agents is degenerate. These families live in an infinite horizon endowment economy without capital which in terms of preferences, technology and trading arrangements is closer to an infinite horizon replication of a Diamond and Dybvig (1983) economy than to conventional macroeconomic models. It is not difficult to integrate this framework into a standard one such as the Aiyagari (1994) incomplete markets model and then to explore policy implications quantitatively, since the price level is determined in a large class of incomplete market models as I show in Section 2.[6] An advantage of using the simpler framework, besides enabling the researcher to better understand the monetary and fiscal transmission mechanism, is that a banking sector can be added to the model. Although beyond the scope of this paper, such an extension will allow the researcher to study the interaction of banking distress, policy, deflation and the real economy in a model where the price level is determinate.

The main result of the theoretical analysis is that the price level is globally uniquely determinate and that it depends on both monetary and fiscal policy. To illustrate the workings of the model I add some key features for a meaningful numerical analysis: labor supply is endogenous, prices are sticky and only a small fraction of government spending is nominally fixed. I then numerically compute impulse responses to monetary and fiscal policy shocks as well as to technology and discount factor shocks. I find that all impulse responses are in line with their empirical counterparts, a finding which is easily explained by the supply/demand logic at the foundation of the determinacy result. An increase in nominal interest rates stimulates saving and therefore lowers consumption demand, implying a drop in prices. An increase in government spending stimulates aggregate demand, implying a rise in prices. An increase in technology raises supply and households' incentives to save, implying a drop in prices. And finally an increase in the discount factor stimulates savings since households are more patient, implying a drop in prices. Quite remarkably, the model delivers these results not only for sticky prices but also when prices are flexible. In particular, price rigidities are not needed for monetary policy to have effects.[7] However, for these effects to be quantitatively consistent with empirical findings, nominal rigidities are likely

[6]For such an exploration see for example Hagedorn et al. (2016) who study the size of the fiscal multiplier.

[7]Garriga et al. (2013), Sterk and Tenreyro (2015) and Buera and Nicolini (2016) among others also find that monetary policy has real effects in an incomplete market models with flexible prices.

needed (Hagedorn et al. (2017)).[8]

In the theory proposed here fiscal policy provides a nominal anchor through setting nominal spending and nominal bonds, making the treasury a key player in determining the price level and *the* key player in determining the long-run inflation rate. The steady-state inflation rate is equal to the growth rate of nominal government spending (minus productivity growth) and therefore is controlled by the treasury. In contrast to conventional wisdom, a tough, independent central bank not only is insufficient to guarantee price stability in the long-run, but also has no direct control over long-run inflation even if it follows an interest rate rule which satisfies the Taylor principle. By controlling the nominal anchor, the treasury always wins out when it comes to long-run inflation. However, if the treasury does not exercise its power, for example if government spending is fixed in real and not in nominal terms, the central bank takes over the job of determining the steady-state inflation rate as conventional wisdom suggests. But here, monetary policy does so in a model with a determinate price level (since government debt is nominal).

The price level is always controlled jointly by fiscal and monetary policies, as already suggested by the impulse responses to increases in spending and in the nominal interest rate. Fiscal policy can raise the price level and stimulate employment through actively increasing spending. But an apparently passive fiscal policy, one that fixes nominal spending, also automatically stabilizes the economy. Consider an increase in the discount factor, which lowers prices and contracts employment. Lower prices automatically lead to higher real government spending since nominal spending is fixed, and thus stimulate aggregate demand which partly offsets the fall in prices and employment. The effectiveness of expansionary fiscal policy in stabilizing the economy depends also on how it is financed, through higher deficits or higher taxes. Not surprisingly, raising taxes is less effective than increasing deficits, because higher taxes lead to lower demand and thus partially offset the stimulative demand effects of higher government spending.

Monetary policy can lower the price level and employment through increasing nominal interest rates and is quite effective in stabilizing the economy. Again as an example, consider an increase in the discount factor. This shock can be fully neutralized through lowering nominal interest rates by the same amount as the increase in the discount factor, which keeps unchanged the effective discounting - given by the product of the discount factor and the nominal interest rate - by households. All variables, including employment, prices and consumption remain at their steady-state values. In contrast, an expansion in government spending cannot stabilize employment and thus output and consumption at the same time, suggesting that monetary policy is the more

[8] Interestingly this finding implies that the equivalence result of Correia et al. (2008) - environments with different price-setting restrictions are equivalent - is likely to break down in incomplete market models with a determinate price level. Correia et al. (2008) results are derived from the simple finding that, under flexible prices, producer prices are constant in any equilibrium in complete market models, whereas they are not in complete market models.

effective option for stabilizing the economy.

However, the ZLB makes monetary policy ineffective. This is the case if the increase in the discount factor is so large that stabilizing the economy only through nominal interest rates requires setting these at a negative value, which is impossible (the ZLB binds). In this scenario, expansionary fiscal policy can stabilize aggregate demand and completely offset the contractionary effects of the discount factor increase, such that prices and employment remain at their steady-state values.

A stimulative policy initiated by the treasury to raise employment and prices requires coordination with monetary policy. In the absence of coordination, a central bank committed only to price stability can raise nominal interest rates and cancel out the price and employment effects of the fiscal stimulus. The result of this (attempted) stimulative fiscal policy and the response of monetary policy is: unchanged employment and prices but higher government spending, taxes and debt. It is obvious that policy coordination extends beyond this example, simply because monetary and fiscal policy jointly determine demand and prices in this framework. Political economy questions like these naturally come up in a framework where the price level is determinate, and they can be answered because the price level is determinate.

While monetary policy can neutralize short-run inflationary fiscal policy, taming inflation in the medium and long runs requires constraining fiscal policy from running an inflationary spending plan. Although the central bank does not set the spending itself, and therefore the treasury can control medium and long-run inflation if it wants to, control of the nominal interest rate is an effective tool for making an inflationary policy quite costly. Raising the nominal interest rate raises the interest payments on government debt, which can sharply constrain government spending. High nominal interest rates may force the treasury into a less expansionary fiscal policy, and thus indirectly lead to a lower inflation rate. Since there is no upper bound on nominal interest rates, there is no limit on the cost the central bank can inflict on the treasury. But the central bank can also support an expansionary fiscal policy through lowering nominal interest rates, if it considers inflation to be too low. However, the ZLB puts a limit on the budgetary support the central bank can provide. The independence of central banks guarantees that those interest rate decisions are taken through monetary and not fiscal policy. As central banks arguably put more weight on price stability than treasuries do (treasuries being more interested in taming deficits), independence leads to a more active interest rate policy to curb inflation than if the treasury were to control the interest rate.

This reasoning and the theory set out in this paper offer a different perspective on US inflation history. After experiencing high inflation rates in the 1970s, the 1980s saw success in keeping inflation rates low. In the standard interpretation, central banks eventually recognized that keeping inflation low was their primary objective and as a consequence, were successful in doing so. The framework proposed in this paper suggests that it was not the change in the conduct of monetary policy that kept inflation in check, but a shift to a less expansionary fiscal policy during the Reagan administration, perhaps forced on by the prolonged high nominal interest rates set by central banks under chairman Paul Volcker and resulting high deficits.

Before presenting the model, I start with a graphical analysis in Section 2 to show that the steady-state price level is determinate in a large class of heterogenous agents incomplete market models. In this Section I explain the mechanism behind the determinacy result; why market incompleteness is necessary for determinacy and why complete markets lead to indeterminacy; why fiscal policy has to be partially nominal; why this is not the FTPL; and why adding capital or cash to the model does not alter these conclusions. I also use the graphical analysis to show how the price level responds to monetary and fiscal policy, and to changes in technology and in the need for liquidity, and find these responses to be in line with conventional wisdom. For pedagogical purposes, I then move to a baseline model where labor is supplied inelastically and prices are flexible, in Section 3. In Section 4 I prove price level determinacy for arbitrary sequences of monetary and fiscal policy. I show that monetary and fiscal policy together determine the price level and the inflation rate, and how these variables respond to policy changes and shocks. I derive the determinacy properties of monetary and fiscal rules in Section 4.6 first for the simple model of Section 3, and then extend the analysis to the more general incomplete market models of Section 2. In Section A.III, I describe the extension to the case of elastic labor supply and sticky prices. I also present some numerical exercises to illustrate the workings of the model, computing impulse responses, government spending as an automatic stabilizer, coordination of monetary and fiscal policy and policies at the ZLB. Section 5 concludes.

2 The Price Level and Incomplete Markets: A Graphical Analysis

In this Section I provide a graphical analysis to argue that the steady-state price level is determinate in a large class of incomplete market models. I start with an endowment economy with uninsurable idiosyncratic labor income risk, based on Huggett (1993), where only one asset - a nominal government bond - can be traded subject to exogenously imposed borrowing limits. The key features sufficient for determinacy of the steady-state price level are that steady-state asset demand depends on the real interest rate, and that fiscal policy is partially nominal.

One way to understand why the price level in complete market models is indeterminate is to notice that the number of endogenous variables exceeds the number of equilibrium conditions by one. The Fiscal Theory of the Price Level (FTPL) provides an additional equation as it assumes that the government budget constraint is satisfied by only one price level. In this paper I pursue a different approach. Instead of using the government budget constraint as an additional equation, I assume that the government balances the budget for all price levels and show that the asset market clearing condition is the needed additional equation in incomplete market models. In this intuitive Section, I focus on steady-states in order to explain the additional equation argument most clearly. The biggest challenge is to show determinacy when the nominal interest rate is constant so that the Sargent and Wallace (1975) critique fully kicks in. Once this step is accomplished, agents' long-run price expectations are determined and coordinated on the unique long-run steady-state price level. Although this is the main step in the determinacy proof it is not sufficient for global determinacy, for two main reasons. First, I have to show that prices are in steady state when monetary and fiscal policy are stationary, which is equivalent to ruling out inflationary and deflationary price spirals. Note that this argument rules out hyper-inflations only if nominal variables are growing at a constant rate, but not when policy decides for an explosive path of nominal variables. The theory thus allows for policy induced hyper-inflations. The second step is to consider arbitrary non-steady-state policies which can result from some fiscal or interest rate rules with and without commitment, and to establish price level determinacy in these cases too.[9] It is important to point out that depending on the fiscal and monetary policy implemented, the economy may experience a hyperinflation, a deflation or price stability, but in each scenario the price level is determinate.

I provide a simple no-liquidity Huggett economy (as in Werning (2015)) example in Section A.I (appendix) which allows for an explicit solution of the price level and illustrates how prices are determined outside steady states. I also provide some intuitive arguments for the general Huggett economy with positive government debt, but a complete proof of global determinacy requires tractable dynamics outside of the steady state. I therefore use a simple incomplete markets model in Sections 3 and 4 to prove global determinacy. As it turns out, once the steady-state hurdle is overcome, showing determinacy outside steady states does not involve any further conceptual issues but is a matter of tractability only.

Equilibrium in a Huggett Economy

I consider a cashless economy (Woodford (2003)) where monetary policy sets the nominal interest rate which in steady state equals $R = 1+i$. This allows for arbitrary interest rate rules responding to inflation. Fiscal policy sets nominal government spending G, nominal taxes T and nominal bonds B such that the government nominal budget constraint holds, $iB + G = B' - B + T$.[10]

[9] As argued above, a constant nominal interest rate unresponsive to inflation increases is not only the key step in the determinacy proof but might be an effective monetary policy to accommodate a fiscal stimulus, as I illustrate below.

[10] The government budget constraint is assumed to involve nominal variables only to make clear that it holds independent of the price level. Note that this is in contrast to the FTPL where the price level is such that it clears the government budget constraint. As I explain below, price level determinacy does not require that fiscal policy

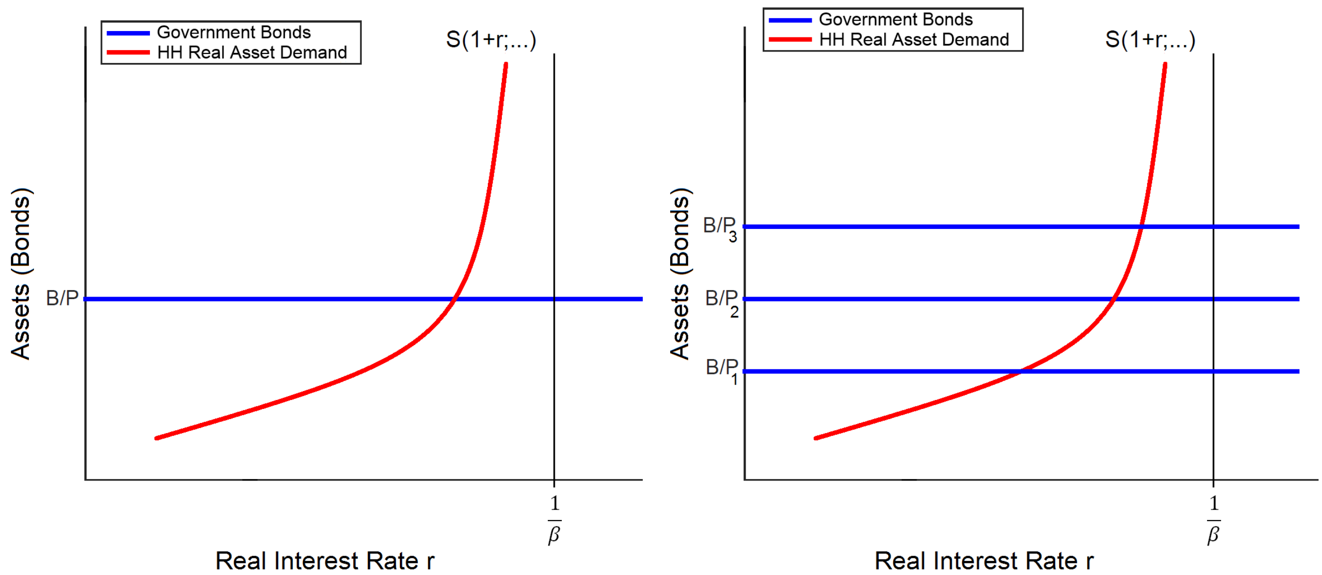


Figure 1: Asset Market in Huggett economy

This definition allows for arbitrary fiscal policy rules such as a rule for nominal bond issuance and nominal government spending

$$B'(\text{B, P, output, other variables of interest}) \tag{1}$$

$$G(\text{B, P, output, other variables of interest}) \tag{2}$$

where taxes are set to balance the budget

$$T := (1+i)B + G(\ldots) - B'(\ldots). \tag{3}$$

I assume only that households transversality condition is satisfied for every price level and fiscal policy is not fully indexed.[11] As stated above I focus on stationary policies in this Section, that is, a particular equilibrium where policy rules prescribe a stationary policy. I allow for policy rules in Section 4.6.

As is well known the Huggett economy is in equilibrium if aggregate asset supply (households' savings) equals real aggregate asset demand (government bonds), which can be represented by the well-known Figure 1 (left panel). Households' asset demand $S(1+r, ...)$ is an upward sloping function of the real interest rate $1+r$.[12] Other variables such as taxes, transfers, and properties of

is fully nominal, only partially so.

[11]Kocherlakota and Phelan (1999) (KP) impose an additional condition on the government budget (equation (25) in KP) to rule out the FTPL. In their complete markets model this condition is equivalent to the household transversality condition which has to be satisfied in any equilibrium. But the KP complete markets condition is not equivalent to the incomplete markets transversality condition, which for example and in contrast to complete markets is consistent with a real interest rate below the growth rate of the economy. Therefore KP does not apply to incomplete markets models and does not indicate whether the FTPL is operating or not.

[12]Typically researchers impose an exogenous credit constraint instead of the natural borrowing limit to obtain a realistic distribution of net wealth and MPCs. None of the arguments in this paper depend on this choice as

the income process shift the asset demand function. Real asset supply by the government equals B/P, where B is nominal bonds and P is the price level.[13] The equilibrium condition is

$$S(1+r,...) = \frac{B}{P}. \tag{4}$$

This is one equation with two unknowns, the real interest rate $1+r$ and the price level P. This suggests that a continuum of price levels (associated with a continuum of real interest rates), e.g. P_1, P_2, P_3, clears the asset market as illustrated in the right panel of Figure 1. I will now argue that equation (4) nevertheless determines the price level, since the real interest rate is determined by monetary and fiscal policy.

How Monetary and Fiscal Policy Determine the Steady-State Real Interest Rate:

In both complete and incomplete market models, a Fisher relation between the steady-state nominal interest i_{ss}, real interest rate r_{ss} and inflation π_{ss} holds:

$$1+r_{ss} = \frac{1+i_{ss}}{1+\pi_{ss}}. \tag{5}$$

Monetary policy sets the steady-state nominal interest rate i_{ss}. Fiscal policy sets the growth rate of nominal spending (G), nominal tax revenues (T) and nominal debt (B). In a steady state, real government spending, real tax revenue and real government debt are constant, such that the steady-state condition for fiscal policy is that the growth rates of nominal spending, nominal tax revenues and nominal debt all are equal to the inflation rate (in the absence of economic growth),[14]

$$1+\pi_{ss} = \frac{G'-G}{G} = \frac{T'-T}{T} = \frac{B'-B}{B}. \tag{6}$$

To be clear about the interpretation of these steady-state conditions: If fiscal policy decides for a 2% nominal growth rate in government spending, $\frac{G'-G}{G}$, then the steady-state condition that steady-state real government expenditures are constant requires that the steady-state inflation rate equals 2% as well. The steady-state further requires that nominal tax revenues T and nominal government debt B also grow at 2%. It is important to note that these considerations do not determine the levels of real spending, real taxes and real debt except in the sense that these are unchanging over time in a steady state. In particular, the price level is not yet determined.

Equation (6) means that the inflation rate is set by fiscal policy and is equal to the growth rate

both generate nontrivial precautionary savings and an asset demand $S(1+r,...)$, which is sufficient for price level determinacy.

[13] With positive inflation rate π, bonds in a steady state at time t equal $B(1+\pi)^t$ and the price level equals $P(1+\pi)^t$ for initial values B and P, so that the term $(1+\pi)^t$ term cancels when computing the real value of bonds B/P.

[14] With real economic growth of rate g, $(1+\pi_{ss})(1+g) = \frac{G'-G}{G} = \frac{T'-T}{T} = \frac{B'-B}{B}$.

of nominal government spending, implying that the equilibrium real interest rate is determined jointly by monetary and fiscal policy.[15] These conclusions about the steady-state inflation rate are valid even if monetary policy implements an interest rate rule such as

$$i_t = \max(\bar{i} + \phi(\pi_t - \pi^*), 0), \tag{7}$$

for an inflation target π^*, an intercept $\bar{i}$ and $\phi > 0$. In this case inflation is still determined by equation (6) and the steady-state nominal interest rate equals[16]

$$i^{ss} = \max(\bar{i} + \phi(\frac{B' - B}{B} - \pi^*), 0). \tag{8}$$

Note that this line of reasoning requires that there is a continuum of potential steady-state real interest rates, and not just one equal to $1/\beta$ as in complete market models. Therefore this logic to determine the long-run inflation rate does not apply if markets are complete.

Price Level Determinacy

I can now use equation (4) to determine the price level. Using the result that $(1 + r_{ss}) = \frac{1+i_{ss}}{1+\pi_{ss}}$ is set by policy to eliminate the real interest rate from the list of unknowns, equation (4) now has just one unknown, the price level P^*:

$$S(\frac{1 + i_{ss}}{1 + \pi_{ss}}, ...) = \frac{B}{P^*}, \tag{9}$$

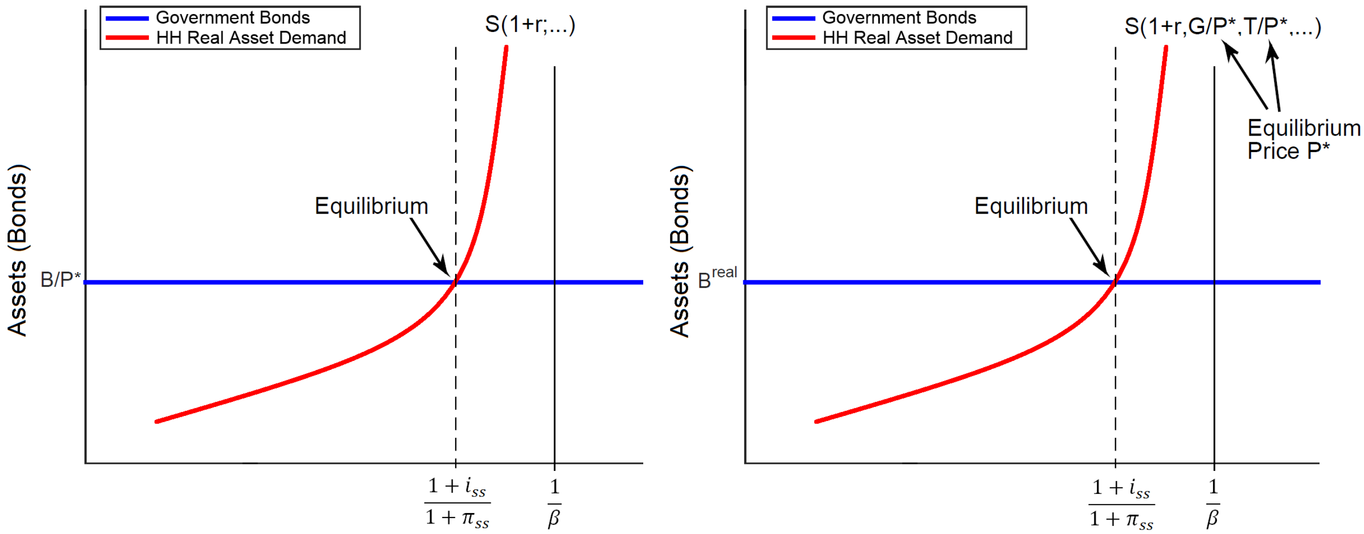
which serves to determine the unique price level, as illustrated in the left panel of Figure 2.

There are two key assumptions to obtain price level determinacy. First, fiscal policy is nominal. Without this assumption fiscal policy would be specified fully in real terms, and the equilibrium in the asset market would not depend on the price level such that the equilibrium condition cannot be used to determine the price level. Second, there is a steady-state aggregate asset demand function, which depends on the real interest rate. This is a standard result in models with heterogeneous agents and market incompleteness. I explain below in detail why the arguments for the Huggett economy do not apply in complete market environments.

Price Level Determinacy with Fully Indexed Bonds

[15] Monetary and fiscal policy cannot implement any arbitrary steady-state real interest rate, only one that is consistent with a steady state. In particular, as in any incomplete markets model, $\beta(1 + r_{ss}) < 1$ has to hold since otherwise asset demand would become infinite.

[16] For example if $\bar{i} = 0.02$, $\phi = 1.5$, debt grows at $\frac{B'-B}{B} = 0.02$ and the inflation target $\pi^* = 0$ then the steady-state inflation is 2% and the nominal interest rate equals $i^{ss} = 0.02 + 1.5 * 0.02 = 0.05$. In the (less realistic) case that the inflation target of monetary policy $\pi^* = 0.04$ exceeds the 2% that follows from fiscal policy, the steady-state nominal interest rate equals $i^{ss} = \max(0.02 + 1.5(0.02 - 0.04), 0) = 0$ and inflation is still 2%.



(a) Nominal Bonds

(b) Price-Indexed Bonds

Figure 2: Asset Market Equilibrium: a) Nominal Bonds; b) Price-Indexed Government Debt B^{real}.

I now generalize the discussion of price level determinacy in the Huggett economy. For illustrative purposes, I make the extreme assumption that the real value of government bonds is fixed at B^{real} and that government spending and taxes are fully nominal.[17]

In the above discussion I assumed that bonds are nominal but did not have households' real asset demand depend on nominal variables as well,

$$S(1 + r, G/P, T/P, \ldots). \tag{10}$$

The reason why households' real asset demand depends on the level of real taxes, T/P, for a fixed real interest rate is again explained by heterogeneity and incomplete markets. Those features imply the failure of the permanent income hypothesis and that agents, as a result of this failure, engage in precautionary savings: A lower steady-state level of real taxes increases both demand and (precautionary) savings. This reasoning extends to changes in the price level which translate one-for-one into changes in real taxes, since the nominal level of taxes is fixed.

The intuition is straightforward. A higher steady-state price level lowers real government consumption since fiscal policy is nominal, and at the same time lowers the tax burden for the private sector by the same amount. Households, however, do not spend all of the tax rebate on consumption but instead use some of the tax rebate to increase their precautionary savings. This less than one-for-one substitution of private sector demand for government consumption implies a drop in aggregate demand (households plus government demand) and an increase in households'

[17]This assumption also makes clear that the theory in my paper is different from the Fiscal Theory of the Price Level (FTPL), where the price level is determined such that the real value of bonds clears the government present value budget constraint. Obviously the FTPL has no bite if the real value of bonds is fixed and nominal taxes are set to balance the budget every period.

asset demand. This would require an adjustment of the real interest rate to stimulate demand and lower savings such that both the goods and the asset markets clear. As in the scenario considered above, however, the steady-state real interest rate cannot adjust to equate supply and demand because it is pinned down by the nominal interest rate set by monetary policy, and by the inflation rate, which is equal to the growth rate of nominal government spending. Therefore the price level must adjust such that demand equals supply when the real interest rate equals $1 + r_{ss} = \frac{1+i_{ss}}{1+\pi_{ss}}$,

$$S(\frac{1+i_{ss}}{1+\pi_{ss}}, G/P^*, T/P^*, ...) = B^{real}, \tag{11}$$

as the right panel of Figure 2 illustrates.

Price Level Determinacy: General Case

In the general case that all fiscal variables, T, G and B are nominal the equilibrium condition determining the price level P^* is

$$B/P^* = S(\frac{1+i_{ss}}{1+\pi_{ss}}, T/P^*), \tag{12}$$

where I used that the stationary distribution of assets and desired aggregate savings can be computed once households know the real interest rate and the real taxes they have to pay. Again there is one unknown, the price level P^* and one equation, asset market clearing.

Instead of writing savings as a function of the real interest rate, one can also write the real interest rate as a function of assets, so that the equilibrium in the asset market can be represented equivalently as

$$\frac{1+i_{ss}}{1+\pi_{ss}} = (1 + r_{ss}(T/P^*, B/P^*, \ldots)) \tag{13}$$

which again determines the steady-state price level P^*. The same arguments made above imply that the real interest rate r_{ss} depends on B/P and T/P.[18]

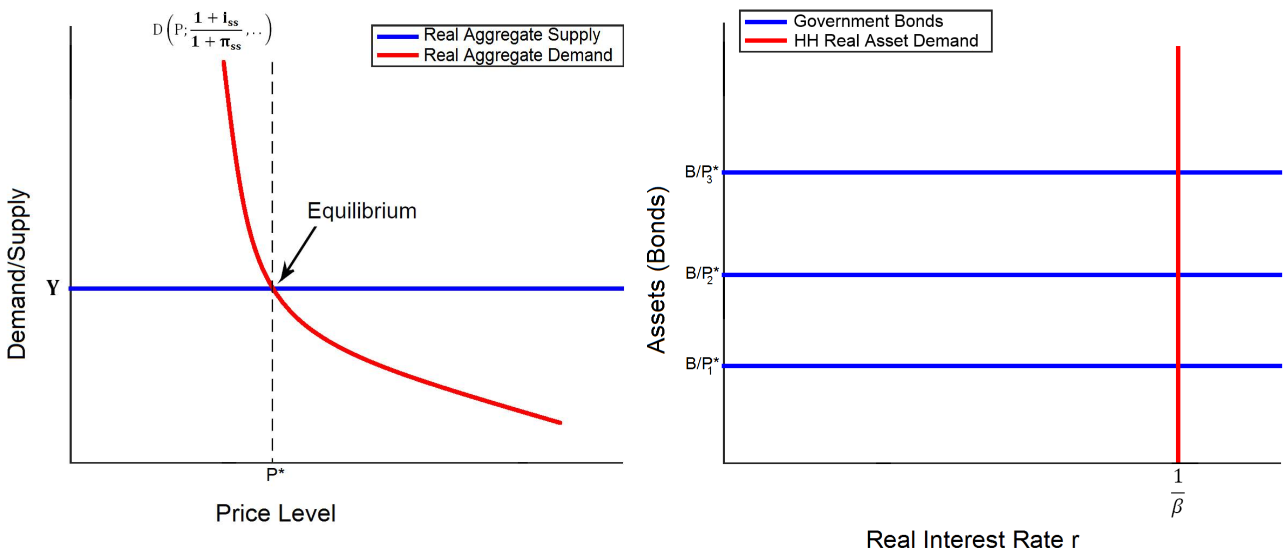
Price Level and Aggregate Demand

There is an equivalent characterization of the price level as clearing the goods market, which not surprisingly mirrors the above characterization as clearing the asset market. Private steady-state real demand equals

$$D(1 + r, T/P, ...) = Y + \frac{RB - T}{P} - S(1 + r, T/P, ...), \tag{14}$$

which is a function of the real interest rate and the price level and where Y is real income. Demand depends on the real interest rate and real taxes T/P (and thus on P since T is nominal) because

[18] The interpretation is that this is the real interest rate that makes households willing to hold B/P real assets in steady state.



(a) Goods Market (b) Representative Agent

Figure 3: a) Price Level Determination in Goods Market; b) Representative Agent: Indeterminate Price Level

savings do. Using the government budget constraint $RB - T = B - G$,

$$D(1+r, P, ..) = Y + \frac{B-G}{P} - S(1+r, T/P, ...), \tag{15}$$

so that aggregate demand, the sum of private demand D and government consumption G/P,

$$D(1+r, P, ..) + \frac{G}{P} = Y + \frac{B}{P} - S(1+r, T/P, ...), \tag{16}$$

which, since private and public consumption are not perfect substitutes, is downward sloping in the price level,

$$\frac{\partial D(1+r, P, ..) + \frac{G}{P}}{\partial P} < 0, \tag{17}$$

determining a unique steady-state price level as equating supply and demand,

$$D(\frac{1+i_{ss}}{1+\pi_{ss}}, P^*, ..) + \frac{G}{P^*} = Y. \tag{18}$$

The left panel of figure 3 illustrates how the price level is determined as clearing the goods market.

Price Level Indeterminacy and the Representative Agent

The above reasoning does not extend to representative agent environments so that the price level is indeterminate if markets are complete. The key implication of complete markets is that the steady-state real interest rate is determined by the discount factor only, $(1 + r_{ss})\beta = 1$, whereas in incomplete market models the real interest rate depends on virtually all model primitives. The counterpart to equation (13) in a representative agent model is thus

$$\frac{1+i_{ss}}{1+\pi_{ss}} = 1 + r_{ss} = 1/\beta, \tag{19}$$

which no longer depends on the price level, and the price level is therefore indeterminate. The right panel of Figure 3 illustrates the indeterminacy, depicting supply and demand in the asset market as before with incomplete markets, but with the difference that now the steady-state savings curve is a vertical line at the steady-state interest rate $1/\beta$. With incomplete markets, it is an upward sloping curve. The vertical asset demand curve with complete markets reflects the result that the real interest rate is independent of the quantity of real bonds such that a continuum of price levels, e.g. P_1^*, P_2^*, P_3^*, satisfies all equilibrium conditions.

The same conclusion is reached if government spending G is nominal. Again, a continuum of equilibrium price levels exists. Different price levels P_1 and P_2 lead to different levels of real government consumption, G/P_1 and G/P_2, and different levels of household consumption, $Y - G/P_1$ and $Y - G/P_2$, respectively, where Y is output. Although the division of output between private and government consumption varies for different price levels, all those divisions of output are equilibria. If fiscal policy is specified in real terms then the price level matters not at all (complete dichotomy) and any price level can be the equilibrium price level.

In Section A.I (appendix) I explore additional aspects of price level determinacy, by:

- Explaining price level indeterminacy in Hand-to-Mouth consumer models, perpetual youth models and representative agent models with aggregate risk.
- Showing that the determinacy result holds in a model with capital and where households have a non-trivial demand for money.
- Showing that the price level responses to monetary and fiscal policy, and to changes in technology and in the need for liquidity, are in line with conventional wisdom and with their precise characterization in Sections 3 and A.II.1.
- Discussing Global Determinacy: First, that prices are in steady state when monetary and fiscal policy are. This in particular requires ruling out inflationary and deflationary spirals. Second, determinacy must be established when policy is not constant but is conducted according to some policy rule. I also
  - illustrate the arguments in a simple Huggett economy,
  - provide a closed-form solution for the price level.

3 Model

Households in this economy are infinitely-lived and heterogenous in their spending needs. In terms of preferences and trading frictions each period resembles a Diamond and Dybvig (1983) economy. However, to keep the heterogeneity analytically tractable, households are members of large families which pool all family assets at the beginning of each period. As in the Bewley-Imrohoroglu-Huggett-Aiyagari incomplete market model households can acquire one interest-bearing liquid asset.[19] I will not distinguish between money and bonds but show in Section 4.5, following the analysis of a cashless economy in Woodford (2003), that a demand for money can be added to the model, so that households can acquire cash and the liquid asset. The demand for money then determines the quantity of money that the central bank will need to supply in order to implement its nominal interest rate target. In the theoretical analysis in this and the next section, prices are flexible and labor supply is inelastic since these assumptions, by themselves, do not imply determinacy or indeterminacy of the price level. To provide numerical illustrations of the workings of the model I relax both assumptions in Section A.III where prices are sticky and labor supply is elastic.

3.1 Households

Time is discrete and extends from $t = 0, \ldots, \infty$. There is a continuum of measure one of households. Each period $t \geq 0$ is divided into two distinct and successive sub-periods t_1 and t_2. The only source of uncertainty is an idiosyncratic i.i.d. emergency expenditure shock in the spirit of Diamond and Dybvig (1983), which realizes only in period t_2. The sub-periods t_1 and t_2 are not different points in real time but instead this type of modeling just reflects informational assumptions within the period. This captures the important underlying assumption that households participate in the asset market before they learn the shock, inducing a demand for liquidity. The timing of events is as follows: In sub-period t_1, before the realization of the the risk household h consumes C_t^h and consumes c_t^h in the second sub-period t_2.

Households are exposed to liquidity risks at t_2, which leads to heterogeneity in consumption and asset holdings. To keep the model tractable, I make the assumption that each household is a family consisting of a continuum of individuals of measure one. Each member of the household

[19] Although some model elements resemble those of a cash-in-advance model where money pays interest the two models are far from being equivalent. The main reason is that here households can use all available assets whereas households are restricted to use one particular asset, cash, for some purchases in cash-in-advance models, implying that different first-order conditions characterize the dynamic equilibrium. As it turns out and I explain as I go along, the simple incomplete market model in this section is therefore much more tractable than monetary models discussed in Woodford (1994).

has a need for spending in t_2 which is governed by the i.i.d. shock

$$\theta \in [\underline{\theta}, \infty] \sim F, \tag{20}$$

where $\underline{\theta} \geq 0$ and corresponding pdf f. The infinite support for θ ensures that some individuals are always liquidity constrained, simplifying the analysis quite a bit. An individual who experiences a shock θ and consumes C_t^h in period t_1 and c_t^h at t_2 derives utility

$$u(C_t^h) + \theta v(c_t^h) \tag{21}$$

in period t. Each individual's demand for consumption at t_2 is increasing in the idiosyncratic value of θ. Because the household has a continuum of members the distribution of θ across the members of a household is given by the distribution F.

As I will later add elastic labor supply, I assume here a linear technology which transforms each individual's inelastically supplied unit of labor $h^i = 1$ into Ah^i units of output y:

$$y = Ah, \tag{22}$$

so that each household produces A consumption goods in period t.

As in Diamond and Dybvig (1983), because the expenditures needs at t_2 are sudden, I assume that a liquid asset (bonds) is necessary for these expenditures. The interpretation is that each member of the household has to acquire period t_2 consumption from the market and cannot obtain it from his or her own family because members are spatially separated.[20]

Since sub-periods t_1 and t_2 are not real points in time, sellers cannot target their sales to a particular point in time, in particular sellers cannot separate t_1 and t_2 consumers. As the goods sold at t_1 and t_2 are identical and produced with the same technology they are sold at the same price P_t at time t_1 and t_2.

In period t_1 each household chooses consumption in period t_1, C_t^h, consumption at t_2 as a function of θ, $c_t^h(\theta)$, and how many nominal bonds to acquire, b_t^h, so that b_t^h/P_t is the value of bonds in terms of consumption goods, where P_t is the price level at time t. The return on bonds is R_{t+1}. Since all members of a household are identical, each has the same level of consumption at t_1 and enters period t_2 with the same amount of bonds. During period t_2, each member has access only to his or her own bonds to be spend on consumption $c_t^h(\theta)$,

$$c_t^h(\theta) \leq b_t^h/P_t. \tag{23}$$

[20]A previous version allowed households to obtain credit up to some limit without any consequences on the results. I therefore omit it now for simplicity.

Excess bonds not needed for emergency expenditures, $\min(b_t^h - P_t c_t^h(\theta), b_t^h)$, are returned to the family.

The household's budget constraint at t_1 is:

$$P_t C_t^h + b_t^h = P_t A h_t - T_t + R_t b_{t-1}^h - P_t \overline{C}_t^h, \tag{24}$$

where T_t are the households' nominal tax obligations of to be paid at t_1, and $\overline{C}_t^h$ is the sale of household consumption goods to members of other families who need consumption in period t_2. It is important to notice that $\overline{C}_t^h$ (how much is sold at t_2) is not a decision variable of the household. The household sells his $Ah = A$ units of endowments, taking the price P_t as given. How much the households sells at t_1 relative to t_2 is exogenous to the households. Consistency then requires that the revenue obtained from t_1 customers can be used for households consumption at t_1 but the revenue from t_2 customers cannot. What is endogenous is how much t_1 and t_2 consumption households demand and market clearing implies that this demand has to be equal to supply, determining equilibrium sales.[21]

Since θ is distributed according to F in all families, expected spending on consumption in period t_2 is equal to sales in period t_2 in a symmetric equilibrium

$$E_\theta(c_t^h(\theta)) = \overline{C}_t^h, \tag{25}$$

so that the amount of bonds owned by a household at the end of period t_2 equals

$$b_{t_2}^h = E_\theta(b_t^h - P_t c_t^h(\theta)) + P_t \overline{C}_t^h = b_t^h. \tag{26}$$

The household's flow budget constraints simplifies to

$$P_t C_t^h + E_\theta(P_t c_t^h(\theta)) + b_t^h = P_t A h_t - T_t + R_t b_{t-1}^h. \tag{27}$$

The decision problem of a household with initial period bond holdings b_{t-1}^h is

$$V_t(b_{t-1}^h) = \max_{b_t^h, C_t^h, c_t^h(\theta)} \{u(C_t^h) + E_\theta \theta v(c_t^h(\theta)) + \beta E_t[V_{t+1}(b_t^h)]\} \tag{28}$$

subject to the flow budget constraint (27) and the liquidity constraint (23). Note that $\overline{C}_t^h$ not

[21] It is straightforward to allow $\overline{C}_t^h$ to be a decision variable without affecting my results. In this case the price Q_t of period t_2 consumption - now different from the period t_1 price P_t - ensures market clearing, so that for example households do not target all their sales to period t_1. This extension does not affect the determinacy result but just adds an additional price, Q_t, and one extra equation (optimal division of sales between t_1 and t_2) with no additional relevant insights.

being a decision variable allows me to simplify the exposition and to use the equilibrium condition (25) already when formulating the household decision problem.

The optimal decision for $c_t(\theta)$ is described through the threshold $\hat{\theta}_t$, which solves

$$\hat{\theta}_t v'(b_t^h/P_t) = u'(C_t) \tag{29}$$

such that

$$\begin{aligned} &i) \quad c_t(\theta) \text{ solves } \theta v'(c_t(\theta)) = u'(C_t) && \text{if } \theta \leq \hat{\theta}_t, \text{ i.e. } v'(b_t^h/P_t) \leq u'(C_t), \\ &ii) \quad c_t(\theta) = b_t^h/P_t && \text{if } \theta > \hat{\theta}_t, \text{ i.e. } v'(b_t^h/P_t) > u'(C_t). \end{aligned}$$

In case $i)$, θ is below the threshold where households do not use all their bonds for emergency expenditures at t_2. In case $ii)$, θ is above the threshold where all bonds acquired at t_1 are used for consumption, so that overall consumption spending equals b_t^h/P_t, Note that in the absence of a binding liquidity constraint, households would choose the first best allocation $c_t^{FB}(\theta)$ solving $\theta v'(c_t^{FB}(\theta)) = u'(C_t)$, which is not feasible for values of θ with $c_t^{FB}(\theta) > b_t^h/P_t$.

To guarantee existence of an equilibrium and that the precautionary demand for savings never vanishes, I make the same assumption as in the Huggett economy in Section A.I (equation (A37)):

$$\lim_{x \to 0} x v'(x) > 0, \tag{30}$$

which is satisfied by log-utility and CRRA with risk aversion large than one. As in the Huggett economy this assumes states of the world which force the household to engage in precautionary savings.[22]

As an alternative proof (see also Section A.I), the literature (Obstfeld and Rogoff (1983, 2017)) has provided a very elegant solution to rule out hyper-inflations in monetary models which applies here as well. If the government provides some fractional real backing of debt/transfers, that is the government trades bonds/transfers for real consumption at a (very high) price $\bar{P}$ then the same arbitrage argument as in the monetary literature eliminates this "perverse" hyper-inflation equilibrium. Prices cannot rise above $\bar{P}$ ruling out ruling out $P = \infty$ and thus this type of

[22]Models with money-in-the-utility function make a similar assumption on the utility derived from monetary services. The interpretation in Huggett type economies is however quite different. First, in Huggett type incomplete market models agents are free to use all their assets for consumption and not only one, cash. Second, here the standard assumption (30) guarantees a state of the world where precautionary savings are essential. It is also important to keep in mind that here as in all of the recent literature in monetary economics the central bank sets the nominal interest rate on short-term bonds and not money supply, which gives rise to price level indeterminacy in the first place (Sargent and Wallace (1975)). In addition to setting the nominal return on bonds, the central bank can also pay interest rates on reserves. But this does not overcome the indeterminacy issue and only changes the opportunity costs of holding money. For determinacy, setting money supply and paying interest rates on reserves is equivalent to not setting the nominal return on bonds but setting money supply instead.

hyperinflation.

What a proof has to rule out is basically only that the price level is infinite, $P = \infty$. The previous two line of arguments work if prices are flexible. Another alternative emerges in models with some price rigidities as under this (realistic) assumption it is straightforward to show that $P = \infty$ is not possible. In models with sticky prices firms set prices and jumping to $P = \infty$ would incur infinite Rotemberg (1982) price adjustment costs which is clearly not an equilibrium. With Calvo price setting the fraction of non-adjusting firms would charge a finite price and would absorb all demand whereas the $P = \infty$ firms would face no demand, implying that setting $P = \infty$ is not optimal and that the output weighted price level would be finite.

I do not follow these routes here since one might (incorrectly) think that the first one is some form of the FTPL and for the second one I would have to add price stickiness to the model. Those who find assumption (30) implausible can think of mz flexible price economy as a close approximation of the same economy but with a very small degree of price rigidities, which virtually does not change the analysis but only rules out $P = \infty$.

The remaining decision on the quantity of bonds to acquire is characterized through the first-order condition:

$$u'(C_t) = \int_{\hat{\theta}_t}^{\infty} \theta v'(b_t^h/P_t) dF(\theta) + F(\hat{\theta}_t) E_t[\frac{R_{t+1}P_t}{P_{t+1}} \beta u'(C_{t+1})]. \tag{31}$$

Observe that if an individual's expenditure needs are strong enough, $\theta_t > \hat{\theta}_t$, which is the case with probability $1 - F(\hat{\theta}_t)$, there is a shortage of the liquid asset. The infinite support of θ ensures the existence of a finite $\hat{\theta}_t$ such that there is shortage of the liquid asset with positive, potentially small, probability. The transversality condition is

$$\lim_{T\to\infty} \beta^T u'(C_T) \frac{R_T B_T}{P_T} = 0. \tag{32}$$

3.2 Fiscal and Monetary Policy

The aim of this paper is to show how monetary policy, fiscal policy, and their interaction determine the price level. A standard way to represent monetary policy is through setting a sequence of nominal interest rates,

$$\mathcal{R} = R_0, R_1, R_2, \ldots, R_t, \ldots. \tag{33}$$

This flexible and unrestricted way of describing monetary policy also captures interest rate rules. An equilibrium is then a fixed point where the interest rate rule maps a sequence of prices to nominal interest rates and the theory laid out in this paper maps these nominal interest rates, without using the Taylor principle, back into a unique sequence of prices. I will prove the properties of economies with monetary and fiscal policy rules in Section 4.6.[23]

Fiscal policy is represented by a sequence of nominal government spending

$$\mathcal{G} = G_0, G_1, \ldots, G_t, \ldots, \tag{34}$$

which needs to be financed by levying nominal lump-sum taxes T_t,

$$\mathcal{T} = T_0, T_1, \ldots, T_t, \ldots. \tag{35}$$

The government's flow budget constraint has to be satisfied at any point in time, which implicitly defines a sequence of nominal bonds[24]

$$B_t = R_t B_{t-1} + G_t - T_t. \tag{36}$$

Since fiscal and tax policies are expressed in nominal terms, this constraint holds for *all* sequences of prices,

$$\mathcal{P} = P_0, P_1, \ldots, P_t, \ldots. \tag{37}$$

In particular the price level is not determined such that the government budget constraint holds. Finally, define the sequence of bonds

$$\mathcal{B} = B_0, B_1, \ldots, B_t, \ldots. \tag{38}$$

3.3 Competitive Equilibrium

Definition 1. *Given sequences of nominal interest rates $\mathcal{R}$, nominal government spending $\mathcal{G}$, nominal taxes $\mathcal{T}$ and nominal bonds $\mathcal{B}$ a competitive symmetric equilibrium are sequences of consumption spending $\{C_t\}_{t=0}^{\infty}$ at t_1 and $\{c_t(\theta)\}_{t=0}^{\infty}$ at t_2, bonds purchases $\{b_t^h\}_{t=0}^{\infty}$ and prices $\mathcal{P}$, such*

[23]A similar argument applies to the case without commitment, for example when political economy considerations kick in. The political process maps prices (and other variables) into nominal interest rates and into fiscal variables and the theory laid out in this paper maps these monetary and fiscal variables back into prices (and other variables). A political economy equilibrium is again a fix point of these mappings. Note however that I assume that the nominal debt issued in period t is fully honored in period $t+1$ which allows me to abstract from government default.

[24]Note that here no limiting condition such as $\lim_{t\to\infty} B_t \prod_{s=0}^{t-1} \frac{1}{R_s} = 0$ needs to be imposed once households transversality condition (TVC) is satisfied. Indeed this limiting condition would not be satisfied in a steady state where $R \equiv 1$ (ZLB), $B > 0$ and $G = T$ although the government budget constraint is clearly satisfied. Such a scenario would violate the TVC in complete market models but not necessarily in incomplete market models where households could be willing to hold on to bonds forever while the real interest rate in nonpositive.

that for all t, the following holds:

1. *Households take prices and policies as given and choose $\{C_t, c_t(\theta), b_t^h\}_{t=0}^{\infty}$ to maximize utility.*
2. *Given prices, firms choose $\{h_t\}_{t=0}^{\infty}$ to maximize profits.*
3. *The government budget constraint holds (36).*
4. *Market clearing and resource constraint:*
   (a) *Bond Market : $B_t = b_t^h$,*
   (b) *Resource Constraint $C_t + E_\theta c_t(\theta) + G_t/P_t = Ah_t$,*
   (c) *Labor Market: $h_t = 1$.*

4 Prices, Inflation and Nominal Demand

This section shows that the price level is determinate as the unique solution where supply equals demand. The main result delivering determinacy is that aggregate demand - the sum of private and government demand - is decreasing in the price level. Key to this result is that households engage in precautionary savings. A fall in government consumption is then not offset one-for-one by an increase in private consumption of the same amount. Instead, households engage in precautionary savings, implying lower aggregate demand for a given price level.

This line of reasoning will be used throughout this section, first to show the existence and uniqueness of a steady-state price level, then to rule out a vanishing or exploding price level, and finally to prove determinacy globally.[25]

4.1 Price Level Determinacy: Steady State

To establish that the steady-state price level is determined, I proceed through several steps. I first characterize the steady state and prove existence and uniqueness of the steady-state price level. Next, I show that the nominal anchor provided by government spending prevents prices from converging to zero. Furthermore, I establish that an exploding price level would lead to insufficient market demand, as private consumption is not fully substituting for government demand, but that households instead engage in precautionary savings, leading to a downward pressure on prices and

[25] The dynamics of the price level out of steady-state can be precisely characterized to be monotone here in contrast to the analysis of models with money in Woodford (1994) where additional assumptions have to imposed to rule bounded endogenous fluctuations and other types of indeterminacies. The main reason is that here the dynamics are derived from one equation only, the (different) first-order condition for bonds, which leads to monotone functions Φ and Γ (introduced below) describing the evolution of prices and other variables. Note also that the infinite support for θ implies that the liquidity-constraint is always binding with positive probability.

thus ruling out such an explosive path. Finally, in Section 4.2, I use the same arguments ruling out explosive and vanishing price paths to prove price level determinacy outside of steady states in Theorem 1.

In a *steady state* nominal interest rates are constant at R; nominal government spending, taxes and nominal bonds are all strictly positive growing at rate γ,

$$G_t = G(1+\gamma)^t, T_t = T(1+\gamma)^t, B_t = B(1+\gamma)^t, \tag{39}$$

consumption C at t_1 and $c(\theta)$ at t_2 are time-invariant, and prices are growing at a constant rate $\frac{P_{t+1}}{P_t} = \pi_{t+1} = \pi$,

$$P_t = P(1+\pi)^t. \tag{40}$$

Since in a steady state C_t, B_t/P_t, $\hat{\theta}_t$ and $\frac{R_{t+1}}{1+\pi_{t+1}}$ are constant, equation (31) implies that

$$\frac{\beta R}{1+\pi} < 1. \tag{41}$$

Permanently higher nominal interest rates would not be consistent with a steady state and would lead instead to exploding asset demand as in the Bewley-Imrohoroglu-Huggett-Aiyagari incomplete markets model.

In a steady state real government spending is constant, implying that the inflation rate π equals the growth rate of nominal spending γ,

$$\pi = \gamma. \tag{42}$$

It is important to stress that this result is not a tautology. Rather, it implies that the steady-state inflation rate is set by fiscal policy. To see this, compare two steady states with the same fiscal policy (the same γ), but with different nominal interest rates $R^H > R^L$. The previous result implies that inflation is the same in the two steady states and that the real interest rate is different. Similarly, two steady states with different fiscal policies, $\gamma^H > \gamma^L$ feature different inflation rates even if nominal interest rates are identical. Note also that this logic breaks down when markets are complete since in this case the real interest rate is equal to $1/\beta$ across all steady states and, in particular, is independent from policy. Note that this is almost the identical logic as used in Section 2 to show that the steady-state inflation rate is equal to the growth rate of nominal debt independently of which interest rate rule or which inflation target is implemented.

Next, I characterize equilibrium consumption C_t at t_1,

$$C_t = A - \frac{G_t}{P_t} - E_\theta c_t(\theta), \tag{43}$$

and how it depends on the price level before moving on to the main result on the price level. Equilibrium consumption C_t equals output A minus government consumption G_t/P_t and expected family consumption $E_\theta c_t(\theta)$ at t_2. Clearly, G/P is falling in the price level, leaving more resources to the private economy and implying that consumption C at t_1 increases in P. Households' aim to also increase spending at t_2 is partly obstructed by the credit constraint (23), which imposes an upper bound on spending at t_2. As a result more households are credit constrained. This constraint becomes binding at a lower value of θ, so that the equilibrium threshold $\hat{\theta}$ falls in P. In the appendix I prove

Proposition 1. *Given a fixed sequence of government policies, consumption C_t in period t_1 can be written as a function of P_t only, $C_t = C(P_t)$, and is increasing in P_t and the threshold $\hat{\theta}_t$ is a function of P_t only, $\hat{\theta}_t = \hat{\theta}(P_t)$ and is decreasing in P_t.*

The steady-state price level P^* then clears the goods market as the solution to

$$u'(C(P^*)) = \int_{\hat{\theta}}^{\infty} \theta v'(B/P^*) dF(\theta) + F(\hat{\theta}(P^*)) \frac{R}{(1+\pi)} \beta u'(C(P^*)), \tag{44}$$

so that the price at time t, $P_t = P^*(1+\pi)^t$. The next proposition summarizes the findings for the steady state

Proposition 2. *A steady exists and is unique. In particular, the steady-state price level is determined uniquely. The steady-state inflation rate is equal to the growth rate of nominal government spending,*

$$\pi = \gamma,$$

which is set by fiscal policy only.

The preceding analysis implies that an equilibrium defines a difference equation in prices, relating prices in periods t and $t+1$,

$$\Phi(\tilde{P}_t) := \frac{u'(C(\tilde{P}_t)) - \int_{\hat{\theta}(\tilde{P}_t)}^{\infty} \theta v'(B_t/\tilde{P}_t) dF(\theta)}{\tilde{P}_t F(\hat{\theta}(\tilde{P}_t))} = \frac{R_{t+1}}{1+\gamma} \beta \frac{u'(C(\tilde{P}_{t+1}))}{\tilde{P}_{t+1}} =: \frac{R_{t+1}}{1+\gamma} \Gamma(\tilde{P}_{t+1}), \tag{45}$$

where I define the detrended price

$$\tilde{P}_t = \frac{P_t}{(1+\gamma)^t}. \tag{46}$$

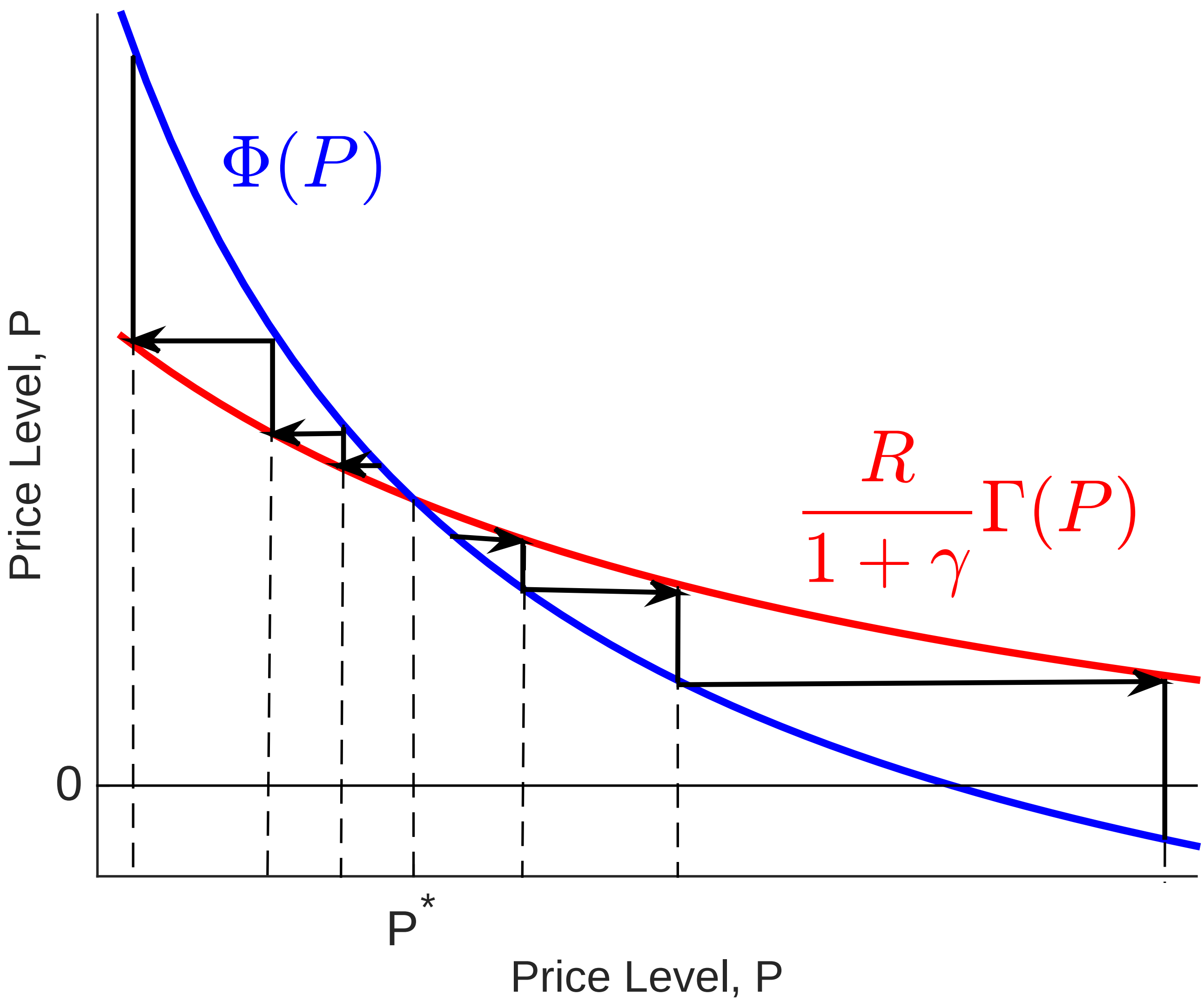


Figure 4: Dynamics of the Price Level

In the steady state, $\tilde{P}_t = P^*$. I now show that this is the only solution by ruling out vanishing and explosive price paths. Since both functions Φ and Γ are decreasing in $\tilde{P}_t$ and $\tilde{P}_{t+1}$ respectively, this equation allows to solve $\tilde{P}_{t+1}$ as an increasing function of $\tilde{P}_t$,

$$\tilde{P}_{t+1} = \Gamma^{-1}\left(\frac{1+\gamma}{R_{t+1}}\Phi(\tilde{P}_t)\right). \tag{47}$$

Figure 4 illustrates the dynamics of the price level, $\tilde{P}_t$, that is implied by equations (45) and (47). Both functions Φ and Γ are downward sloping and from the diagram it is apparent that there is a unique steady state at P^* since Φ is steeper than Γ.

A price level $\tilde{P}_t$ higher than P^* implies lower real government spending and therefore that aggregate supply exceeds aggregate demand. Goods market clearing requires an even higher price

$\tilde{P}_{t+1} > \tilde{P}_t$, i.e. a lower real interest rate, next period to decrease savings and increase private demand such that aggregate demand increases to match period t aggregate supply, which leads by the same argument again to a higher price level in period $t+2$, $\tilde{P}_{t+2}$, and so on. Eventually the exploding price path will drive real government spending to zero. Since precautionary savings do not disappear, private demand will fall short of aggregate supply, establishing that this price sequence is not an equilibrium. The non-existence becomes apparent in the diagram as Φ eventually becomes negative, and the iteration breaks down since $\Gamma > 0$.

A price level $\tilde{P}_t$ lower than P^* implies higher real government spending and therefore that aggregate demand exceeds aggregate supply. Goods market clearing at time t then requires an even lower price $\tilde{P}_{t+1} < \tilde{P}_t$, i.e. a higher real interest rate, to increase savings and lower private demand such that aggregate demand falls to period t aggregate supply, again leading by the same arguments to a lower price level in period $t+2$, $\tilde{P}_{t+2}$, and so on. Eventually the price level will be so low that government demand exceeds output, which clearly cannot be an equilibrium.[26]

The only price sequence which forms an equilibrium is the one where the price is constant and equal to the steady-state price level P^*, that is, the price at time t equals $P^*(1+\gamma)^t$ and aggregate demand equals aggregate supply.

Proposition 3. *For a constant nominal interest rates R, and strictly positive nominal government spending, taxes and nominal bonds which are growing at rate γ, there is a unique equilibrium,*

$$\begin{aligned} \tilde{P}_t &= P^* \\ P_t &= P^*(1+\gamma)^t \end{aligned}$$

so that $C_t = C(P^*)$ and $\hat{\theta}_t = \hat{\theta}(P^*)$.

In particular the nominal anchor set by fiscal policy ensures that both exploding prices and vanishing prices are not an equilibrium.

An equivalent characterization of the dynamics of the price level can be given in terms of the eigenvalue of the first-order difference equation in $\hat{p}_t = \log\left(\tilde{P}_t\right) - \log(P^*)$. The above arguments

[26]One can also iterate on the first-order condition for bonds to obtain

$$\frac{u'(C_t)}{\tilde{P}_t} = \sum_{s=t}^{t+T-1} \left\{ \left[\prod_{k=t}^{s-1} \frac{F(\hat{\theta}_k) R_{k+1}\beta}{1+\gamma} \right] \frac{\int_{\hat{\theta}(\tilde{P}_s)}^{\infty} \theta v'(B_s/\tilde{P}_s + \bar{b}) dF(\theta)}{\tilde{P}_s} \right\} + \left[\prod_{k=t}^{t+T-1} \frac{F(\hat{\theta}_k) R_{k+1}\beta}{1+\gamma} \right] \frac{u'(C_{t+T})}{\tilde{P}_{t+T}}$$

and use an transversality argument which requires that the latter term $\left[\prod_{k=t}^{t+T-1} \frac{F(\hat{\theta}_k)R_{k+1}\beta}{1+g}\right] \frac{u'(C_{t+T})}{\tilde{P}_{t+T}} \to 0$. This rules out deflationary paths since $F(\hat{\theta}_{t+T-1}) \frac{R_{t+T}\tilde{P}_{t+T-1}}{(1+g)\tilde{P}_{t+T}} \beta \to 1$ when $\tilde{P} \to 0$ implying that the transversality condition does not hold along deflationary paths.

imply that $\hat{p}_t$ satisfies

$$\frac{\partial \hat{p}_{t+1}}{\partial \hat{p}_t} > 1, \tag{48}$$

that is the eigenvalue is larger than one. As is well known this is equivalent to local determinacy.

I will argue in section 4.6 that local determinacy can be expected to hold in a wider class of incomplete markets models. The key requirement is that the aggregate asset demand function is increasing in the real interest rate.

4.2 Price Level Determinacy: Non Steady State Policies

The analysis so far has considered stationary policies where the nominal interest rate is constant and fiscal policy is characterized by a constant growth rate. Since I also want to consider impulse responses to shocks to the nominal interest rate or to government spending, the analysis has to go beyond steady states. I now consider arbitrary sequences of nominal interest rates R_t, B_t, T_t and G_t which are assumed to be stationary only after time S, $R_t = R$, $G_t = G(1+\gamma)^{t-S}$, $T_t = T(1+\gamma)^{t-S}$ and $B_t = B(1+\gamma)^{t-S}$ for $t \geq S$. The next proposition establishes that the determinacy result extends to these non-stationary policies with the difference that now, the unique price sequence is no longer constant.

Theorem 1. *The price level is determined for arbitrary sequences of nominal interest rates and nominal government spending. In particular there is a unique price sequence.*

The proof uses similar arguments - non vanishing precautionary savings / government demand cannot exceed output - as those used to establish Proposition 3. The only difference is that now I must rule out prices higher or lower than the unique price sequence; above I had to rule out prices above or below the unique steady state. Figure 5 illustrates the key features of the argument. The aggregate demand curve is downward sloping and intersects the aggregate supply curve at price level P^*. Two components, government and private demand, add up to aggregate demand but the first is decreasing while the latter is increasing in the price level. For high price levels government consumption approaches zero, but a non-vanishing precautionary demand prevents private demand from fully substituting for the fall in government consumption such that it always falls short of aggregate supply. This rules out prices higher than P^*. For low prices real government spending explodes and therefore private consumption falls, such that government spending eventually exceeds output, ruling out prices lower than P^* as well.

The economic mechanism which ensures price determinacy in my simple incomplete markets model is the same as the one described in the graphical analysis in Section 2. To see this I derive

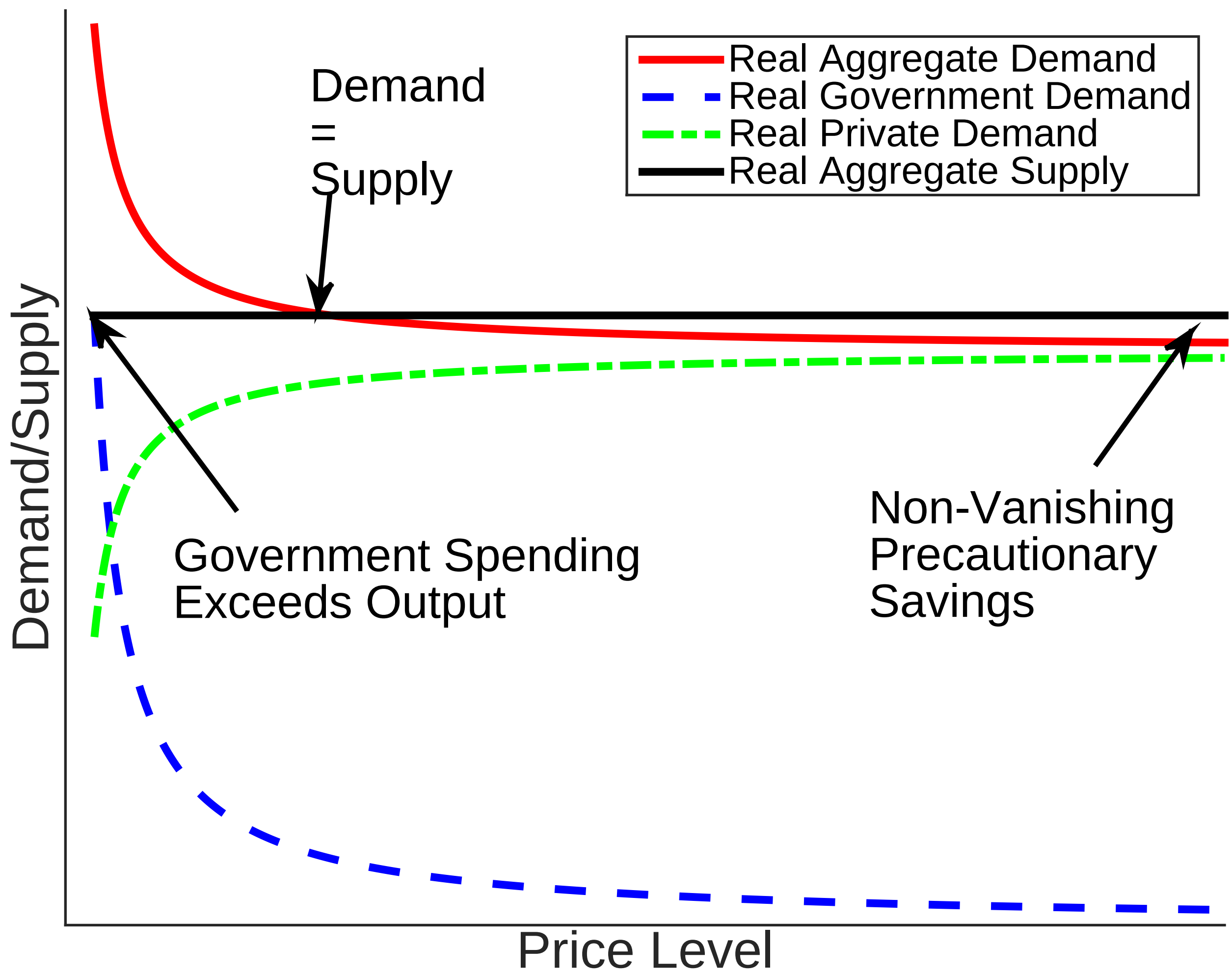


Figure 5: Aggregate Demand = Government Demand + Private Demand = Aggregate Supply

the aggregate savings schedule from the first-order condition for households savings decision,

$$u'(C) = \int_{\hat{\theta}}^{\infty} \theta v'(S) dF(\theta) + F(\hat{\theta}(\tilde{P})) \frac{R}{(1+\pi)} \beta u'(C). \tag{49}$$

Steady-state consumption C and $c(\theta)$ satisfy

$$C + E_{\theta} c(\theta) = A + \frac{R}{1+\pi} S - S' - \frac{T}{\tilde{P}}. \tag{50}$$

where households' savings are S', assets (=last period savings) are S so that $\frac{R}{1+\pi} S$ is real asset income, $\frac{T}{\tilde{P}}$ are real tax obligations and A is labor income. Solving this equation for real savings S' yields the savings $S'(S, \frac{R}{(1+\pi)}, T/\tilde{P}, ...)$ as a function of assets S, real interest rates and real tax obligations. Imposing the steady state condition $S = S'$ and solving for the fixpoint results in households' steady-state savings schedule

$$S(\frac{R}{(1+\pi)}, T/\tilde{P}, ...). \tag{51}$$

as a function of the steady-state real interest rate $\frac{R}{(1+\pi)}$ real tax payments $T/\tilde{P}$. As in Section 2 the unique steady-state price level P^* equates asset supply and demand while the real interest rate is determined by policy,

$$B/P^* = S(\frac{R}{(1+\pi)}, T/P^*, ...). \tag{52}$$

Again an equivalent representation exists where P^* equates demand and supply in the goods market. Private demand for households with $B/\tilde{P}$ real assets

$$D(\frac{R}{(1+\pi)}, T/\tilde{P}, ...) = A + \frac{R}{1+\pi}\frac{B}{\tilde{P}} - \frac{T}{\tilde{P}} - S' \tag{53}$$

Using the government budget constraint $T = G + \frac{R}{(1+\pi)}B - B'$, where B is existing debt and B' is newly issued debt,

$$D(\frac{R}{(1+\pi)}, T/\tilde{P}, ...) = A + \frac{B'}{\tilde{P}} - S'(B/\tilde{P}, \frac{R}{(1+\pi)}, T/\tilde{P}, ...) - \frac{G}{\tilde{P}}, \tag{54}$$

where as above $S'(B/\tilde{P}, \frac{R}{(1+\pi)}, T/\tilde{P}, ...)$ is savings as a function of assets $B/\tilde{P}$, real interest rates and real tax obligations. Aggregate demand, the sum of private demand D and government consumption $G/\tilde{P}$, equals

$$D(\frac{R}{(1+\pi)}, T/\tilde{P}, ...) + \frac{G}{\tilde{P}} = A + \frac{B'}{\tilde{P}} - S'(B/\tilde{P}, \frac{R}{(1+\pi)}, T/\tilde{P}, ...), \tag{55}$$

which is downward sloping in the price level,

$$\frac{\partial D(\frac{R}{(1+\pi)}, T/\tilde{P}, ...) + \frac{G}{\tilde{P}}}{\partial \tilde{P}} < 0. \tag{56}$$

The equilibrium price clears the goods market

$$D(\frac{R}{(1+\pi)}, T/\tilde{P}, ...) + \frac{G}{\tilde{P}} = A, \tag{57}$$

which not surprisingly is equivalent to clearing the asset market,

$$\frac{B}{\tilde{P}} = \frac{B'}{\tilde{P}} = S'(B/\tilde{P}, \frac{R}{(1+\pi)}, T/\tilde{P}, ...) = S. \tag{58}$$

Without a precautionary demand for savings, consumption equals $C = A - G/\tilde{P}$ and aggregate demand is equal to aggregate supply $C + G/\tilde{P} = A$ for all price levels,

$$\frac{\partial D(\tilde{P}) + \frac{G}{\tilde{P}}}{\partial \tilde{P}} = 0. \tag{59}$$

Therefore in this case the price level would not be determined as equating demand and supply but would simply determine the size of the government, $G/\tilde{P}$.

4.3 Impulse Responses

In the appendix A.II.1 I characterize the model impulse responses of prices which turn out to be consistent with the responses of steady-state prices to monetary and fiscal policy, technology and liquidity shocks in Figures 8 - 9 in the graphical analysis in Section A.I: tightening monetary policy, increases in productivity and a higher demand for liquidity lower prices, and expansionary fiscal policy increases prices. The tractability of my incomplete markets model allows me to go beyond comparing steady states and to characterize the impulse response paths of prices in Section A.II.1.

4.4 How and when monetary policy controls the inflation rate

A main result of this paper is that fiscal policy is what determines the steady-state inflation rate through controlling nominal spending. If nominal government consumption, nominal taxation and nominal government debt grow at rate γ, then steady-state inflation $\pi = \gamma$. In this case monetary policy has no control over the long-run inflation rate, since fiscal policy does control the nominal anchor.

Yet, monetary policy, as shown above, can still affect the price level. I also show in Section A.III that monetary policy can quite effectively stabilize prices around the steady state. An attempt by fiscal policy to stimulate the economy through generating above steady-state temporary inflation and employment can be neutralized completely by monetary policy. This results in higher government spending, but prices and employment remain at their steady-state values.

In different scenarios monetary policy can affect the long-run inflation rate. One such scenario is where fiscal policy takes the initial level of nominal government debt, B, prices P and monetary policy as given, and fixes the level of real government consumption, g, and the real tax revenue, τ. Some simple algebra can then illustrate how this scenario leads to a different conclusion about long-run inflation.

Fiscal policy has to make two choices. It decides about $s = \tau - g$, the real constant primary surplus, and about the growth rate of nominal government debt, $\frac{B_{+1}-B}{B}$, such that the steady-state flow government constraint holds

$$\frac{Ps}{B} = -\frac{B_{+1} - B}{B} + (R - 1), \tag{60}$$

taking as given P, B and R. Note that in the Fiscal Theory of the Price Level the government effectively picks P through committing to some s and $\frac{B_{+1}-B}{B}$ which satisfy the budget constraint for only one price level P. Generically this makes the price level overdetermined in my incomplete markets model since one variable, the price level, has to satisfy both equations, the government budget constraint and goods market clearing equations, at the same time. Here, in contrast, the budget constraint is satisfied for all P and the price level is uniquely determined as clearing the goods market.

If fiscal policy commits to a real surplus s, then equation (60) implies the debt issuance necessary to balance the budget. In this scenario the growth rate of nominal debt and thus the long-run inflation rate are jointly determined, by fiscal policy through setting s and by monetary policy by setting R:

$$1 + \pi = \frac{B_{+1} - B}{B} = (R - 1) + \frac{Ps}{B}, \tag{61}$$

where the price level clears the good market.

The reason why now monetary policy and fiscal policy determine the long-run inflation rate jointly is simple. The general principle, that it is fiscal policy which de facto determines steady-state inflation, still holds as it is the evolution of the nominal anchors that matters. However, a government insisting on a certain level of real expenditures and tax revenues has to finance all deficits through issuing debt with a return determined by monetary policy. An increase in the nominal interest rate therefore will lead to an increase in nominal government debt, which with flexible prices materializes immediately in higher inflation rates. In this scenario it is monetary policy and fiscal policy that determine the growth rate of nominal government debt, and therefore the growth rate of G, T and prices P. Future research will show whether and in which episodes monetary or fiscal policy determined the inflation target.

But although monetary policy cannot fully assume control of steady-state inflation, the reasoning illustrates that it still has a large impact on government debt. This channel of monetary policy can be used to make a debt-financed expansionary fiscal policy very expensive for the government. In practice it could be an effective way for monetary policy to prevent inflationary policies.

4.5 Robustness: Adding money to the model

So far I made the standard assumption of a cashless economy. To show that this assumption is inconsequential for the results I now add a motive to hold cash to the model, an extension that does not affect my conclusions. Setting nominal interest rates and fiscal policy is sufficient to determine the price level. Households' nominal money holdings are then endogenously determined to satisfy real money demand. To show this I assume that the transaction services provided by real money balances are represented as an argument of the utility function,

$$u(C_t^h) + \chi(M_t/P_t) + \theta v(c_t^h), \tag{62}$$

where M_t is a family's end of period t_1 money holdings. The family's budget constraint then equals

$$P_t C_t + E_\theta(P_t c_t(\theta)) + b_t^h + M_t = P_t A h_t - T_t + R_t b_{t-1}^h + M_{t-1}, \tag{63}$$

where M_{t-1} is period t initial money holdings carried over from period $t-1$. The government collects seigniorage, which is rebated to households through lower taxation so that its flow budget constraint equals

$$B_{t+1} + M_{t+1} = R_t B_t + M_t + G_t - T_t. \tag{64}$$

The remaining features of the model are unchanged.

The decision problem of a household with initial period bond holdings b_t^h and money holdings M_{t-1} is

$$V_t(b_{t-1}^h, M_{t-1}) = \max_{b_t^h, C_t, c_t(\theta), M_t} \quad \{u(C_t^h) + \chi(M_t/P_t) + E_\theta \theta v(c_t^h) + \beta E_t[V_{t+1}(b_t^h, M_t)]\} \tag{65}$$

subject to the flow budget constraint (63) and the liquidity constraint (23).

Money holdings M_t appear only in the first-order condition of M_t (the small seignorage gains are rebated lump-sum to households),

$$\frac{u'(C_t) - \chi'(M_t/P_t)}{P_t} = \frac{u'(C_{t+1})}{P_{t+1}}, \tag{66}$$

implying that prices P_t and P_{t+1} and consumption C_t and C_{t+1} can be solved independently from M_t. The first-order condition for money is then used to solve for M_t. The only purpose of adding a demand for money to the model is thus to determine the quantity of money that the central bank will need to supply in order to implement its nominal interest rate target.

4.6 Policy Rules

The previous sections establish price level determinacy without aggregate shocks and when monetary and fiscal policy is described as an exogenous sequence of policy variables (see Section 3.2). I now extend the analysis and allow for two features common to many monetary model: time-varying policy rules and aggregate productivity is fluctuating. I then establish conditions on the policy rule which deliver local determinacy. Interestingly, in nominal incomplete market models policy rules do not overcome indeterminacy but instead may induce it.

I assume a standard interest rate rule

$$(1+i_{t+1}) = (1+\bar{i}_{t+1})(\frac{P_t}{\bar{P}})^{\rho_p^i}(\frac{A_t}{\bar{A}})^{\rho_A^i}, \tag{67}$$

a rule for nominal debt

$$B_t = \bar{B}_t(\frac{P_t}{\bar{P}})^{\rho_p^B}(\frac{A_t}{\bar{A}})^{\rho_A^B}, \tag{68}$$

a nominal government spending rule

$$G_t = \bar{G}_t(\frac{P_t}{\bar{P}})^{\rho_p^G}(\frac{A_t}{\bar{A}})^{\rho_z^G}, \tag{69}$$

and set taxes to balance the budget

$$T_t := (1+i_t)B_{t-1} + G_t - B_t, \tag{70}$$

where $\bar{A}$ and $\bar{P}$ are the target levels and $\rho_p^{\cdot}, \rho_A^{\cdot} \geq 0$.

In a steady-state $P_t = P^*$, $A_t = A_{ss}$, $\bar{i}_t = \bar{i}$, $\bar{B}_t = \bar{B}$ and $\bar{G}_t = \bar{G}$, so that

$$(1+i_{ss}) = (1+\bar{i})(\frac{P^*}{\bar{P}})^{\rho_p^i}(\frac{A_{ss}}{\bar{A}})^{\rho_A^i}, \tag{71}$$

$$B_{ss} = \bar{B}(\frac{P^*}{\bar{P}})^{\rho_p^B}(\frac{A_{ss}}{\bar{A}})^{\rho_A^B}, \tag{72}$$

$$G_{ss} = \bar{G}(\frac{P^*}{\bar{P}})^{\rho_p^G}(\frac{A_{ss}}{\bar{A}})^{\rho_A^G}, \tag{73}$$

$$T_{ss} = i_{ss}B_{ss} + G_{ss}. \tag{74}$$

Proposition 4. *An equilibrium with aggregate fluctuations and policy rules exists. If the economy and policies are in a steady-state as described in (71) - (74) then the steady state price level P^* is* *as in Proposition 3 and solves*

$$u'(C(P^*)) = \int_{\hat{\theta}}^{\infty} \theta v'(B_{ss}/P^*)dF(\theta) + F(\hat{\theta}(P^*))\frac{1+i_{ss}}{(1+\pi)}\beta u'(C(P^*)), \tag{75}$$

for $C(P^*) = A_{ss} - \frac{G_{ss}}{P^*} - E_\theta c_t(\theta)$.

I consider now local determinacy at a steady-state where log deviations of prices are denoted $\hat{p} = \log(P) - \log(P^*)$. Note that prices are the only endogenous variables and are not predetermined so that an eigenvalue larger than one is equivalent to local determinacy.

Proposition 5. *If policy parameters* $\rho_p^G, \rho_p^B < 1$ *then the equilibrium is locally determinate, i.e. the eigenvalue of prices*

$$\frac{\partial \hat{p}_{t+1}}{\partial \hat{p}_t} > 1. \tag{76}$$

No restriction on monetary policy is required.

The intuition why determinacy can be ensured only if parameter restrictions on fiscal policy are imposed is straightforward. Consider first debt policy and suppose that $P_t > P^*$. If debt policy is not responding to prices, $\rho_p^B = 0$, $P_t > P^*$ implies a fall in the real value of debt, so that households require a lower real interest rate or equivalently a higher inflation rate ($P_{t+1} > P_t$) to absorb less real debt. If debt policy is aggressive, $\rho_p^B > 1$, this reasoning does not work since in this case $P_t > P^*$ implies an increase in the real value of debt. Households then require a higher real interest rate to be willing to absorb more real debt. If the nominal interest rate is not responding to prices this requires a fall in prices, $P_{t+1} < P_t$, that is the eigenvalue is smaller than one. Similarly $\rho_p^G > 1$ implies that real government spending increases if prices increase, implying that aggregate demand is increasing in prices. Equilibrium in goods market then requires an increase in the real interest rate to contract private demand. Again, if the nominal interest rate is not responding to prices, this requires a fall in prices, $P_{t+1} < P_t$, that is the eigenvalue is smaller than one.

Note that there are no restrictions on monetary policy. This is not surprising since already a constant nominal interest rate implies determinacy. A higher price $P_t > P^*$ implies a fall in real debt and requires a decrease in the real interest rate, which is equivalent to $P_{t+1} > P_t$, that is the eigenvalue is larger than one. If the nominal interest rate increases in response to higher prices then an even larger increase in prices is necessary to lower the real interest rate, that is the eigenvalue gets even larger, again implying determinacy.

The intuition when fiscal policy induces local determinacy independently of how monetary policy is conducted did not use much model details and can thus be expected to be valid in richer incomplete market models as well. A criterion that combines conditions on monetary and fiscal policy can be derived as well but this is necessarily more model dependent as it requires to compare the demand effects of monetary and fiscal policy. A key number in this endeavour is naturally the intertemporal elasticity of substitution. For the clarity of exposition I assume that

$$u(C) = \frac{C^{1-\sigma}}{1-\sigma} \tag{77}$$

$$v(c) = \eta\frac{c^{1-\sigma}}{1-\sigma}. \tag{78}$$

Another complication is that the fraction of individuals running into liquidity constraint may change. To abstract from this model detail I assume that $f(\theta(P^*)) = 0$. I can then derive an extended condition for local determinacy.

Proposition 6. *Suppose utility functions are* $u(C) = \frac{C^{1-\sigma}}{1-\sigma}$ *and* $v(c) = \eta\frac{c^{1-\sigma}}{1-\sigma}$*. If*

$$-\rho_p^i - \sigma\min\{(1-\rho_p^B),(1-\rho_p^G)\}\frac{\frac{G_{ss}}{AP_{ss}} + \frac{B_{ss}}{AP_{ss}}}{1 - \frac{G_{ss}}{AP_{ss}} - \frac{B_{ss}}{AP_{ss}}} < 0 \tag{79}$$

then the equilibrium is locally determinate, i.e. the eigenvalue of prices

$$\frac{\partial\hat{p}_{t+1}}{\partial\hat{p}_t} > 1. \tag{80}$$

If one realistically assume that the sum of government spending and liquid assets is less than 50% of GDP then $\frac{\frac{G_{ss}}{AP_{ss}}+\frac{B_{ss}}{AP_{ss}}}{1-\frac{G_{ss}}{AP_{ss}}-\frac{B_{ss}}{AP_{ss}}} < 1$ so that the determinacy condition simplifies to

$$-\rho_p^i - \sigma\min\{(1-\rho_p^B),(1-\rho_p^G)\} < 0, \tag{81}$$

which includes the conditions of proposition 5 but goes beyond those. An expansive fiscal policy ($\rho_p^B, \rho_p^G > 1$) now induces determinacy but monetary policy has to be sufficiently contractionary, that is ρ_p^i has to be sufficiently high as stated in the proposition. The intuition for this result builds on the explanations given above for the determinacy of monetary and fiscal policy. Suppose again $P_t > P^*$ and $\rho_p^B > 1$. Again real debt increases so that households require a higher real interest rate to be willing to absorb more real debt. If monetary policy was passive this would require a fall in P_{t+1} (relative to P_t). But if the nominal interest rate increases more than the required real interest rate, then $P_t = P_{t+1}$ would imply that the real interest rate is too high. As a consequence next period's price P_{t+1} again has to increase to bring the real interest rate down. A similar intuition applies to an aggressive spending policy, $\rho_p^G > 1$.

To see that similar conditions apply to the more general incomplete markets models of Section

2, consider a log-linear approximation of the asset market clearing condition

$$\frac{B_t}{P_t} = S_t(1 + r_{t+1}, T_t/P_t, (1 + i_t)\frac{B_{t-1}}{P_t}), \tag{82}$$

which is

$$\hat{b}_t - \hat{p}_t = \epsilon_{S,r}\hat{r}_{t+1} + \epsilon_{S,t}(\hat{t}_t - \hat{p}_t) + \epsilon_{S,b}[(1 + \bar{i})(\hat{b}_{t-1} - \hat{p}_t) + \hat{i}_t\bar{b}], \tag{83}$$

where ˆ variables are the log deviations from steady-state values. Using the Fisher equation and policy rules for nominal bonds $\hat{b}_t = \rho^b\hat{p}_t$, for nominal taxes $\hat{t}_t = \rho^t\hat{p}_t$ and the nominal interest rate $\hat{i}_{i+1} = \rho^i\hat{p}_t$, the approximation simplifies to

$$(\rho^b - 1)\hat{p}_t = \epsilon_{S,r}((1 + \rho^i)\hat{p}_t - \hat{p}_{t+1}) + \epsilon_{S,t}((\rho^t - 1)\hat{p}_t) + \epsilon_{S,b}[(1 + \bar{i})(\rho^b\hat{p}_{t-1} - \hat{p}_t) + \rho^i\bar{b}\hat{p}_{t-1}] \tag{84}$$

and after re-ordering yields a second-order difference equation

$$\hat{p}_{t+1} = \underbrace{\big[1 + \rho^i + \frac{1 - \rho^b + \epsilon_{S,t}(\rho^t - 1) - \epsilon_{S,b}(1 + \bar{i})}{\epsilon_{S,r}}\big]}_{=:\gamma_1}\hat{p}_t + \underbrace{\frac{\epsilon_{S,b}[(1 + \bar{i})\rho^b + \rho^i\bar{b}]}{\epsilon_{S,r}}}_{=:\gamma_2}\hat{p}_{t-1}. \tag{85}$$

The solution is

$$\hat{p}_t = c_0\lambda_0^t + c_1\lambda_1^t, \tag{86}$$

for two eigenvalues

$$\lambda_{0,1} = \frac{\gamma_1}{2} \pm \sqrt{\left(\frac{\gamma_1}{2}\right)^2 + \gamma_2} \tag{87}$$

and coefficients c_0, c_1 solving initial conditions (for an initial price $\hat{p}_0$)

$$\hat{p}_0 = c_0 + c_1, \tag{88}$$

$$\gamma_1\hat{p}_0 = c_0\lambda_0 + c_1\lambda_1. \tag{89}$$

If policy is passive, $\rho^i = \rho^b = \rho^t = 0$ then $\hat{p}$ follows a simple first-order difference equation

$$\hat{p}_{t+1} = \big[1 + \frac{1 - \epsilon_{S,t} - \epsilon_{S,b}(1 + \bar{i})}{\epsilon_{S,r}}\big]\hat{p}_t. \tag{90}$$

Since higher taxes lower savings, $\epsilon_{S,t} < 0$, and higher wealth increases savings, $\epsilon_{S,b} > 0$, and $1 - \epsilon_{S,t} - \epsilon_{S,b}(1 + \bar{i})$ can be expected to be positive, the eigenvalue is larger than one iff $\epsilon_{S,r} > 0$, reflecting the intuition given above which is based on the assumption that households require a higher real interest rate to be willing to absorb a higher amount of real bonds.

The general setup of this section shows that the real interest rate elasticity of asset demand, $\epsilon_{S,r}$, is the key number for determinacy in all incomplete markets models, in the simple model of Section 3 and in the Huggett economy. However, as in propositions 5 and 6 and in contrast to the complete markets case, policy can induce (and not remove) indeterminacy in the general model. Indeterminacy arises if both eigenvalues $\lambda_{0,1}$ are smaller than one in absolute value.[27] Therefore since $\gamma_2 > 0$ a sufficient condition for determinacy is

$$\gamma_1 = 1 + \rho^i + \frac{1 - \rho^b + \epsilon_{S,t}(\rho^t - 1) - \epsilon_{S,b}(1 + \bar{i})}{\epsilon_r} > 1. \tag{91}$$

Consistent with the intuition above, a more aggressive monetary policy (a higher ρ^i) increases γ_1 and makes determinacy more likely whereas more aggressive fiscal policy (higher ρ^b, ρ^t) decreases γ_1 and thus makes determinacy less likely. Note however that γ_2 is increasing in ρ^b and thus makes determinacy more likely.[28] Taking this into account makes the determinacy region of ρ^b values larger than implied by condition (91). A similar argument, enlarging the determinacy region, applies to ρ^i and γ_2.

5 Conclusion

This paper has shown that the price level is globally determinate in models which incorporate the simple empirical finding that a permanent income increase leads to a less than one-for-one increase in consumption, and at the same time to an increase in precautionary savings.

The simplicity of my theoretical model keeps the analysis tractable and enables the researcher to better understand the monetary and fiscal transmission mechanism. A key finding is that the price level is determined jointly by monetary and fiscal policy, with long-run inflation determined by the growth rate of nominal government spending even if monetary policy is operating an interest rate rule with a different inflation target. The nominal anchor - nominal government spending - is controlled by fiscal policy, which therefore has the power to set the long-run inflation rate.

Applied to recent attempts by the ECB to increase inflation in the Euro area, the findings in this paper suggest that these efforts are unlikely to be successful. Instead higher inflation would require an expansion of nominal fiscal spending by Euro area member states to stimulate nominal

[27] If $\gamma_2 \neq 0$, then both coefficients $c_0, c_1 \neq 0$, so that the larger eigenvalue determines the dynamic (explosive) behavior of prices.

[28] The intuition is straightforward. The higher is ρ^b, the more assets households hold, the higher is asset demand, the larger will be the required reduction in the real interest rate and thus the larger will be the price increase.

demand, assigning an important role to large countries such as Germany. A fiscal stimulus by a small country would have very little impact on inflation, as it has only a negligible effect on area-wide demand, but would lead to a real appreciation (with likely adverse economic consequences) for this small country.

Applied to the growing concerns that the US or the world economy may be stuck in a liquidity trap with zero nominal and real interest rates for an extended time, the findings in this paper suggest an easy solution. Although the ZLB prevents further cuts of the nominal interest rate, fiscal policy can increase the growth rate of nominal spending and therefore the inflation rate, leading to lower real interest rates, provided that this policy is sufficiently persistent and credible. If instead fiscal policy continues its current austerity plan bringing low inflation rates to around zero, then the real interest rate will hover around zero too, even in the long run.

In a numerical exercise, I show that impulse responses to monetary and fiscal policy shocks, as well as to technology and discount factor shocks, line up with empirical evidence and conventional wisdom. I establish that forward guidance has only small effects, technological regress does not increase output and the fiscal multiplier gets smaller when prices become more flexible. I also establish how government spending serves as an automatic stabilizer, and how monetary and fiscal policy interact, and discuss stabilization policies at the ZLB.

The model used to conduct the quantitative exercises lacks many elements which are likely to be necessary to obtain more precise estimates of the policy effects. A full quantitative macroeconomic model would: use the Aiyagari (1994) incomplete market model with capital and elastic labor supply as a starting point; model the consumption behavior better such that the MPC aligns with the data; allow for distortionary taxation such that the cost of an expansionary policy is more realistic; and allow for long-term government debt to discuss quantitative easing. We do so in Hagedorn et al. (2016). Building on the insights in this paper, we use a full Aiyagari (1994) incomplete market model to quantitatively assess the size of the fiscal multiplier in a model with a determinate price level.

Furthermore, a determinate price level not only allows for policy analyses when the zero lower bound is binding, but more generally for studying policy when the nominal interest rate is pegged or the interest rate rule does not satisfy the Taylor principle. In standard complete market models the nominal interest rate has to respond to inflation aggressively enough to guarantee a locally determinate inflation rate. Policy analysis is now not restricted to this subset of aggressive monetary policies to avoid indeterminacy but instead monetary policy can be represented by an arbitrary exogenous sequence of nominal interest rates or an interest rate rule, because for each sequence and each rule determinacy of the equilibrium is ensured. In particular monetary policy can coordinate with fiscal policy and is not bound to respond aggressively to any stimulative inflationary policy conducted by the treasury.

Bibliography

Aiyagari, R. S. (1994): "Uninsured Idiosyncratic Risk and Aggregate Saving," *Quarterly Journal of Economics*, 109, 659–684.

Angeletos, G.-M. and C. Lian (2016): "A (Real) Theory of Aggregate Demand," Working paper.

Attanasio, O. and S. J. Davis (1996): "Relative Wage Movements and the Distribution of Consumption," *Journal of Political Economy*, 104, 1227–1262.

Attanasio, O. P. and N. Pavoni (2011): "Risk Sharing in Private Information Models With Asset Accumulation: Explaining the Excess Smoothness of Consumption," *Econometrica*, 79, 1027–1068.

Auclert, A. (2016): "Monetary Policy and the Redistribution Channel," Working paper.

Bayer, C., R. Lütticke, L. Pham-Dao, and V. Tjaden (2015): "Precautionary Savings, Illiquid Assets, and the Aggregate Consequences of Shocks to Household Income Risk," Working paper, revise and resubmit, Econometrica.

Blanchard, O. J. (1985): "Debt, Deficits, and Finite Horizons," *Journal of Political Economy*, 93, 223–247.

Blundell, R., L. Pistaferri, and I. Preston (2008): "Consumption Inequality and Partial Insurance," *American Economic Review*, 98, 1887–1921.

Buera, F. and J. P. Nicolini (2016): "Liquidity Traps and Monetary Policy: Managing a Credit Crunch," mimeo.

Bénassy, J.-P. (2005): "Interest rate rules, price determinacy and the value of money in a non-Ricardian world," *Review of Economic Dynamics*, 8, 651 – 667.

——— (2008): "The Fiscal Theory of the Price Level: A Non-Ricardian View," *Macroeconomic Dynamics*, 12, 31?44.

Campbell, J. and A. Deaton (1989): "Why is Consumption So Smooth?" *The Review of Economic Studies*, 56, 357–373.

Clarida, R., J. Gali, and M. Gertler (2000): "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory," *Quarterly Journal of Economics*, 115, 147–180.

Cochrane, J. H. (2015): "The New-Keynesian Liquidity Trap," Working paper, University of Chicago Booth School of Business.

Correia, I., J. Nicolini, and P. Teles (2008): "Optimal Fiscal and Monetary Policy: Equivalence Results," *Journal of Political Economy*, 116, 141–170.

Den Haan, W., P. Rendahl, and M. Riegler (2015): "Unemployment (Fears) and Deflationary Spirals," CEPR Discussion Paper 10814.

Diamond, D. W. and P. H. Dybvig (1983): "Bank Runs, Deposit Insurance and Liquidity," *Journal of Political Economy*, 91, 401–419.

Galí, J. (2017): "Monetary Policy and Bubbles in a New Keynesian Model with Overlapping Generations," Tech. rep.

Garriga, C., F. E. Kydland, and R. Sustek (2013): "Mortgages and Monetary Policy," Working Paper 19744, National Bureau of Economic Research.

Gornemann, N., K. Kuester, and M. Nakajima (2012): "Monetary Policy with Heterogeneous Agents," Working paper 12-21, Federal Reserve Bank of Philadelphia.

Guerrieri, V. and G. Lorenzoni (2015): "Credit Crises, Precautionary Savings, and the Liquidity Trap," mimeo.

Hagedorn, M., I. Manovskii, and K. Mitman (2016): "The Fiscal Multiplier," mimeo, University of Oslo.

——— (2017): "Monetary Policy in Incomplete Market Models: Theory and Evidence," mimeo, University of Oslo.

Huggett, M. (1993): "The risk-free rate in heterogeneous-agent incomplete-insurance economies," *Journal of Economic Dynamics and Control*, 17, 953 – 969.

Kaplan, G., B. Moll, and G. Violante (2016): "Monetary Policy According to HANK," Working paper.

Kaplan, G. and G. L. Violante (2014): "A Model of the Consumption Response to Fiscal Stimulus Payments," *Econometrica*, 82, 1199–1239.

Kocherlakota, N. and C. Phelan (1999): "Explaining the Fiscal Theory of the Price Level," *Federal Reserve Bank of Minneapolis Quarterly Review*, 14–23.

Kocherlakota, N. R. (2016): "Fragility of Purely Real Macroeconomic Models," mimeo.

Lütticke, R. (2015): "Transmission of Monetary Policy with Heterogeneity in Household Portfolios," Working paper.

Mankiw, N. G. and R. Reis (2002): "Sticky Information versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve," *The Quarterly Journal of Economics*, 117, 1295–1328.

McKay, A., E. Nakamura, and J. Steinsson (2015): "The Power of Forward Guidance Revisited," Working paper, forthcoming, American Economic Review.

McKay, A. and R. Reis (2016): "The Role of Automatic Stabilizers in the U.S. Business Cycle," *Econometrica*, 84, 141–194.

Obstfeld, M. and K. Rogoff (1983): "Speculative Hyperinflations in Maximizing Models: Can We Rule Them Out?" *Journal of Political Economy*, 91, 675–687.

——— (2017): "Revisiting Speculative Hyperinflations in Monetary Models," Working paper, London, Centre for Economic Policy Research.

Oh, H. and R. Reis (2012): "Targeted transfers and the fiscal response to the great recession," *Journal of Monetary Economics*, 59, Supplement, 50 – 64.

Ravn, M. O. and V. Sterk (2013): "Job Uncertainty and Deep Recessions," Working paper, conditionally accepted, Journal of Monetary Economics.

Rotemberg, J. J. (1982): "Sticky prices in the United States," *The Journal of Political Economy*, 1187–1211.

Sargent, T. J. and N. Wallace (1975): "Rational Expectations, the Optimal Monetary Instrument, and the Optimal Money Supply Rule," *Journal of Political Economy*, 83, 241–254.

——— (1981): "Some unpleasant monetarist arithmetic," *Federal Reserve Bank of Minneapolis Quarterly Review*, 1–17.

Sterk, V. and S. Tenreyro (2015): "The Transmission of Monetary Policy through Redistributions and Durable Purchases," Working paper.

Werning, I. (2015): "Incomplete Markets and Aggregate Demand," Working paper.

Woodford, M. (1994): "Monetary policy and price level determinacy in a cash-in-advance economy," *Economic Theory*, 4, 345–380.

——— (2003): *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton: Princeton University Press.

Yaari, M. E. (1965): "Uncertain Lifetime, Life Insurance, and the Theory of the Consumer," *The Review of Economic Studies*, 32, 137–150.

APPENDIX

A.I Appendix: The Price Level and Incomplete Markets: A Graphical Analysis

In this Section I discuss additional aspects of price level determinacy, supplementing the arguments in Section 2, by:

- Explaining price level indeterminacy in Hand-to-Mouth consumer models, perpetual youth models and representative agent models with aggregate risk.
- Showing that the determinacy result holds in a model with capital and where households have a non-trivial demand for money.
- Showing that the price level responses to monetary and fiscal policy, and to changes in technology and in the need for liquidity, are in line with conventional wisdom and with their precise characterization in Sections 3 and A.II.1.
- Discussing Global Determinacy: First, that prices are in steady state when monetary and fiscal policy are. This in particular requires ruling out inflationary and deflationary spirals. Second, determinacy must be established when policy is not constant but is conducted according to some policy rule. I also
  - illustrate the arguments in a simple Huggett economy,
  - provide a closed-form solution for the price level.

Price Level Indeterminacy: Hand-to-Mouth Consumers

The same basic arguments for the representative agent economy apply to models where a fraction of households is always hand-to-mouth and the remaining ones behave according to the permanent income hypothesis (PIH). Since hand-to-mouth consumers do not participate in the asset market, the real interest rate is determined by the discount factor of PIH households only, $(1 + r_{ss})\beta = 1$, and equilibrium in the asset market is again characterized through

$$\frac{1 + i_{ss}}{1 + \pi_{ss}} = 1 + r_{ss} = 1/\beta, \tag{A1}$$

which does not depend on the price level, implying that the price level is indeterminate. This model shows that it is not heterogeneity by itself that delivers the result. Rather it is the combination of heterogeneity and market incompleteness that leads to precautionary savings and a well-defined aggregate savings function, implying price level determinacy. By the same argument, permanent heterogeneity in productivity but otherwise complete markets will not lead to price level determinacy either, since again $(1 + r_{ss})\beta = 1$ in a steady state.

Price Level Indeterminacy: Perpetual youth model

Similar arguments hold in "perpetual youth" models (Yaari (1965), Blanchard (1985)) since the steady-state interest rate is again equal to the discount rate, but now adjusted for the probability of death or retirement, so that $(1 + r_{ss})\tilde{\beta} = 1$ in a steady state for the adjusted discount rate $\tilde{\beta}$. Again, the steady-state real interest rate is independent of the price level and only the change in prices, $1 + \pi_{ss}$, but not the level itself is determined.

In this class of models, this is however not the only equilibrium if the Samuelson dynamic inefficiency condition is satisfied. In this case both a bubbleless as well as a continuum of bubbly equilibria exist, a scenario explored in recent work by Galí (2017). Whereas most papers assume that the bubble is a real asset affecting the stock market or housing, a monetary bubble may coexist so that money has value as in Samuelson's work. As a result there is a continuum of equilibria each associated with a different value of money (= different size of the monetary bubble) and each associated with a different price level. As an example, suppose that a bubble exists that has a real value of one. In one equilibrium nominal money has a value of one, the price level is one and thus there are no real bubbles. In another equilibrium the price level is two and a real bubble with value one half exists. Or the price level is three and the real bubble has a value of one third. Or the real bubble has a value of one and money has no value.

Bénassy (2005, 2008) make a particular choice on the size of the monetary bubble through ruling out real bubbles (the first case in the previous example) and conditional on this choice find a unique bubbly price level. This approach however does not overcome the indeterminacy problem in the "bubbleless" equilibrium and it rules out other bubbly equilibria with different price levels by assumption.[29] Bénassy (2005, 2008) need to make this equilibrium selection to obtain a well-defined demand for money (or more generally for nominal government liabilities) since the Samuelson logic only delivers existence of a monetary equilibrium but not uniqueness. This shows again, as in the Hand-to-Mouth economy, that the failure of Ricardian equivalence is a necessary but not a sufficient condition for price level determinacy.

Price Level Indeterminacy: Representative agent and aggregate risk

Price level indeterminacy also arises in representative agent economies with aggregate risk. Suppose there are n aggregate shocks $s_1, \dots, s_n$ with associated consumption levels of the representative household $c_1, \dots, c_n$ and marginal utilities of consumption $u_1, \dots, u_n$. The FOC for

[29] Bénassy (2005, 2008) implicitly assume a particular strong dynamic inefficiency condition - the population growth rate exceeds the real interest rate (which exceeds $1/\beta$) - since consumption of the initial generation would eventually exceed GDP otherwise.

nominal bonds are therefore

$$v_i := \frac{u_i}{\tilde{P}_i} = \beta \frac{1+i_{ss}}{1+\gamma} \sum_{j=1}^{n} q_{ij} \frac{u_j}{\tilde{P}_j},$$

where $q_{ij} = Prob(s_j \mid s_i)$, $\tilde{P}_i$ is the price level in state i as a deviation from the trend γ, which is equal to the constant growth rate of nominal debt. In matrix from the FOCs read

$$\underbrace{\begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix}}_{v} = \beta \frac{1+i_{ss}}{1+\gamma} \underbrace{\begin{pmatrix} q_{11} & q_{12} & \cdots & q_{1n} \\ \vdots & \ddots & & \vdots \\ q_{n1} & \cdots & & q_{nn} \end{pmatrix}}_{Q} \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix}$$

The vector v is an eigenvector with eigenvalue one of the matrix $\beta \frac{1+i_{ss}}{1+\gamma} Q$ and the largest eigenvalue of Q is one with eigenvector $(1, 1, \ldots, 1)^{tr}$. Since $\beta \frac{1+i_{ss}}{1+\gamma} \leq 1$ it follows that $\beta \frac{1+i_{ss}}{1+\gamma} = 1$. Therefore for each $\kappa > 0$ the vector

$$\begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} = \begin{pmatrix} \kappa \\ \vdots \\ \kappa \end{pmatrix}$$

is a solution to the FOCs. This continuum of solutions corresponds to different price levels $\tilde{P}_i > 0$, establishing the indeterminacy. Adding aggregate risk to an economy with PIH-households and hand-to-mouth households also does not overcome the indeterminacy problem. The same arguments for the representative agent now apply to the PIH households.

Price Level Determinacy with Capital and Money (Incomplete Markets)

The determinacy result holds in a model with capital K and where households have a non-trivial demand for money. Denote by $L(1+i, ..)$ the aggregate demand for real balances as a function of the nominal interest rate i, and let M be nominal money supply. It is important that M is an endogenous variable, adjusted by the central bank to satisfy whatever money demand households have given the nominal interest rate set by the central bank.[30] The steady-state price level P^* and money M are determined as solutions to

$$\frac{M}{P^*} = L(1+i_{ss}, ..) \tag{A2}$$

$$K + \frac{B}{P^*} = S\left(\frac{1+i_{ss}}{1+\pi_{ss}}, ...\right), \tag{A3}$$

[30] The same argument holds when the central bank pays an interest rate i^m on money holdings. Again the central bank has to satisfy households money demand.

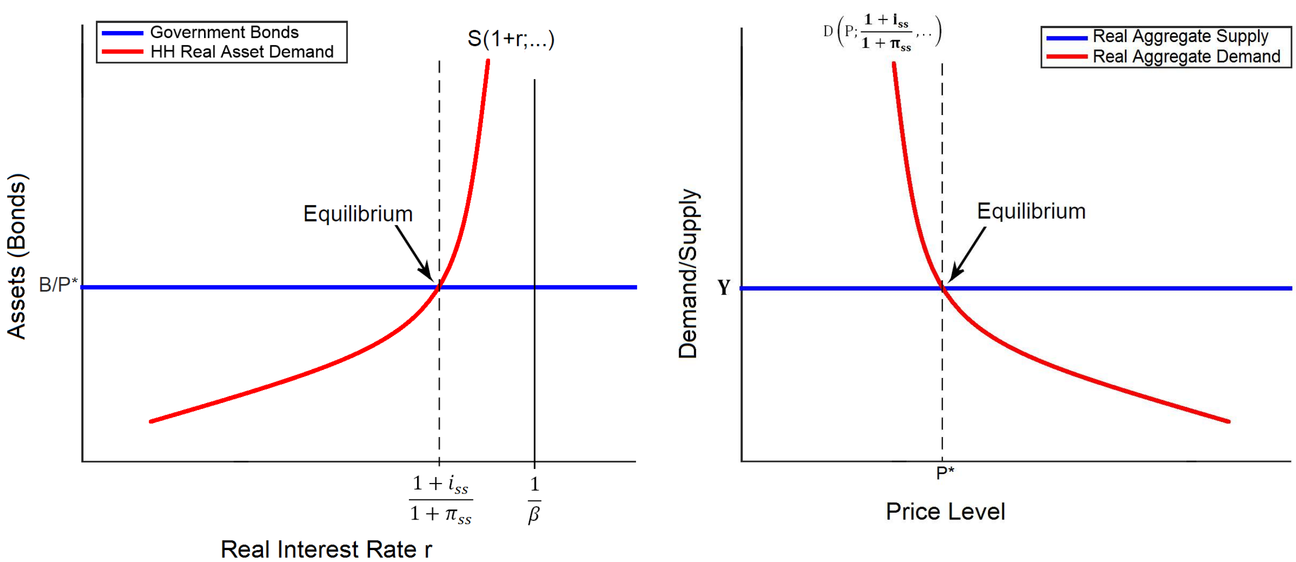
now two equations in two unknowns M and P^*, where B and π_{ss} are set by fiscal policy, i is set by monetary policy and K is such that the marginal product of capital is equal to the real interest rate. Here the assumption is that households exchange consumption goods for money. If one assumes instead that households obtain money through open market operations, then P^* and M solve

$$\frac{M}{P^*} = L(1 + i_{ss}, ..) \tag{A4}$$
$$K + \frac{B - M}{P^*} = S(\frac{1 + i_{ss}}{1 + \pi_{ss}}, ...), \tag{A5}$$

again two equations in two unknowns. Clearly, equation (A4) alone does not determine the price level since the central bank sets i and not M, which adjusts endogenously to satisfy the quantity equation. It is the asset market clearing condition that determines the price level, which depends on fiscal variables G, T and B *and* on i.

Price Level: Monetary and Fiscal Policy, Technology, Liquidity

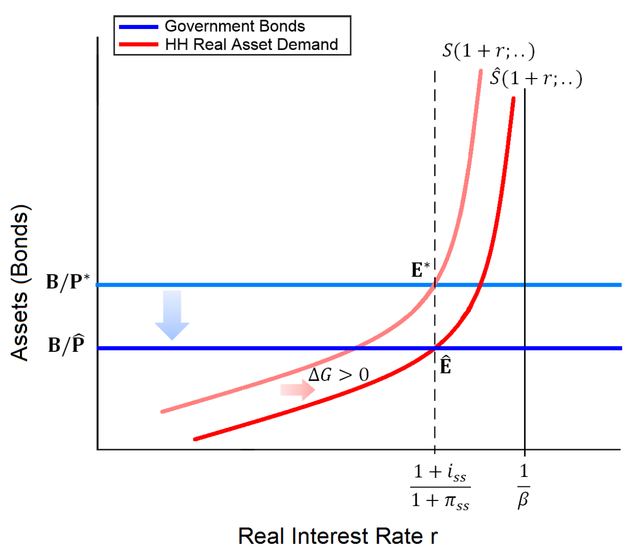
Finally to illustrate the mechanism of price level determination, I use the graphical analysis to show how the price level responds to monetary and fiscal policy as well as to changes in technology and in the need for liquidity. These responses are in line with conventional wisdom and with their precise characterization in Section A.II.1. For each of these four experiments, I will show diagrams both for the asset market and for the goods market to derive how prices move and compare these new steady states to the pre-experiment steady state in the asset market and in the goods market in Figure 6.



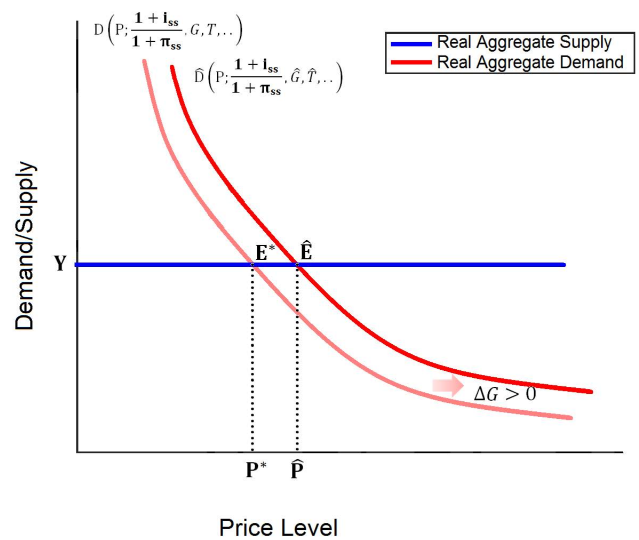
(a) Asset Market (b) Goods Market

Figure 6: Steady State : a) Asset Market b) Goods Market

i) Fiscal Policy (Figure 7) A tax-financed increase in government spending from G to $\hat{G}$ shifts the aggregate demand curve up and leads to a higher price level since private and public consumption are not one-for-one substitutes. For the same reason aggregate savings shift down, the economy moves from E^* to $\hat{E}$, and the price increases from P^* to $\hat{P}$.
ii) Monetary Policy (Figure 8) An increase in the nominal interest rate from i_{ss} to $\hat{i}$ increases asset demand and moves the economy up the savings curve from E^* to E', leading to a fall in the price level. The lower price level increases the real tax burden and thus shifts the savings curve down so that the new equilibrium is at $\hat{E}$ with a lower price $\hat{P} < P^*$ to restore equilibrium in the asset market. A higher nominal interest rate also contracts demand (shifts down the demand curve) such that the price level has to fall to ensure an equilibrium in the goods market.
iii) Liquidity (Figure 10) A higher liquidity demand (e.g. due to higher idiosyncratic uncertainty $\Delta\sigma > 0$) increases savings and depresses demand. The savings curve shifts up, the demand curve shifts down. The economy moves from E^* to $\hat{E}$ and the price decreases from P^* to $\hat{P}$.

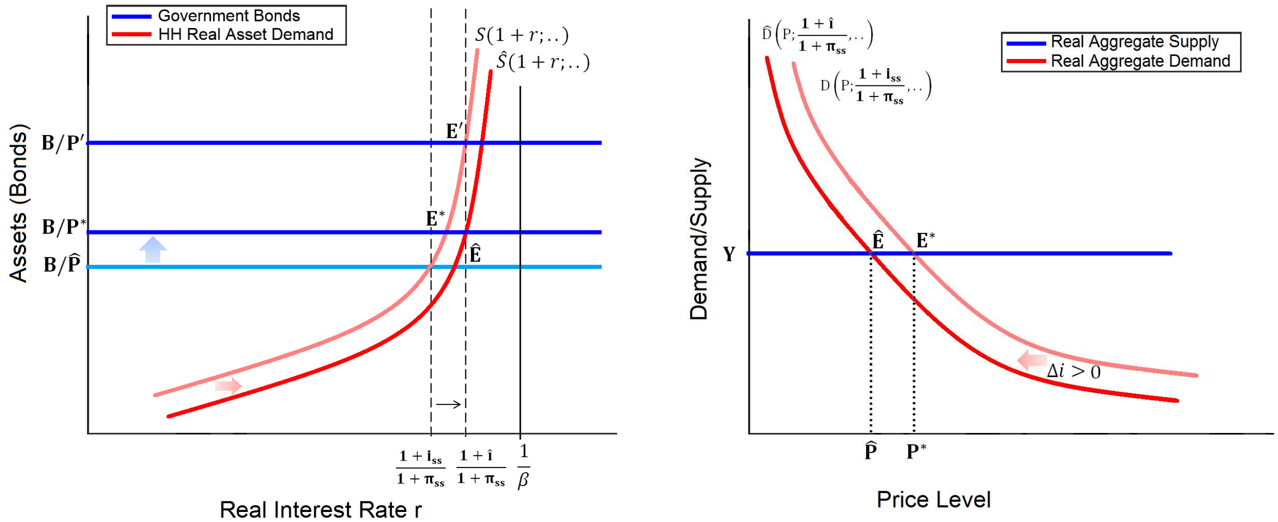


(a) Asset Market



(b) Goods Market

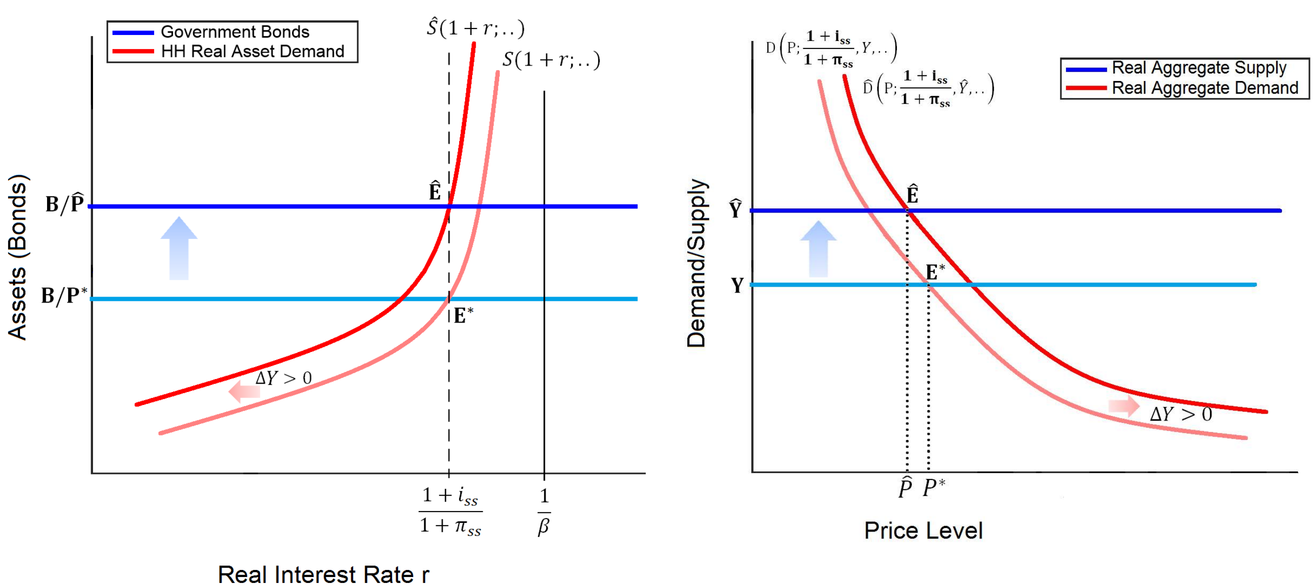
Figure 7: Steady State : Expansionary Fiscal Policy $\Delta G > 0$



(a) Asset Market (b) Goods Market

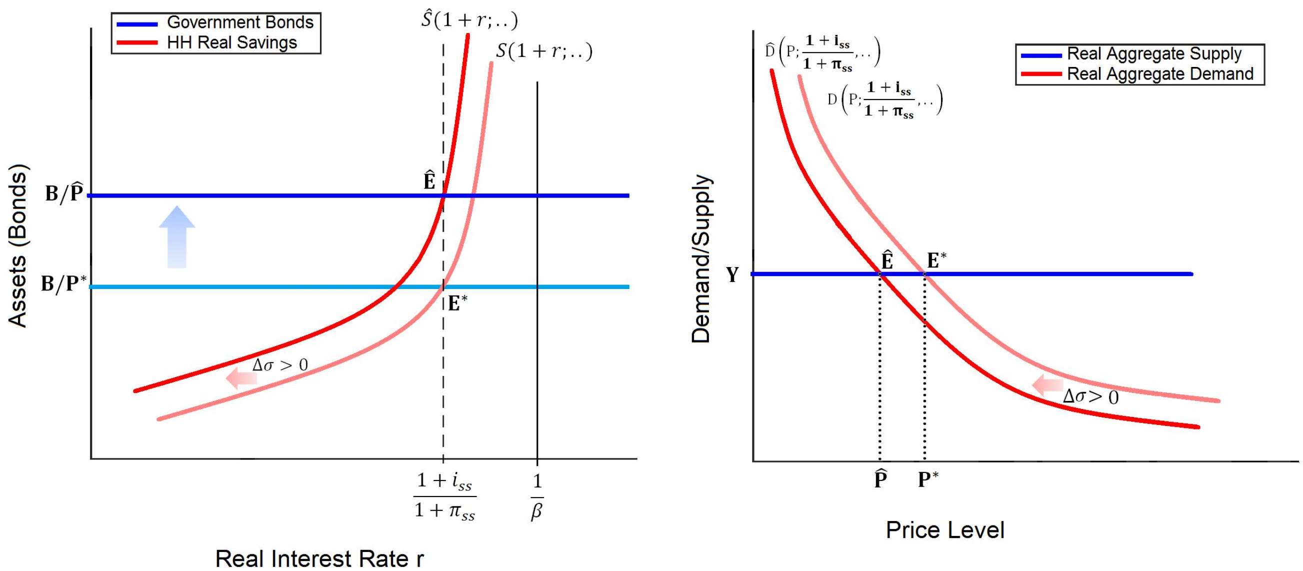
Figure 8: Steady State : Tighter Monetary Policy $\Delta i > 0$

iv) Technology (Figure 9) An increase in productivity leads to an increase in households' income, of which a fraction is spent on consumption and a fraction is saved. As a result, the asset demand curve shifts up and the price level drops. In the goods market, both the supply and the demand curve shift up, but the latter by less, since a fraction of the additional income is saved.



(a) Asset Market (b) Goods Market

Figure 9: Steady State : Productivity Increase $\Delta Y > 0$



(a) Asset Market (b) Goods Market

Figure 10: Steady State : Higher Liquidity Demand $\Delta\sigma > 0$

To summarize, I can obtain price level determinacy when fiscal policy is partially nominal and heterogeneity and incomplete markets lead to precautionary savings and a well-defined aggregate savings function. The determinacy result of the steady-state price level therefore holds in a large class of incomplete markets models. The response of the price level to monetary and fiscal policy and to technology and liquidity shocks is in line with conventional wisdom.

Global Determinacy

Showing determinacy of the steady-state price level although the key step is not equivalent to global determinacy though. Two more results need to be established. First, that prices are in steady state when monetary and fiscal policy are. This in particular requires to rule out inflationary and deflationary spirals. Second, determinacy has to be established when policy is not stationary. I now show these results in a simplified Huggett economy without government bonds, which allows for an explicit solution for the price level even outside steady-states an when monetary and fiscal policy operate according to some policy rules (Section 4.6). To show these results for the general case with positive liquidity, I developped a simple incomplete markets model in Section 3. There I have to use a simpler model since the dynamics of the Huggett economy with government bonds outside the steady state are not tractable at all, such that e.g. cycles etc. cannot be ruled out. But I provide some intuition for the Huggett economy with positive liquidity at the end of this Section

Price Level Determinacy in a simple Huggett Economy

I consider a Huggett economy where I make several assumptions not for realism but because these render the model tractable and even allow to solve explicitly for the price level outside steady states.

There are N real income levels $y_1 = 0 < y_2 < y_3 \ldots < y_N$ with associated probabilities $q_1, q_2, \ldots, q_N$. Shocks are i.i.d. No bonds are available and households except the y_N type cannot obtain any credit, that is there is complete illiquidity as in Werning (2015). As a result households neither save nor obtain credit in equilibrium and thus equilibrium consumption equals income for each household.[31] Fiscal policy is just redistributing here. A household with income level y has to pay nominal taxes $T(y)$, where $T < 0$ is a transfer. The period government budget constraint is

$$\sum_{i=1}^{N} q_i T(y_i) = 0. \tag{A6}$$

The government budget constraint is clearly satisfied for all price levels in all periods. Also any limiting behavior for government debt that one might want to impose is satisfied since government debt is equal to zero here. The FTPL is thus not operating here.

I only consider monotonically increasing tax schemes T and price levels P which satisfy an incentive-compatibility constraint

$$y_N - T(y_N)/P > y_{N-1} - T(y_{N-1})/P > \ldots > y_2 - T(y_2)/P > y_1 - T(y_1)/P > 0, \tag{A7}$$

which delivers tractability as it ensures that only the high income y_N household's consumption Euler equation is binding whereas the consumption Euler equation of all other agents is slack as they are credit constrained. Note that this does not solve the indeterminacy problem as it only rules out low prices but a continuum of prices is still feasible: whenever a tax scheme T and price P are incentive-compatible then any price $\tilde{P} > P$ is incentive-compatible as well. I also assume that the tax scheme is not regressive,

$$\frac{\partial \frac{T(y)}{y}}{\partial y} \geq 0, \tag{A8}$$

I make this assumption as it implies that aggregate demand for a fixed real interest rate is falling in the price level, consistent with the intuition provided in the paper before. A non-regressive nominal redistribution scheme guarantees this as a higher price then means lower real transfers from low MPC to high MPC households, which as a result decreases real aggregate demand.

[31] Note that while y_N households could obtain credit, there is no real interest rate at which y_N households are creditors and other households are savers, so that consumption is equal to income for each household.

The steady-state inflation rate is zero so that the constant nominal interest rate $1+i < 1/\beta$ and the real interest rate $1 + r_t = (1+i)\frac{P_t}{P_{t+1}}$, which is determined by the consumption Euler equation for the y_N households:

$$u'(y_N - T(y_N)/P_t) = \beta(1+i)\frac{P_t}{P_{t+1}} \sum_{i=1}^{N} q_i u'(y_i - T(y_i)/P_{t+1}), \tag{A9}$$

which for $u = \log$ is equivalent to

$$\frac{1}{y_N - T(y_N)/P_t} = \beta(1+i)\frac{P_t}{P_{t+1}} \sum_{i=1}^{N} q_i \frac{1}{y_i - T(y_i)/P_{t+1}}. \tag{A10}$$

Note that the FOC for the y_N household is binding as these households are not credit-constrained. The steady-state price level P^* solves

$$\underbrace{\frac{1}{y_N - T(y_N)/P^*}}_{=:H(P^*)} = \underbrace{\beta(1+i)\sum_{i=1}^{N} q_i \frac{1}{y_i - T(y_i)/P^*}}_{=:G(P^*)}. \tag{A11}$$

This is the additional equation in incomplete market models, the asset market clearing condition in this simple model, which determines the price level.
Existence of a price level P^* follows from the intermediate value theorem since

$$\lim_{P \searrow \frac{T(y_n)}{y_n}} H(P) - G(P) > 0, \tag{A12}$$

and

$$\lim_{P \to \infty} H(P) - G(P) < 0. \tag{A13}$$

For uniqueness note that for every steady-state price level P

$$-\frac{\partial H(P)P}{\partial P} = \frac{y_n}{(Py_n - T(y_n))^2} \tag{A14}$$

$$= \frac{y_n}{(Py_n - T(y_n))}\beta(1+i)\sum_{i=1}^{N} q_i \frac{1}{Py_i - T(y_i)} \tag{A15}$$

$$= \beta(1+i)\sum_{i=1}^{N} q_i \frac{y_n}{(Py_n - T(y_n))(Py_i - T(y_i))} \tag{A16}$$

$$> \beta(1+i)\sum_{i=1}^{N} q_i \frac{y_i}{(Py_i - T(y_i))(Py_i - T(y_i))} \tag{A17}$$

$$= -\frac{\partial G(P)P}{\partial P}, \tag{A18}$$

where the inequality follows from $\frac{T(y_n)}{y_n} > \frac{T(y_i)}{y_i}$ since

$$\frac{y_n(Py_i - T(y_i))}{y_i(Py_n - T(y_n))} = \frac{P - \frac{T(y_i)}{y_i}}{P - \frac{T(y_n)}{y_n}} > 1. \tag{A19}$$

This implies that $H(P) - G(P) = 0$ for only one P (since at every steady-state this curve "cuts zero from above" whereas a second steady state would require "cutting zero from below".)

Note that in complete market models equation (A11) reduces to, denoting steady-state consumption by c,

$$\frac{1}{c} = \beta \underbrace{(1+i)\frac{P_t}{P_{t+1}}}_{=1+r=1/\beta} \frac{1}{c} = \beta(1+i)\frac{1}{c} = \frac{1}{c}, \tag{A20}$$

which is independent of the price level and therefore cannot not be used to determine it.

This result eliminates all those equilibria that, as Cochrane (2015) shows, are feasible in New Keynesian complete market models since these equilibria correspond to a continuum of steady-state price levels on which agents can coordinate their long-run beliefs. This however does not yet rule out that there are other equilibria, where the price is not constant, that is not equal to its steady-state value. Note that this still needs to be shown although I maintain the assumption that fiscal policy is constant, that is both the transfer schedule T and the nominal interest rate i are constant. Of course, this does not establish determinacy for other cases when fiscal or monetary policy are responding to the price level but only for the case when policy is not responding. I allow for time-varying policies below and I discuss how policy rules affect determinacy in section 4.6.

To establish global uniqueness ($=P^*$ is the only equilibrium price level) when policy is constant, it therefore remains to be shown that hyper-inflations and hyper-deflations cannot be equilibria when fiscal policy is constant.[32]

Suppose that the price level $P_t > P^*$, then P_{t+1} solves

$$\underbrace{\frac{u'(y_N - T(y_N)/P_t)}{P_t}}_{=:\Phi(P_t)} = \underbrace{\frac{\beta(1+i)\sum_{i=1}^{N} q_i u'(y_i - T(y_i)/P_{t+1})}{P_{t+1}}}_{=:\Gamma(P_{t+1})}. \tag{A21}$$

Iterating using this first-order condition yields a sequence $P_{t+2}, P_{t+3}, \ldots$ as solutions to

$$\Phi(P_{t+1}) = \Gamma(P_{t+2}) \tag{A22}$$

$$\Phi(P_{t+2}) = \Gamma(P_{t+3}) \tag{A23}$$

$$\vdots \qquad \vdots$$

To show this I will use the uniqueness of a steady-state price level and $\frac{\partial(\Phi(P)-\Gamma(P))}{\partial P}|_{P=P^*} < 0$ implies that $\Phi(P) - \Gamma(P) < 0$ for $P > P^*$ and $\Phi(P) - \Gamma(P) > 0$ for $P < P^*$. Since $\Phi(P_t) < \Gamma(P_t)$ and $\frac{\partial \Gamma(P)}{\partial P} < 0$, it follows that P is a monotonically increasing sequence, $P_{t+1} > P_t$. Uniqueness of a steady-state implies that this sequence converges to infinity. This is not an equilibrium since

$$\lim_{P\to\infty} \Gamma(P) = \beta(1+i)\frac{q_1}{-T(y_1)} > 0 = \lim_{P\to\infty} \Phi(P). \tag{A24}$$

As an alternative proof, the literature (Obstfeld and Rogoff (1983, 2017)) has provided a very elegant solution to rule out hyper-inflations in monetary models which applies here as well. If the government provides some fractional real backing of debt/transfers, that is the government trades bonds/transfers for real consumption at a (very high) price $\bar{P}$ then the same arbitrage argument as in the monetary literature eliminate this "perverse" hyper-inflation equilibrium. Prices cannot rise above $\bar{P}$ ruling out this type of hyperinflation. I do not follow this route here since one might (incorrectly) think that this is some form of the FTPL.

What a proof has to rule out is basically only that the price level is infinite, $P = \infty$. The previous two line of arguments work if prices are flexible. Another alternative emerges in models with some price rigidities as under this (realistic) assumption it is straightforward to show that $P = \infty$ is not possible. In models with sticky prices firms set prices and jumping to $P = \infty$ would incur infinite Rotemberg (1982) price adjustment costs which is clearly not an equilibrium. With Calvo price setting the fraction of non-adjusting firms would charge a finite price and would absorb all demand whereas the $P = \infty$ firms would face no demand, implying that setting $P = \infty$ is not optimal and that the output weighted price level would be finite.

[32]It is important to point out that this does not eliminate hyperinflations induced by policies, for example if nominal government obligations explode, but only when policy is stationary/constant.

Now move to rule out hyper-deflations. Therefore similarly suppose that the price level $P_t < P^*$. Iterating using the first-order condition (A21) yields a sequence $P_{t+1}, P_{t+2}, P_{t+3}, \ldots$ as solutions to

$$\Phi(P_t) = \Gamma(P_{t+1}) \tag{A25}$$

$$\Phi(P_{t+1}) = \Gamma(P_{t+2}) \tag{A26}$$

$$\Phi(P_{t+2}) = \Gamma(P_{t+3}) \tag{A27}$$

$$\vdots \qquad \vdots$$

The argument is simple. Suppose that $P_t < P^*$ then P_{t+1} solves $\Phi(P_t) = \Gamma(P_{t+1})$. Since $\Phi(P_t) > \Gamma(P_t)$ and $\frac{\partial \Gamma(P)}{\partial P} < 0$, it follows that P is now a monotonically decreasing sequence, $P_{t+1} < P_t$. Uniqueness of a steady-state implies that this sequence converges to zero. This is not an equilibrium since prices lower than $T(y_n)/y_n$ are not feasible. The argument is basically that real fiscal variables - bonds, transfers, taxes and government spending - become infinite if prices converge to zero, which is not an equilibrium. An alternative to my proof is to use a standard transversality argument to rule out hyper-deflations when policy is constant. The logic is again simple. If $B > 0$ and $P \to 0$ the real value of bonds converges to infinity which requires that the real interest rate converges to $1/\beta$, violating households transversality condition in incomplete market models (consumption is then a martingale and assets and consumption converge to infinity in expectation).

I can conclude that P^* is the globally uniquely determined price level.

A closed-form solution

For $N = 2$ an explicit solution exists. The steady-state price level P^* solves (remember that $y_1 = 0$)

$$\frac{1}{y_N - T/P^*} = \beta(1+i)[(1-q)\frac{1}{T/P^*} + q\frac{1}{y_N - T/P^*}], \tag{A28}$$

where I set $q = q_N$ and $T = T(y_N)$. The incentive-compatibility constraint is now $P > T/z_N$. Solving for P^* yields

$$P^* = \frac{T}{y_N}\frac{1 - \beta(1+i)(1-2q)}{\beta(1+i)(1-q)}. \tag{A29}$$

This example also allows to compute the unique current price level outside of steady states. The price level satisfies a first-order difference equation

$$\frac{1}{P_t y_N - T_t} = \beta(1+i_{t+1})[(1-q)\frac{1}{T_{t+1}} + q\frac{1}{P_{t+1}y_N - T_{t+1}}], \tag{A30}$$

which can be rewritten, defining $v_t := \frac{1}{P_t y_N - T_t}$ and $\alpha_t := \frac{\beta(1+i_{t+1})(1-q)}{T_{t+1}}$, as

$$v_t = \alpha_t + \beta(1 + i_{t+1})qv_{t+1}. \tag{A31}$$

Solving forward yields[33]

$$v_t^* = \sum_{s=t}^{\infty} \alpha_s (q\beta)^{s-t} \prod_{k=t+1}^{s} (1 + i_k), \tag{A32}$$

and thus

$$P_t^* = (\frac{1}{v_t^*} + T_t)/y_N, \tag{A33}$$

which simplifies to the steady-state price level $P^* = \frac{T}{y_N} \frac{1-\beta(1+i)(1-2q)}{\beta(1+i)(1-q)}$ if policy is constant.

It is also straightforward to add an interest rate rule

$$(1 + i_{t+1}) = (1 + \bar{i}_{t+1})\big(\frac{P_t}{\bar{P}}\big)^{\rho_p}\big(\frac{z_t}{\bar{z}}\big)^{\rho_z} \tag{A34}$$

and a transfer rule

$$T_t = \bar{T}_t\big(\frac{P_t}{\bar{P}}\big)^{\phi_p}\big(\frac{z_t}{\bar{z}}\big)^{\phi_z}, \tag{A35}$$

which yields the same difference equation for P_t and v_t and the the same solution as before. However α_t and i_{t+1} depend on the sequence of prices P, so that a closed form solution is not available and fixpoint arguments are needed for existence. This and the dynamic properties induced by these rules are discussed in Section 4.

To better understand how the price level is determined through iteration outside a steady state, suppose that the economy is in steady state in period $S + 1$, so that $P_{S+1} = P^*$. If $T_S = T$ and $i_S = i$ then the price in period S is in steady state as well, $P_S = P^*$. If either $T_S \neq T$ or $i_S \neq i$ then $P_S \neq P^*$ and P_S is obtained from the FOC (A30). Iterating this way, where in each step the FOC is used to obtain the price level in period k knowing the price in period $k + 1$, one obtains the full sequence of price levels, which after some simple algebra yields expression (A33). The same iteration applies in the presence of policy rules.

Some intuition for outside steady state dynamics can also be gained for the general Huggett economy with positive government debt whereas a full treatment is only possible in the simple incomplete markets model in Section 3. An important part is to rule out exploding price paths

[33]The sum is finite if on average $(1+i)q\beta < (1-\epsilon)$ for some $\epsilon > 0$. This puts an upper bound on nominal interest rates since $(1 + i)q\beta > 1$ eliminates any equilibrium. Note that $v_t^* = \infty$ is not an equilibrium since the associated price level, $P = T/y_N$, does not satisfy the incentive-compatibility constraint.

although nominal bonds B, nominal government spending G and nominal taxes T are constant. It is easy to rule out deflationary paths where prices converge to zero as this implies that real fiscal variables converge to infinity, violating resource and budget constraints. Prices converging to infinity can be ruled out if there is always a positive demand for precautionary savings, as I will illustrate using an extreme but therefore clear example.

It is easy to provide a sufficient condition that precautionary savings never disappear in a Huggett economy, e.g. by assuming a state s^{death} without any income, a binding credit constraint such that all consumption needs to be financed through savings and that the utility of zero consumption is minus infinity (satisfied by the standard assumption of log utility and CRRA with risk aversion large than one). Consider now a household who faces a probability $1 > \pi^{death} > 0$ of being in state s^{death} next period. The consumption Euler equation is

$$\frac{u'(c_t)}{P_t} = \beta(1+i)\Big\{\pi^{death}\frac{u'(\frac{b_{t+1}}{P_{t+1}})}{P_{t+1}} + (1-\pi^{death})E[\frac{u'(c_{t+1})}{P_{t+1}} \mid s_{t+1} \neq s^{death}]\Big\} \tag{A36}$$

for period t and $t+1$ prices P_t and P_{t+1}, a utility function u, today's consumption c_t, tomorrow's state-dependent consumption c_{t+1} and nominal bonds b_{t+1} acquired in period t. The standard assumption of

$$u(c) = log(c) \tag{A37}$$

or of CRRA with risk aversion large than one now implies that $b_{t+1} > 0$ since otherwise consumption is zero and utility is minus infinity and that

$$\lim_{P_{t+1}\to\infty} \frac{u'(\frac{b_{t+1}}{P_{t+1}})}{P_{t+1}} > 0. \tag{A38}$$

Applying this to the Euler equation (A36) shows that it does not hold when $P \to \infty$:

$$0 = \lim_{P_t\to\infty}\frac{u'(c_t)}{P_t} < \lim_{P_{t+1}\to\infty}\beta(1+i)\Big\{\pi^{death}\frac{u'(\frac{b_{t+1}}{P_{t+1}})}{P_{t+1}} + (1-\pi^{death})E[\frac{u'(c_{t+1})}{P_{t+1}} \mid s_{t+1} \neq s^{death}]\Big\} > 0,$$

implying a positive precautionary demand for savings and a non-clearing asset market.[34]

The important economic assumption for global determinacy is that the demand for precau-

[34] Note that here the household chooses $b_{t+1} > 0$ to avoid zero consumption in state s^{death} so that the Euler equation is binding. That is not essential for the argument as the limiting argument also shows that a non-binding credit constraint,

$$\frac{u'(c_t)}{P_t} > \beta(1+i)\Big\{\pi^{death}\frac{u'(\frac{b_{t+1}}{P_{t+1}})}{P_{t+1}} + (1-\pi^{death})E[\frac{u'(c_{t+1})}{P_{t+1}} \mid s_{t+1} \neq s^{death}]\Big\} \tag{A39}$$

is violated as the LHS < RHS when prices converge to infinity.

tionary savings does not disappear outside the steady state. The assumptions made above is just one example to ensure this. I make a similar assumption in terms of utility in Section 3 and show that it also implies a non-vanishing precautionary demand for savings. This assumption, combined with tractable global dynamics, allows me to prove determinacy.

A.II Appendix Section 4

A.II.1 Impulse Responses

Theorem 1 allows a consideration of the economy's impulse response to monetary and fiscal shocks. While a full analysis will be conducted in a model with sticky prices in Section A.III, it is still instructive to compute the impulse responses in the flexible price version of the model.

A theoretical reason why this usually is not done, is that the price level is not determined in both standard sticky and flexible price models when monetary policy is described as setting a sequence of exogenous nominal interest rate. As this Section has established, this theoretical obstacle is overcome, allowing to conduct such policy experiments.

I consider two policy experiments, a persistent increase in the nominal interest rate and an increase in nominal government spending. I also consider the response of prices to a discount factor and a technology shock. Figures 11, 12 and 13 in Section A.III show not only the impulse responses in the sticky price model but also, as a benchmark, for the flexible price economy. A remarkable feature of these flexible price impulse responses is that prices adjust sluggishly and that monetary policy affects prices without the assumption of prices being sticky.

A.II.1.1 Monetary Policy

For the monetary policy experiment, I characterize the response of prices to an initial unexpected increase in interest rates $R_0 > R$, which then dies out over time and is back at the steady-state level R from time S onwards, $R_t = R$ for $t \geq S$, that is the interest rate sequence equals

$$R_0 \geq R_1 \geq \ldots \geq R_{S-1} \geq R_S = R, R, \ldots. \tag{A40}$$

The resulting price sequence can be precisely characterized:

Proposition 7. *The detrended price sequence in response to a monetary policy as in (A40) is*

$$\tilde{P}_0 \leq \tilde{P}_1 \leq \ldots \leq \tilde{P}_{S-1} \leq \tilde{P}_S = P^*, \tilde{P}_{S+1} = P^*, \ldots, \tag{A41}$$

where the inequality between prices at t and $t+1$ is strict whenever it is strict for nominal interest *rates,* $R_t > R_{t+1}$*. For the non-detrended prices*

$$\frac{P_{t+1}}{P_t} \geq (1+\gamma) \tag{A42}$$

with strict inequality if $R_t > R_{t+1}$*.*

A more persistent or larger impulse to the nominal interest rate leads to uniformly lower prices.

Proposition 8. *Consider two interest rate sequences*

$$R_0^a \geq R_1^a \geq \ldots \geq R_{S-1}^a \geq R_S^a = R, R, \ldots, \tag{A43}$$

$$R_0^b \geq R_1^b \geq \ldots \geq R_{S-1}^b \geq R_S^a = R, R, \ldots, \tag{A44}$$

with $R_t^a \geq R_t^b$*. Then the prices* $\tilde{P}^a$ *for policy* R^a *are uniformly lower than the prices* $\tilde{P}^b$ *for policy* R^b*,*

$$\tilde{P}_t^a \leq \tilde{P}_t^b. \tag{A45}$$

A.II.1.2 Fiscal Policy

For the fiscal policy experiment, I characterize the response of prices to an initial unexpected increase in nominal government spending for S periods by x percent financed by an increase in taxation, so that

$$\begin{aligned} \hat{G}_t = & \quad (1+x)G_t \quad \text{for } 0 \leq t < S \\ \hat{G}_t = & \quad\quad G_t \quad\quad \text{for } t \geq S. \end{aligned} \tag{A46}$$

The response of prices can be precisely characterized:

Proposition 9. *The detrended price sequence in response to a fiscal policy as in (A46) is*

$$\tilde{P}_0 \geq \tilde{P}_1 \geq \ldots \geq \tilde{P}_{S-1} \geq \tilde{P}_S = P^*, \tilde{P}_{S+1} = P^*, \ldots, \tag{A47}$$

and for the non-detrended prices

$$\frac{P_{t+1}}{P_t} \leq (1+\gamma). \tag{A48}$$

A more expansive or persistent fiscal policy $G_t^a \geq G_t^b \geq G_t$ *leads to stronger price increases*

$$\tilde{P}_t^a \geq \tilde{P}_t^b. \tag{A49}$$

A.II.1.3 Discount Factor Shock

I consider a discount factor shock which increases β for S periods:

$$\begin{aligned} \hat{\beta} > & \quad \beta \quad \text{for } 0 \leq t < S \\ \hat{\beta} = & \quad \beta \quad \text{for } t \geq S. \end{aligned} \tag{A50}$$

The next proposition shows that prices persistently fall.

Proposition 10. *The detrended price sequence in response to a discount factor shock as in (A50) is*

$$\tilde{P}_0 \leq \tilde{P}_1 \leq \ldots \leq \tilde{P}_{S-1} \leq \tilde{P}_S = P^*, \tilde{P}_{S+1} = P^*, \ldots, \tag{A51}$$

A larger discount factor shock, $\hat{\beta}^a > \hat{\beta}^b$ leads to stronger price decreases

$$\tilde{P}_t^a < \tilde{P}_t^b. \tag{A52}$$

With one important caveat, discount factor shocks can be neutralized fully by monetary policy through keeping $R\beta$ constant, that is decreasing R by the size of the shock $\hat{\beta}/\beta$. The caveat is that the zero lower bound prevents large cuts in the nominal interest rates in response to large increases in β.

A.II.1.4 Productivity Shock

Finally I consider a persistent shock to the productivity A:

$$A_0 \geq A_1 \geq \ldots \geq A_{S-1} \geq A_S = A, A, \ldots. \tag{A53}$$

The response of prices to this productivity innovation can be characterized precisely as a persistent drop in prices:

Proposition 11. *The detrended price sequence in response to a technology shock as in (A53) is*

$$\tilde{P}_0 \leq \tilde{P}_1 \leq \ldots \leq \tilde{P}_{S-1} \leq \tilde{P}_S = P^*, \tilde{P}_{S+1} = P^*, \ldots. \tag{A54}$$

A.III Monetary and Fiscal Policy with Sticky Prices

This section adds two features to the basic model: sticky prices and elastic labor supply. These two features allow me to illustrate the workings of the model and to compute the response of the price level and employment to a tightening of monetary policy and to a fiscal demand stimulus.

A.III.1 A Sticky Price Model

I now assume that firms are constrained in their price setting to see whether the model can (qualitatively) produce standard impulse responses. I follow Mankiw and Reis (2002) and assume that information is sticky, and I follow the literature in modeling a competitive final good and a monopolistically competitive intermediate sector.

Labor Supply Households provide h_t hours in a perfectly competitive labor market at a fully flexible real wage w_t, which they receive at t_1. The disutility from working h hours is described through GHH preferences:

$$\log\left(C_t^h - \kappa h_t^{1+\phi}\right) + \theta\eta \log\left(c_t^h\right), \tag{A55}$$

which implies that an increase in government spending has no negative wealth effect leading to an increase in labor supply. The benefit of this choice of preferences is that an effect of government spending on labor supply is due to demand effects as in the previous sections, and is not confounded with well known wealth effects.

Final Goods

The perfectly competitive, representative, final good producing firm combines a continuum of intermediate goods $Y_t(j)$ indexed by $j \in [0,1]$ using the technology

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{1}{1+\lambda}} dj\right)^{1+\lambda}. \tag{A56}$$

Here $\lambda > 0$ and $\frac{1+\lambda}{\lambda}$ represents the elasticity of demand for each intermediate good. The final goods firm takes intermediate good prices $P_t(j)$ of $Y_t(j)$ and output prices P_t of the final output Y_t as given. Profit maximization of intermediate firms implies that the demand for intermediate goods is

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\frac{1+\lambda}{\lambda}} Y_t. \tag{A57}$$

The relationship between intermediate goods prices and the price of the final good is

$$P_t = \left(\int_0^1 P_t(j)^{-\frac{1}{\lambda}} dj\right)^{-\lambda}. \tag{A58}$$

Intermediate Goods.

Intermediate good j is produced by a monopolist who has access to a linear production technology which uses labor $H(j)$ as the only input:

$$Y_t(j) = AH_t(j). \tag{A59}$$

Labor market clearing requires

$$h_t = \int_0^1 H_t(j)dj. \tag{A60}$$

Price Setting As in Mankiw and Reis (2002) prices adjust slowly since information disseminates slowly and processing information takes time. In each period, a fraction ω of firms obtains full information, so that their subjective expectation of the contemporaneous values of aggregate variables coincides with the actual values of these variables. The remaining firms base their expectations and decisions on outdated information.

Taking as given nominal wages, final good prices, the demand schedule for intermediate products and technological constraints, firm j chooses its labor inputs $H_t(j)$ and the price $P_t(j)$ to maximize profits

$$\mathbb{E}_t^j \left(\frac{P_{t+s}(j)}{P_{t+s}} Y_{t+s}(j) - \frac{1}{P_{t+s}} W_{t+s} H_{t+s}(j) \right), \tag{A61}$$

where the superscript j in the expectation operator $\mathbb{E}_t^j$ indicates that this expectation is formed using firm $j's$ information.

Combining the production technology and the demand schedule implies

$$H_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\frac{1+\lambda}{\lambda}} \frac{Y_t}{A_t}$$

Thus, the firm maximizes the following objective function with respect to $P_t(j)$:

$$\mathbb{E}_t^j \left(\frac{P_t(j)}{P_t} \left(\frac{P_t(j)}{P_t} \right)^{-\frac{1+\lambda}{\lambda}} Y_t - \frac{W_t}{P_t} \left(\frac{P_t(j)}{P_t} \right)^{-\frac{1+\lambda}{\lambda}} \frac{Y_t}{A_t} \right), \tag{A62}$$

resulting in a price setting rule

$$\mathbb{E}_t^j \left(\frac{P_t(j)}{P_t} = \frac{W_t}{P_t A_t} (1 + \lambda) \right). \tag{A63}$$

Each firm uses its information set to set the price as a markup $1 + \lambda$ over real marginal costs. As information is dispersed across firms and only a fraction ω of firms obtain updated information each period, prices are dispersed as well.

A.III.2 Numerical Example

Next, I provide some calibrated examples to illustrate the workings of the model, through computing the same impulse responses as characterized analytically in section A.II.1. Yet here, prices are sticky and labor supply is elastic. I also now allow households to obtain credit up to some limit $\bar{b}$ to pay for their period t_2 consumption, such that

$$c_t^h(\theta) \leq \bar{b} + b_t^h/P_t. \tag{A64}$$

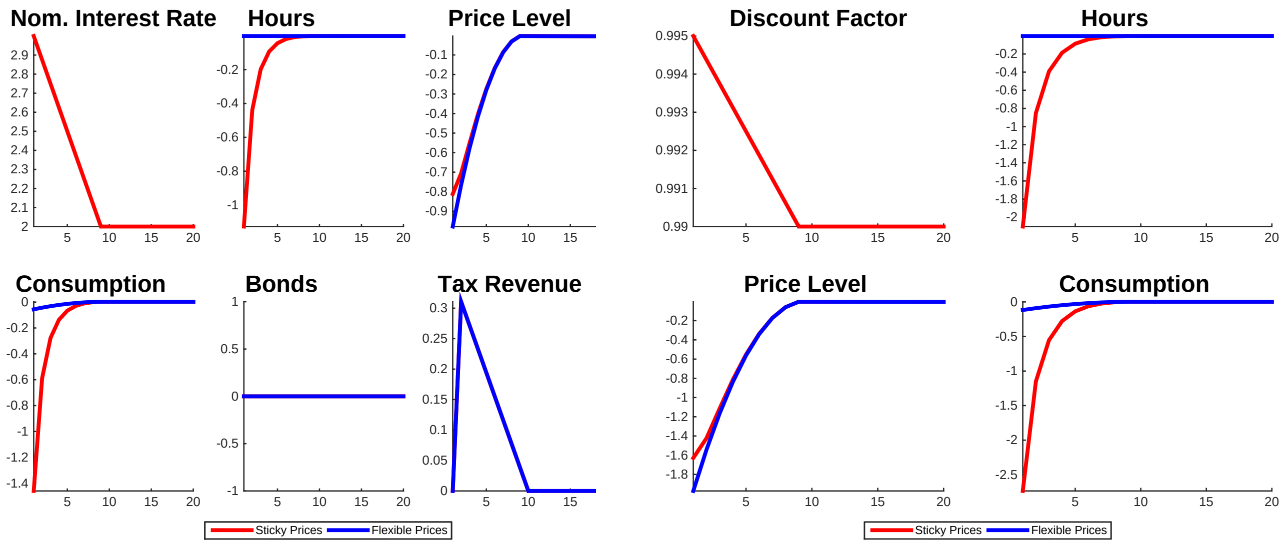
The results should not be considered precise estimates due to the simplifying assumptions of the model, mainly when modeling the consumption and saving decisions.[35]

These numerical illustrations necessitate a choice of parametric forms and parameter values. I choose the following: A model period is a quarter and therefore $\beta = 0.99$. I normalize steady state $A = 1$. The steady-state nominal interest rate is 2% (annualized) and the steady state inflation rate is zero. Following standard choices in the literature I assume $\lambda = 0.2$, $\omega = 0.25$ and $\phi = 2$. I allow households to use a credit line of 10% of their income to pay for emergency consumption, $\bar{b} = 0.1$. The distribution of the preference shocks θ is assumed to be lognormal with parameters $\mu = 0$ and $\sigma = 1$ (the mean and standard deviation of $\log(\theta)$). Kaplan and Violante (2014) document that the median household holds only a small amount of liquid assets, motivating a choice of $B = 0.25$, equal to households' liquid assets to income ratio of 25%. These choices imply that only 1% of households are constrained in emergency expenditures, a fraction much lower than the 17.5% and 35% hand to mouth consumers found in Kaplan and Violante (2014). The model therefore does not overstate the importance of credit constraints. Government expenditures equal $G = 0.2$. Finally I pick the two preference scale parameters $\kappa = 0.833$ and $\eta = 0.079$ to obtain $h = 1$ and $P = 1$ in steady state.

I now use this calibrated model to compute model impulse responses to a monetary policy shock (an increase of R by one (annualized) percentage point), a fiscal policy shock (an increase in nominal government spending G by 1%), a technology shock (an increase of A by 1%) and a discount factor shock (an increase of β from 0.99 to 0.995). All shocks are unexpected. In all experiments except for the fiscal policy one, I assume that 80% of government spending, 0.16, is fixed in real terms to that nominal spending equals $0.16P + 0.04$. Real spending thus equals $0.16 + 0.04/P$, which increases if the price level P falls. To the extent that government spending serves as a nominal anchor, real spending automatically increases in response to price-decreasing shocks. I explore this automatic stabilization feature in more detail in the next section.

The monetary policy shock has the expected negative effects on prices and employment (panel

[35] To obtain analytical tractability I assumed large families with the consequence that the model is not even close to more realistic models of consumption behavior, as for example in Kaplan and Violante (2014). I do so since this assumption has no relevance for the objective of this paper.



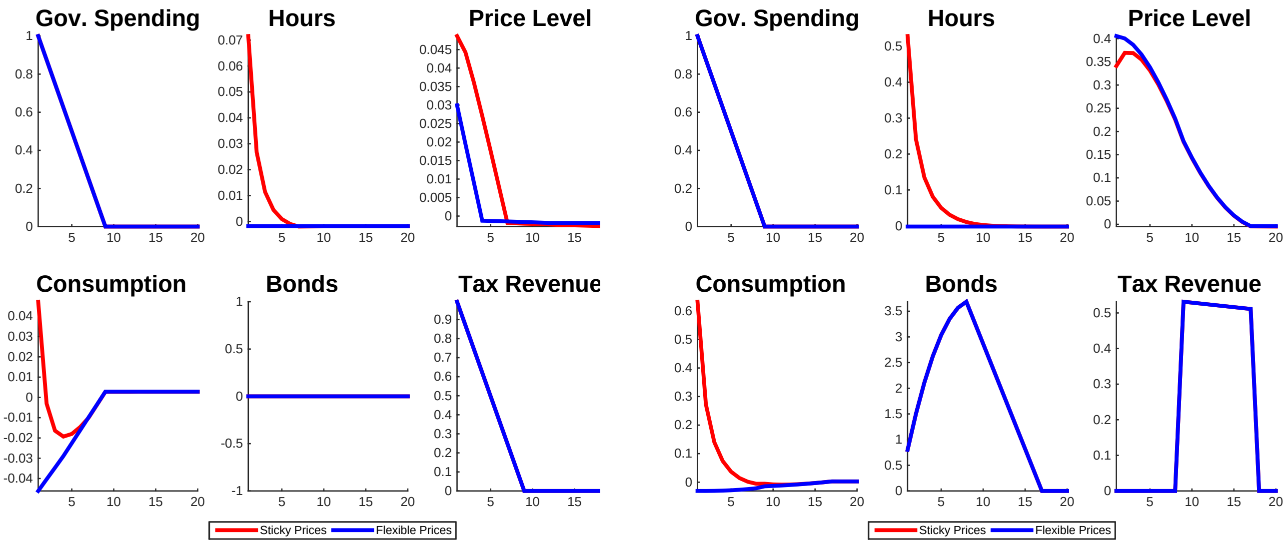
(a) Monetary Policy (b) Discount Factor

Figure 11: Impulse Responses for a) Monetary Policy and b) Discount Factor of hours, consumption C at t_1 and prices for both the sticky and flexible price economies.

a) of Figure 11) since higher nominal interest rates increase the incentives to save and thus decrease consumption. Interestingly, the aggregate price level response is very close with flexible and sticky prices, whereas hours do not move if prices are flexible but drop quite a bit if prices are sticky. This is because with sticky prices many firms charge too high a markup and as a result hire too little. The similar logic applies to the discount factor shock where again prices and hours fall.

The increase in government spending stimulates aggregate demand and therefore leads to an increase in prices, both with flexible and sticky prices. Hours increase only in the latter case, since firms charge too low a markup and as a result hire more to satisfy the additional demand. With flexible prices firms always charge the right markup and as a result employment does not respond. In panel a) of Figure 12 the increase in spending is financed with an increase in taxes only, which by itself contracts private demand. However, the tax increase lowers private demand by less than the increase in government demand. Total demand still increases, but the positive effects on prices and hours are muted. In panel b) of Figure 12 the increase in spending is financed by issuing government debt which is paid back only later through higher taxes. The initial deficit financing avoids the contractionary demand effect of the tax financing as shown in panel a). As a result the response of hours and prices is, larger, almost by an order of magnitude, with deficit spending than with tax financing.

The technology policy shock has negative effects on prices and employment (panel a) of figure 13) operating through two channels. First, the increase in technology raises aggregate supply and the price level has to fall to stimulate aggregate demand. Second, a non-permanent increase in technology leads to higher savings to smooth consumption between initial periods when technology



(a) Fiscal Policy (Tax financed)

(b) Fiscal Policy (Debt financed)

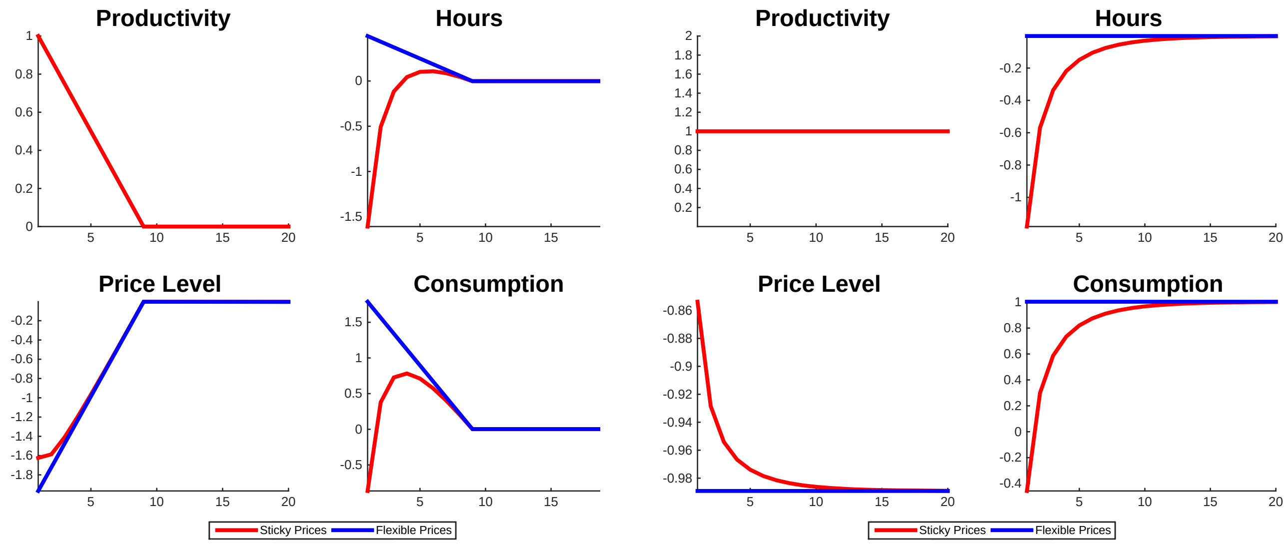
Figure 12: Fiscal Policy: Impulse Responses of hours, consumption C at t_1 and prices for both the sticky and flexible price economies.

is high and later periods when the shock is dying out. The savings channel is quite strong such that even though prices are pretty sticky the aggregate price level response when prices are sticky is very close to the flexible prices case. Hours increase instead if prices are flexible, but drop quite a bit if prices are sticky. This is again because with sticky prices many firms charge too high a markup and then hire too little, whereas for firms with flexible prices the increase in productivity leads to more employment. When the technology shock is permanent, the savings channel is eliminated and the impulse responses change accordingly as shown in panel b) of figure 13. In this experiment I make balanced growth assumptions[36] such that with flexible prices the price level drops permanently by one percent and aggregate nominal supply is unchanged. With sticky prices the absence of consumption-smoothing incentives makes prices and hours move less than when the technology shock is temporary. Prices would not adjust at all if fiscal policy keeps the spending output ratio, G/Y, and the debt GDP ratio, B/Y, constant in response to an increase in Y as this equates nominal demand and supply.

A.III.3 Nominal Fiscal Anchor as Automatic Stabilizer

As explained above, when computing the impulse responses I assumed that 80% of government spending is real and only 20% is nominal, so that changes in the price level move only 20% of real government spending. The larger the fraction of fixed nominal spending is, the larger is the change in real spending as an automatic response to changes in prices. The larger the nominal

[36] The utility function is $\log\left(C_t^h - A_t \kappa h_t^{1+\phi}\right) + \theta\eta \log(c_t^h)$, credit constraints are $A_t\bar{b}$ and government policy is fully nominal.



(a) Technology (b) Technology (Permanent)

Figure 13: Technology Shock: Impulse Responses of hours, consumption C at t_1 and prices for both the sticky and flexible price economies.

anchor, the larger then the automatic demand response. I established above that a fiscal policy stimulus increases prices and employment. Not surprisingly, the size of this automatic demand response matters for the reaction of the economy.

I now consider the same discount factor shock as above, and again compute the impulse response of consumption, prices and hours but with one change. Government spending is now fully nominal, so that real spending as a function of the price level equals G/P. Since the discount factor induces a fall in prices, G/P increases automatically and significantly more than in the benchmark where only 20% of spending is nominal. This additional demand increase partly offsets the discount-factor shock induced drop in demand, and therefore also alleviates the negative consequences on hours worked. The result is shown in Figure 14.

A.III.4 Liquidity Trap: Disappearance of Puzzles

Cochrane (2015) describes several puzzles in New Keynesian models during a liquidity trap: the forward guidance puzzle, technological regress is expansionary (output increases) and the fiscal multiplier becomes larger if prices are less sticky. Cochrane (2015) then shows that these results are artifacts of equilibrium selection and that this selection is necessary because of price level indeterminacy. In incomplete market models, such as the one in this paper, the price level is determinate so that a selection is not necessary and I now show that as a result of this these puzzles disappear. The forward guidance puzzle disappears as commitment to future monetary policy has only negligible effects here, technological regress decreases output and the size of the fiscal multiplier becomes smaller if prices are less sticky.

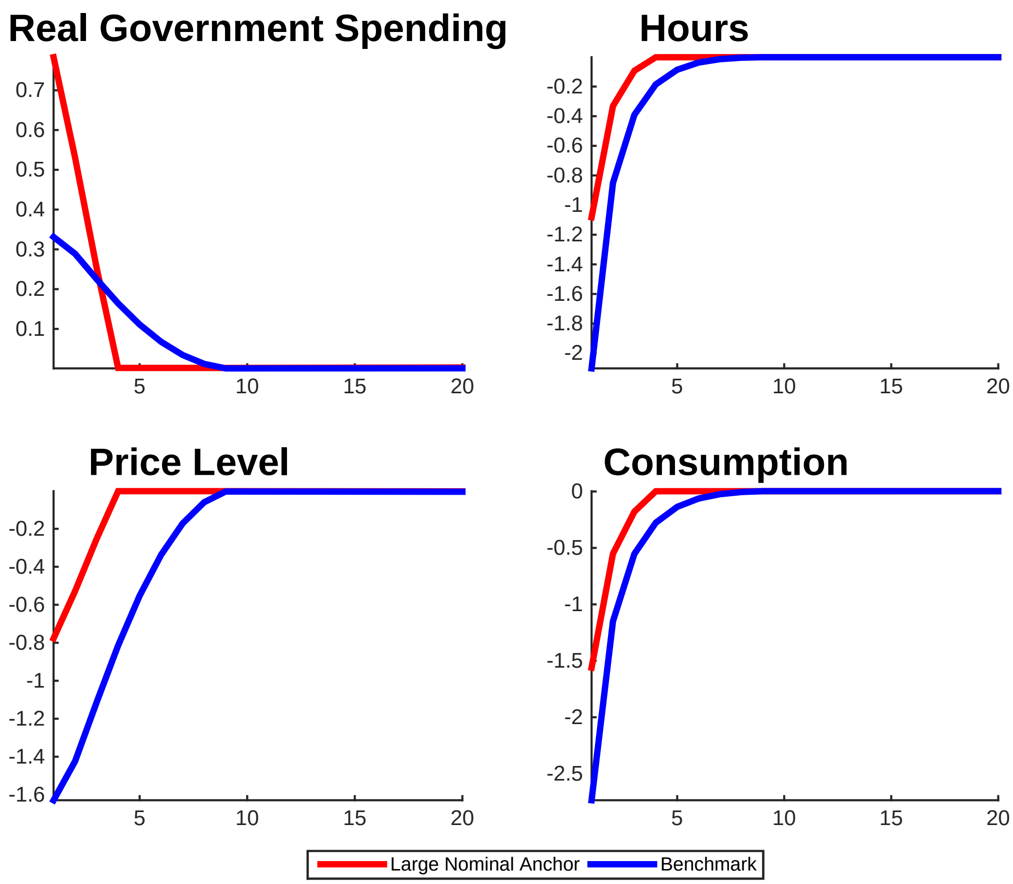


Figure 14: Response to Discount Factor Shock: a) Benchmark (Partial Nominal Anchor) and b) Full Nominal Anchor

A.III.4.1 Liquidity Trap: Fiscal Multiplier and Degree of Price Rigidity

In New Keynesian models, as explained in Cochrane (2015), fiscal multipliers increase as price stickiness is reduced for the standard equilibrium selection - zero inflation at the time of escape from the liquidity trap. In contrast Figure 15 shows that the output gets smaller and the fiscal multiplier decreases if price stickiness is reduced in the incomplete market model.

A.III.4.2 Liquidity Trap: Output Effects of Technological Progress

The standard equilibrium selection in New Keynesian models also implies that technological regress is expansionary. Again in contrast, Figure 16 shows that this is not the case in the incomplete markets model. Technological progress is expansionary and technological regress is contractionary.

A.III.4.3 Liquidity Trap: Forward Guidance

Forward guidance refers to the idea that commitment of the central bank to keep the nominal interest rate low during a period after the liquidity trap leads to large output gains already during

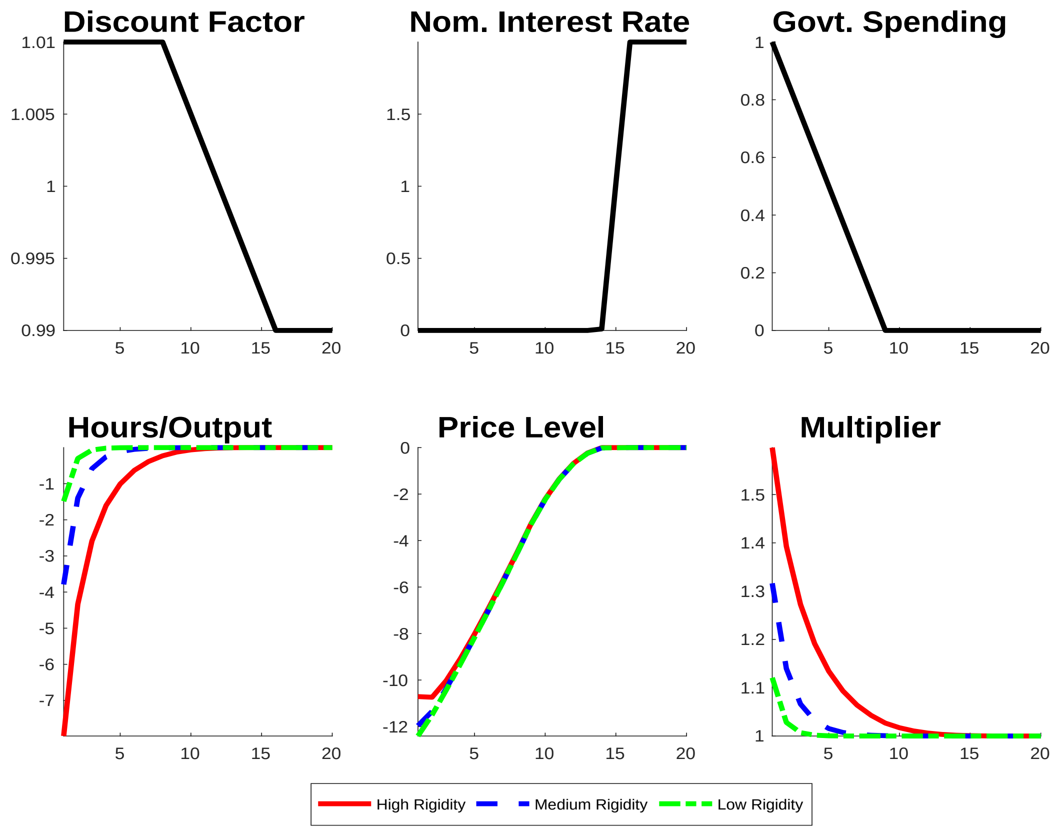


Figure 15: Fiscal Multiplier in a Liquidity Trap for different degree of price rigidities.

the liquidity trap. Figure 17 shows again that the large output gains from forward guidance do not occur in the incomplete markets model with a determinate price level. Independently of whether the nominal interest rate is raised earlier or later than in the benchmark, the output path is almost unchanged. The same conclusion is reached if instead of raising the nominal interest rate linearly to its steady-state level an interest rate rule is applied, as Figure 18 demonstrates. Note that is the case although monetary policy has sizeable effects here as panel a) of Figure 11 shows.

A.III.5 Zero Lower Bound and Fiscal Policy

Even when government spending is fully nominal and therefore the automatic stabilizer is most effective, increases in the discount factor still lead to a contraction in employment. There are, however, a number of ways to neutralize the effect of a discount factor increase. As long as the zero lower bound is not binding, the solution is particularly simple. An increase in the discount factor from β to $\hat{\beta}_t$ can be neutralized basically through monetary policy alone, by decreasing the nominal interest rate from R to $R_t = R\frac{\beta}{\hat{\beta}_t}$. Fiscal policy uses the (due to the lower R) lower interest rate payments on government debt to lower taxes. This policy is successful in stabilizing employment, consumption and prices at their steady-state values.

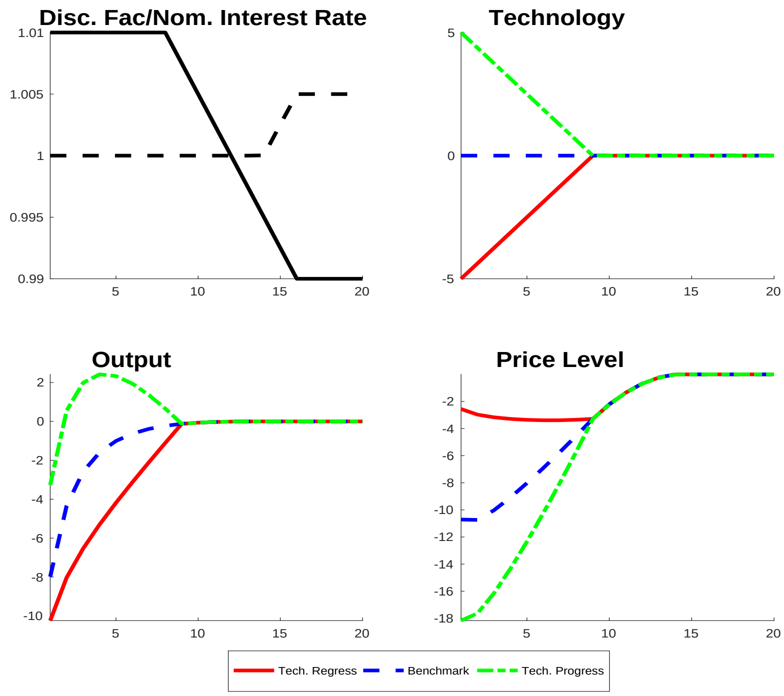


Figure 16: Technology Shocks in a Liquidity Trap

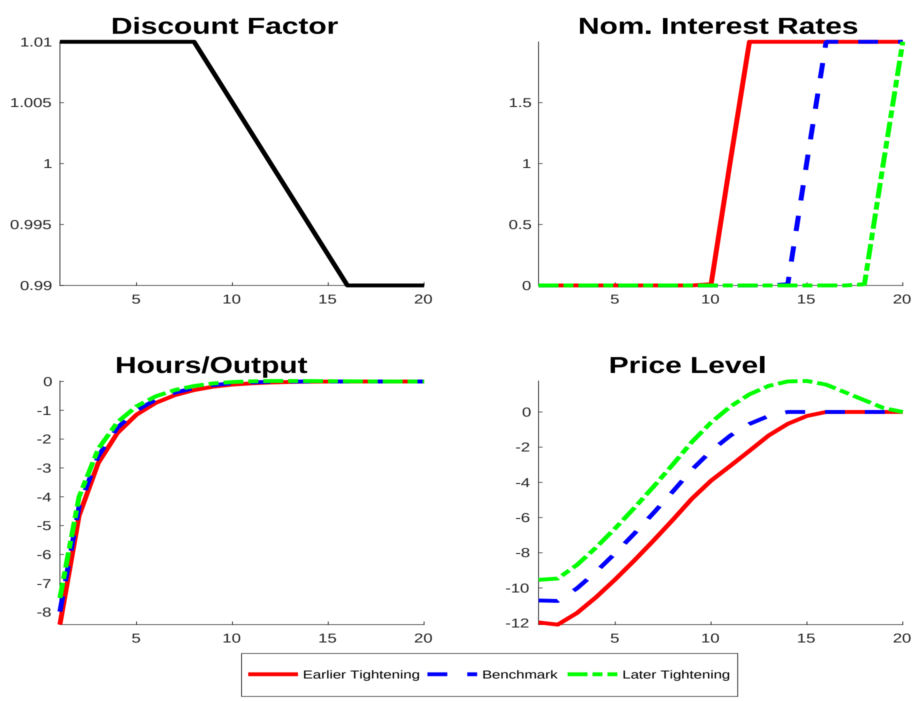


Figure 17: No Forward Guidance Puzzle

If the shock to the discount factor is too large the zero lower bound renders this solution impossible since $R\frac{\beta}{\hat{\beta}_t} < 1$. In this scenario fiscal policy has to step in and increase spending to

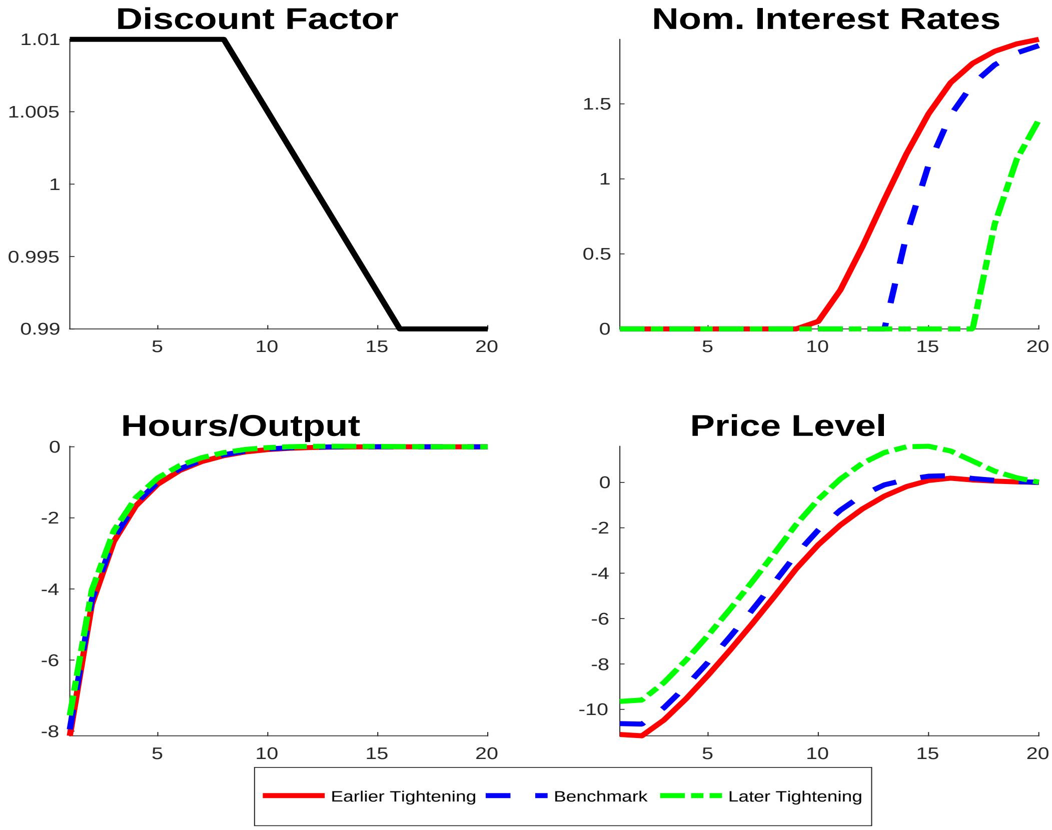


Figure 18: No Forward Guidance Puzzle with Interest rate rule

stabilize employment and prices. Obviously if the goal is employment stabilization, output is stabilized too and therefore private consumption has to fall. This suggests that monetary policy is the preferred policy tool in general as it enables stabilization of all three variables, whereas fiscal policy cannot. Fiscal actions are taken only if the zero lower bound makes monetary policy ineffective.

An increase of $\beta = 0.99$ to 1.01 implies that the zero lower bound is binding. Figure 19 shows the impulse responses when government spending is unchanged and only monetary policy decreases interest rates to zero, as long as the zero lower bound is binding. Since monetary policy is not able to neutralize the increase in β, employment and prices fall. Under these circumstances the goal to stabilize employment and prices requires fiscal policy to become active and increase spending. This policy is successful. Employment and prices remain throughout at their steady-state values. The path of government spending which achieves this is shown in the last panel of Figure 19.

A.III.6 Monetary and Fiscal Policy: Coordination

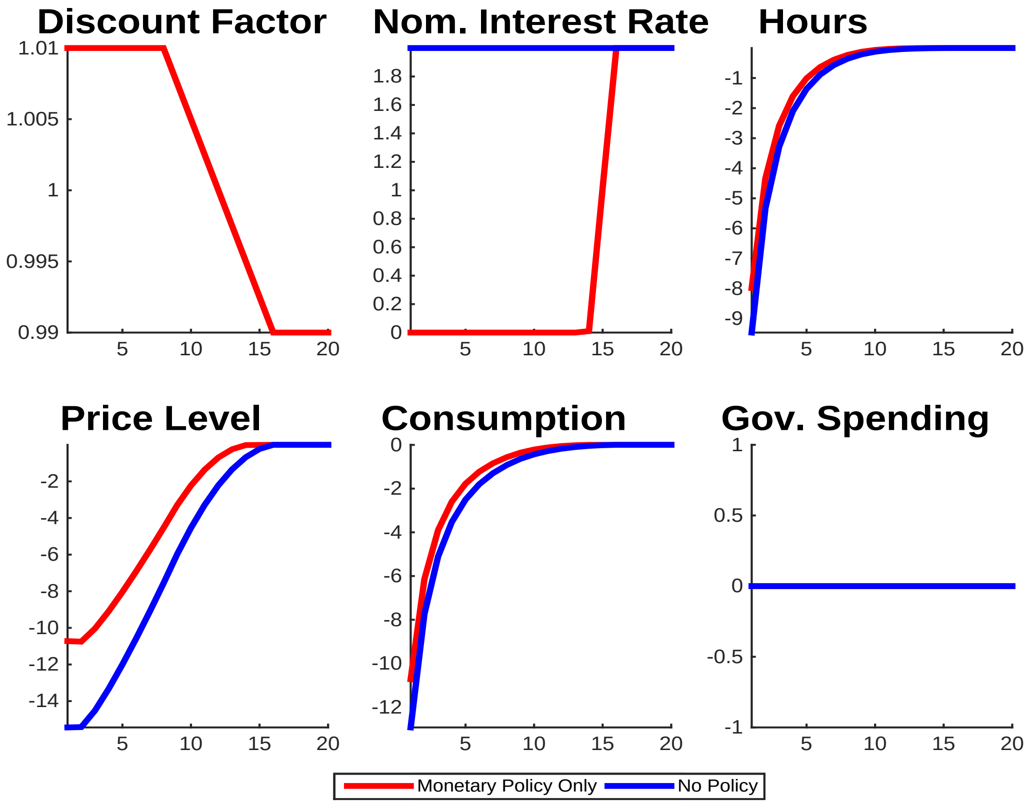
A main result of this paper is that fiscal and monetary policy jointly determine the price level. Understanding how prices move requires a good grasp of the policy coordination by the treasury and the central bank. Since here the price level is determined, policy coordination problems are quite different from those in Sargent and Wallace (1981)'s classic "monetarist arithmetic" or in Leeper (1991)'s active and passive monetary and fiscal policies. In particular the question is not which combination of fiscal and monetary policy leads to local determinacy of the price level and which combination does not. Instead prices are globally unique for any combination of policies.

Policy coordination is therefore very different. Fiscal policy can control the long-run inflation rate through controlling the nominal anchor. I showed in Section 4.4 that monetary policy takes over and sets the steady-state inflation rate when fiscal policy does not exercise its power to control long-run inflation.

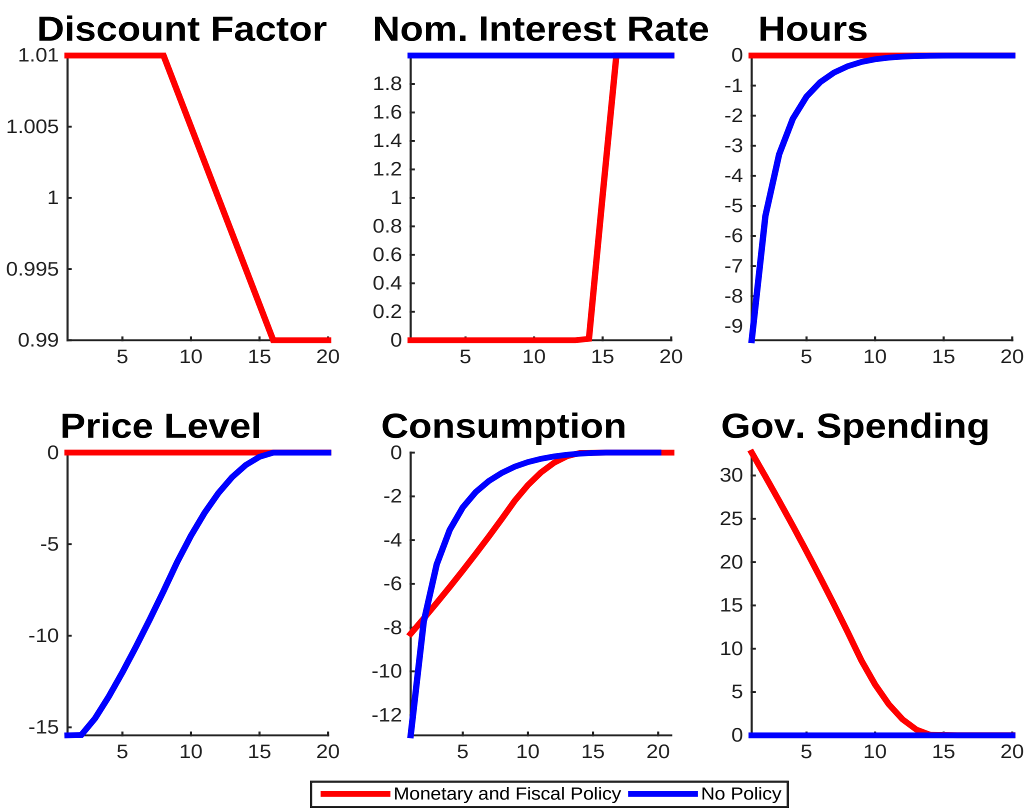
The analysis so far could give the impression that monetary policy is quite powerless and that fiscal policy can always get its way, not only determining fiscal policy but also setting the inflation rate, which in every textbook model is under the control of monetary policy exclusively.

This impression would underestimate the power of a central bank. On the one hand I have shown the effectiveness of monetary policy in stabilizing the economy. I will now illustrate its power by showing that the central bank can undo stimulative policies initiated by the treasury to raise inflation and boost employment. Any such short-run fiscal policy actions not meeting with the central bank's approval, for example because they conflict with the objective of price stability, can be neutralized resulting in no change in prices and employment and only higher taxes or debt. Figure 20 shows the sequence of nominal interest rates, in panel a) for a tax-financing and in panel b) for deficit-financing, which undo the fiscal policy expansion considered in Figure 12. Since a deficit-financed expansionary fiscal policy is more stimulative than a tax-financed one, the offsetting increase in nominal interest rates has to be higher in the former case.

This exercise also reveals another unpleasant consequence of an increase in nominal interest rates for the treasury. In practice this could be the most effective way for the central bank to impose its will on fiscal policy. Increases in nominal interest rates raise the interest payments on government debt, leading to higher debt and eventually higher taxes.

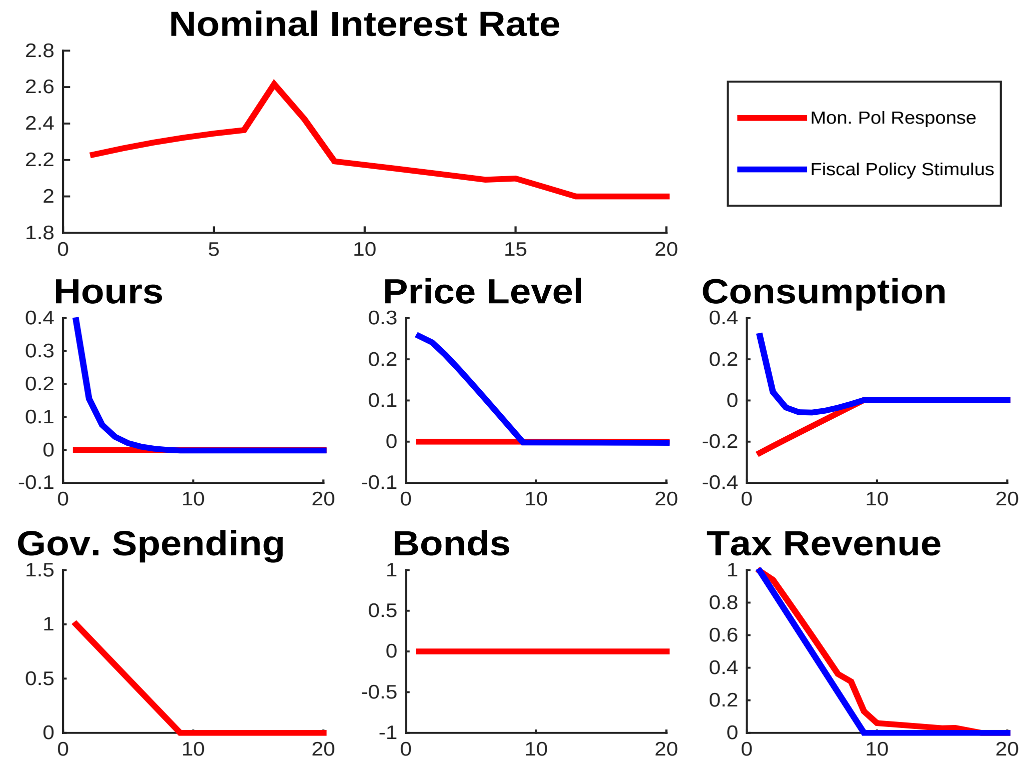


(a) ZLB: Monetary Policy Only



(b) ZLB: Fiscal and Monetary Policy

Figure 19: Zero Lower Bound : a) Monetary Policy Only b) Monetary and Fiscal Policy



(a) Monetary Policy Neutralizing Fiscal Policy

(b) Monetary Policy Neutralizing Fiscal Policy (Debt)

Figure 20: a) Fiscal Policy Stimulus (Tax financed) b) Fiscal Policy Stimulus (Debt financed)

A.IV Proofs and Derivations

A.IV.1 Proofs of Section 4

Proof of Proposition 1 For a given level of consumption C at t_1 and a given threshold level $\hat{\theta}$, consumption in period t_2 equals

$$\begin{array}{rll} i) & c(\theta) = (v')^{-1}(\frac{u'(C)}{\theta}) & \text{if } \theta \leq \hat{\theta}, \\ ii) & c(\theta) = B_t/P_t & \text{if } \theta > \hat{\theta}. \end{array}$$

As a result consumption C at period t_1 solves

$$C = A - \frac{G}{P} - (1 - F(\hat{\theta}))(\frac{B}{P}) - \int_{\underline{\theta}}^{\hat{\theta}} (v')^{-1}(\frac{u'(C)}{\theta}) dF(\theta), \tag{A65}$$

and the threshold level then solves

$$\hat{\theta} = \frac{u'(C)}{v'(\frac{B}{P})}. \tag{A66}$$

This defines $\hat{\theta}$ as a function of P and C. Plugging this expression into the fixpoint equation (A65) for C, shows that C is a function of P only and so is $\hat{\theta}$. To see that C is increasing in P, take derivatives of the fixpoint equation

$$C(P) = A - \frac{G}{P} - (1 - F(\hat{\theta}(P)))(\frac{B}{P}) - \int_{\underline{\theta}}^{\hat{\theta}(P)} (v')^{-1}(\frac{u'(C(P))}{\theta}) dF(\theta). \tag{A67}$$

w.r.t P,

$$\begin{aligned} C'(P) &= \frac{G}{P^2} + (1 - F(\hat{\theta}(P)))\frac{B}{P^2} - [\int_{\underline{\theta}}^{\hat{\theta}(P)} (\frac{u''(C(P))}{\theta v''((v')^{-1}(\frac{u'(C(P))}{\theta}))}) dF(\theta)]C'(P) && \text{(A68)} \\ &+ \hat{\theta}'(P)\underbrace{[(\frac{B}{P})f(\hat{\theta}(P)) - (v')^{-1}(\frac{u'(C(P))}{\theta})f(\hat{\theta}(P))]}_{=0}, && \text{(A69)} \end{aligned}$$

so that

$$C'(P) = \frac{\frac{G}{P^2} + (1 - F(\hat{\theta}(P)))\frac{B}{P^2}}{1 + [\int_{\underline{\theta}}^{\hat{\theta}(P)} (\frac{u''(C(P))}{\theta v''((v')^{-1}(\frac{u'(C(P))}{\theta}))}) dF(\theta)]} > 0, \tag{A70}$$

since both $u'' < 0$ and $v'' < 0$. Having established that $C(P)$ is increasing in P immediately implies that

$$\hat{\theta}(P) = \frac{u'(C(P))}{v'(\frac{B}{P})} \tag{A71}$$

is decreasing in P since the numerator is decreasing in P and the denominator is increasing in P.

Proof of Proposition 2 Existence and uniqueness of the price level solving the FOC equation (44) implies the existence and uniqueness of all other variables, which can all be expressed as functions of the price level. Rewriting equation (44) yields

$$u'(C(P)) - \int_{\hat{\theta}(P)}^{\infty} \theta v'(B/P) dF(\theta) = F(\hat{\theta}(P)) \frac{R}{(1+\pi)} \beta u'(C(P)), \tag{A72}$$

such that the derivatives of both the LHS and the RHS are negative,

$$\begin{aligned}
\frac{\partial LHS}{\partial P} &= u''(C(P)) \frac{\partial C(P)}{\partial P} + \int_{\hat{\theta}(P)}^{\infty} \theta \frac{B}{P^2} v''(B/P) dF(\theta) + f(\hat{\theta}(P)) \hat{\theta}(P) v'(B/P) \frac{\partial \hat{\theta}(P)}{\partial P} \\
\frac{\partial RHS}{\partial P} &= F(\hat{\theta}(P^*)) \frac{R}{1+\pi} \beta u''(C(P)) \frac{\partial C(P)}{\partial P} + f(\hat{\theta}(P)) \frac{R}{1+\pi} \beta u'(C(P^*)) \frac{\partial \hat{\theta}(P)}{\partial P},
\end{aligned}$$

with

$$\frac{\partial LHS}{\partial P} < \frac{\partial RHS}{\partial P} < 0 \tag{A73}$$

since $\frac{\partial \hat{\theta}(P)}{\partial P} < 0$, $\frac{\partial C(P)}{\partial P} > 0$, $\hat{\theta}(P) v'(B/P) = u'(C(P^*)) > \frac{R\beta}{1+\pi} u'(C(P^*))$ and $\frac{R\beta}{1+\pi} F(\hat{\theta}(P^*)) \leq \frac{R\beta}{1+\pi} < 1$. Thus $LHS - RHS$ is a monotonically decreasing function of P and the intermediate value theorem implies that there is at most one P where $LHS(P) = RHS(P)$, that is there is at most one steady state price level.

To establish existence of such a price level, I now show that the LHS is larger than the RHS for small P and the RHS is larger than the LHS for large P, implying a unique P where the LHS is equal to the RHS.

To see this, note that if $P \to 0$, $u'(C(P)) \to \infty$ which since $F(\hat{\theta}(P)) \frac{R}{(1+\pi)} \beta < 1$ and all other terms are bounded implies that $LHS > RHS$ for small P.

If $P \to \infty$, C converges to A and $\hat{\theta}$ converges to its lower bound $\underline{\theta}$. Thus using assumption (30) implies

$$\lim_{P \to \infty} LHS = -\infty < 0 < RHS. \tag{A74}$$

Assumption (30) also justifies to focus on finite steady-state prices. The first-order condition outside steady-states reads

$$\frac{u'(C_t)}{P_t} = \int_{\hat{\theta}_t}^{\infty} \theta \frac{v'(B/P_t)}{P_t} dF(\theta) = F(\hat{\theta}_t) R \beta \frac{u'(C_{t+1})}{P_{t+1}}. \tag{A75}$$

Since C and $\hat{\theta}$ are bounded, the LHS converges to 0 when $P \to \infty$ whereas assumption (30) implies that the RHS converges to a finite number, ruling out an $P \to \infty$ equilibrium.

Proof of Proposition 3

Proposition 2 implies a unique steady state price level such that $\Phi(P) = \frac{R}{1+\gamma}\Gamma(P)$. I now rule out non steady state equilibria, such as cycles etc. To this aim, Step 1 is key as it establishes the monotonicity of Φ and Γ.

Step 1

I first show that both the function

$$\Phi(\tilde{P}_t) = \frac{u'(C(\tilde{P}_t)) - \int_{\hat{\theta}(\tilde{P}_t)}^{\infty} \theta v'(B_t/\tilde{P}_t) dF(\theta)}{\tilde{P}_t F(\hat{\theta}(\tilde{P}_t))} \tag{A76}$$

is decreasing in $\tilde{P}_t$ if $\Phi(\tilde{P}_t) > 0$ and the function

$$\Gamma(\tilde{P}_{t+1}) = \beta \frac{u'(C(\tilde{P}_{t+1}))}{\tilde{P}_{t+1}} \tag{A77}$$

is decreasing in $\tilde{P}_{t+1}$.

This is obvious for Γ with derivative

$$\Gamma'(\tilde{P}) = \beta \frac{u''(C(\tilde{P}))C'(\tilde{P})\tilde{P} - u'(C(\tilde{P}))}{\tilde{P}^2} < 0 \tag{A78}$$

since $u', C' > 0$ and $u'' < 0$.

The derivative of Φ equals

$$
\begin{aligned}
\Phi'(\tilde{P}) &= \frac{u''(C(\tilde{P}))C'(\tilde{P}) + \int_{\hat{\theta}(\tilde{P})}^{\infty} \theta \frac{B}{\tilde{P}^2} v''(B/\tilde{P})dF(\theta) + \hat{\theta}'(\tilde{P})\hat{\theta}(\tilde{P})v'(B/\tilde{P})f(\hat{\theta}(\tilde{P}))}{\tilde{P}F(\hat{\theta}(\tilde{P}))} \\
&- \frac{[u'(C(\tilde{P})) - \int_{\hat{\theta}(\tilde{P})}^{\infty} \theta v'(B/\tilde{P})dF(\theta)][F(\hat{\theta}(\tilde{P})) + \tilde{P}f(\hat{\theta}(\tilde{P}))\hat{\theta}'(\tilde{P})]}{(\tilde{P}F(\hat{\theta}(\tilde{P})))^2} \\
&= \frac{u''(C(\tilde{P}))C'(\tilde{P}) + \int_{\hat{\theta}(\tilde{P})}^{\infty} \theta \frac{B}{\tilde{P}^2} v''(B/\tilde{P})dF(\theta)}{\tilde{P}F(\hat{\theta}(\tilde{P}))} \\
&- \frac{u'(C(\tilde{P})) - \int_{\hat{\theta}(\tilde{P})}^{\infty} \theta v'(B/\tilde{P})dF(\theta)}{\tilde{P}^2 F(\hat{\theta}(\tilde{P}))} \\
&+ \hat{\theta}'(\tilde{P}) \frac{f(\hat{\theta}(\tilde{P}))}{\tilde{P}F(\hat{\theta}(\tilde{P}))^2} \left[\hat{\theta}(\tilde{P})v'(B/\tilde{P})F(\hat{\theta}(\tilde{P})) - u'(C(\tilde{P})) + \int_{\hat{\theta}(\tilde{P})}^{\infty} \theta v'(B/\tilde{P})dF(\theta) \right] \\
&< 0, \qquad \text{(A79)}
\end{aligned}
$$

since

$$
\frac{u'(C(\tilde{P})) - \int_{\hat{\theta}(\tilde{P})}^{\infty} \theta v'(B/\tilde{P})dF(\theta)}{\tilde{P}^2 F(\hat{\theta}(\tilde{P}))} = \Phi(\tilde{P})/\tilde{P} > 0,
$$

$$
\begin{aligned}
& \hat{\theta}(\tilde{P})v'(B/\tilde{P})F(\hat{\theta}(\tilde{P})) - u'(C(\tilde{P})) + \int_{\hat{\theta}(\tilde{P})}^{\infty} \theta v'(B/\tilde{P})dF(\theta) \\
\geq\ & \hat{\theta}(\tilde{P})v'(B/\tilde{P})F(\hat{\theta}(\tilde{P})) - u'(C(\tilde{P})) + \int_{\hat{\theta}(\tilde{P})}^{\infty} \hat{\theta}(\tilde{P})v'(B/\tilde{P})dF(\theta) \\
=\ & \hat{\theta}(\tilde{P})v'(B/\tilde{P})F(\hat{\theta}(\tilde{P})) - u'(C(\tilde{P})) + (1 - F(\hat{\theta}(\tilde{P})))\hat{\theta}(\tilde{P})v'(B/\tilde{P}) \\
=\ & \hat{\theta}(\tilde{P})v'(B/\tilde{P}) - u'(C(\tilde{P})) \\
=\ & 0, \qquad \text{(A80)}
\end{aligned}
$$

and $\hat{\theta}'(\tilde{P}) < 0$.

In steady state $\Phi'(P^*) < \frac{R}{1+\gamma}\Gamma'(P^*)$:

$$\begin{aligned}
& \Phi'(P^*) - \frac{R}{1+\gamma}\Gamma'(P^*) \\
= & \frac{-1}{(P^*)^2 F(\hat{\theta}(P^*))}\Big[u'(C(P^*)) - \int_{\hat{\theta}(P^*)}^{\infty} \theta v'(B/P^*) dF(\theta) - \frac{R}{1+\gamma} F(\hat{\theta}(P^*))\beta u'(C(P^*))\Big] \\
+ & \frac{u''(C(P^*))C'(P^*)}{P^* F(\hat{\theta}(P^*))}[1 - \frac{R}{1+\gamma} F(\hat{\theta}(P^*))\beta] \\
+ & \frac{\int_{\hat{\theta}(P^*)}^{\infty} \theta \frac{B}{(P^*)^2} v''(B/P^*) dF(\theta)}{P^* F(\hat{\theta}(P^*))} \\
+ & \hat{\theta}'(P^*) \frac{f(\hat{\theta}(P^*))}{F(\hat{\theta}(P^*))^2}\left[\hat{\theta}(P^*) v'(B/P^*) F(\hat{\theta}(P^*)) - u'(C(P^*)) + \int_{\hat{\theta}(P^*)}^{\infty} \theta v'(B/P^*) dF(\theta)\right] \\
< & \ 0,
\end{aligned}$$

since the first term is zero at the steady state (the term in square brackets is the FOC and thus zero), the second term is negative since $[1 - \frac{R}{1+\gamma} F(\hat{\theta}(P^*))\beta] > 0$ and $u'' < 0$, the third term is negative since $v'' < 0$ and the last term was shown above to be negative (equation (A80)).

This implies that $\Phi(\tilde{P}) < \frac{R}{1+\gamma}\Gamma(\tilde{P})$ for $\tilde{P} > P^*$ and $\Phi(\tilde{P}) > \frac{R}{1+\gamma}\Gamma(\tilde{P})$ for $\tilde{P} < P^*$ since there is a unique steady-state price level, $\Phi(\tilde{P}) = \frac{R}{1+\gamma}\Gamma(\tilde{P})$ iff $\tilde{P} = P^*$, as shown in Proposition 2.

Step 2

I now show that if $\tilde{P}_t > P^*$ then the subsequent sequence of prices is monotonically increasing, $\tilde{P}_t < \tilde{P}_{t+1} < \ldots < \tilde{P}_{t+k}, \ldots$ and if $\tilde{P}_t < P^*$ then the subsequent sequence of prices is monotonically decreasing, $\tilde{P}_t > \tilde{P}_{t+1} > \ldots > \tilde{P}_{t+k}, \ldots$ before I finally show in Step 3 that such price sequences do not form an equilibrium.

For every $\tilde{P}_s$ at time s the price at time $s+1$, $\tilde{P}_{s+1}$, is defined as solving

$$\Phi(\tilde{P}_s) = \frac{R}{1+\gamma}\Gamma(\tilde{P}_{s+1}) \tag{A81}$$

If $\tilde{P}_s > P^*$ then

$$\frac{R}{1+\gamma}\Gamma(\tilde{P}_s) > \Phi(\tilde{P}_s) = \frac{R}{1+\gamma}\Gamma(\tilde{P}_{s+1}), \tag{A82}$$

which implies that $\tilde{P}_{s+1} > \tilde{P}_s$ since $\Gamma'(\tilde{P}) < 0$.

If $\tilde{P}_s < P^*$ then

$$\frac{R}{1+\gamma}\Gamma(\tilde{P}_s) < \Phi(\tilde{P}_s) = \frac{R}{1+\gamma}\Gamma(\tilde{P}_{s+1}), \tag{A83}$$

which implies that $\tilde{P}_{s+1} < \tilde{P}_s$ since $\Gamma'(\tilde{P}) < 0$.

Step 3

Step 2 shows that a price $\tilde{P}_t$ different from the steady-state price P^* leads to either an monotonically increasing price sequence (if $\tilde{P}_t > P^*$) or a monotonically decreasing price sequence (if $\tilde{P}_t < P^*$).

The monotonically increasing price sequence is unbounded since otherwise the prices would converge. The limit would be a steady state, contradicting the uniqueness of a steady state established in Proposition 2.

Price levels that are too high are not equilibrium prices however, since for all high enough price levels $\tilde{P}$ it holds that $\hat{\theta}(\tilde{P}) = \underline{\theta}$ and $v'(B/\tilde{P})$ is arbitrarily large and thus

$$u'(C(\tilde{P})) - \int_{\hat{\theta}(\tilde{P})}^{\infty} \theta v'(B/\tilde{P}) dF(\theta) < 0, \tag{A84}$$

implying that such a high $\tilde{P}$ does not form an equilibrium since $\frac{R}{1+\gamma}\Gamma > 0$.

The monotonically decreasing price sequence is not bounded from below by some strictly positive number with a positive consumption level, since otherwise the price sequence would converge to a positive price level. The limit would be a steady state, contradicting the uniqueness of a steady state established in Proposition 2.

Price levels that are too low are not equilibrium prices either, since for a low enough price level $\tilde{P}$ it holds that $G/\tilde{P} > A$, that is real government expenditures exceed real output and consumption is non-positive, implying that such a low $\tilde{P}$ is not an equilibrium either.

Altogether this implies that there is no $\tilde{P}_t \neq P^*$ since an $\tilde{P}_t \neq P^*$ would be consistent only with price expectations which are (eventually) not an equilibrium.

In steady state $\Phi'(P^*) < \frac{R}{1+\gamma}\Gamma'(P^*)$:

$$\begin{aligned}
& \Phi'(P^*) - \frac{R}{1+\gamma}\Gamma'(P^*) \\
= \; & \frac{-1}{(P^*)^2 F(\hat{\theta}(P^*))}\Big[u'(C(P^*)) - \int_{\hat{\theta}(P^*)}^{\infty} \theta v'(B/P^*) dF(\theta) - \frac{R}{1+\gamma} F(\hat{\theta}(P^*))\beta u'(C(P^*))\Big] \\
+ \; & \frac{u''(C(P^*))C'(P^*)}{P^* F(\hat{\theta}(P^*))}[1 - \frac{R}{1+\gamma} F(\hat{\theta}(P^*))\beta] \\
+ \; & \frac{\int_{\hat{\theta}(P^*)}^{\infty} \theta \frac{B}{(P^*)^2} v''(B/P^*) dF(\theta)}{P^* F(\hat{\theta}(P^*))} \\
+ \; & \hat{\theta}'(P^*) \frac{f(\hat{\theta}(P^*))}{F(\hat{\theta}(P^*))^2}\left[\hat{\theta}(P^*) v'(B/P^*) F(\hat{\theta}(P^*)) - u'(C(P^*)) + \int_{\hat{\theta}(P^*)}^{\infty} \theta v'(B/P^*) dF(\theta)\right] \\
< \; & 0,
\end{aligned}$$

since the first term is zero at the steady state (the term in square brackets is the FOC and thus zero), the second term is negative since $[1 - \frac{R}{1+\gamma}F(\hat{\theta}(P^*))\beta] > 0$ and $u'' < 0$, the third term is negative since $v'' < 0$ and the last term was shown above to be negative (equation (A80)).

This implies that $\Phi(\tilde{P}) < \frac{R}{1+\gamma}\Gamma(\tilde{P})$ for $\tilde{P} > P^*$ and $\Phi(\tilde{P}) > \frac{R}{1+\gamma}\Gamma(\tilde{P})$ for $\tilde{P} < P^*$ since there is a unique steady-state price level, $\Phi(\tilde{P}) = \frac{R}{1+\gamma}\Gamma(\tilde{P})$ iff $\tilde{P} = P^*$, as shown in Proposition 2.

Proof of Theorem 1 From time S on policies are stationary, i.e. for $t \geq S$, $R_t = R$, $G_t = G(1+\gamma)^{t-S}$, $T_t = T(1+\gamma)^{t-S}$ and $B_t = B(1+\gamma)^{t-S}$. The results shown above imply that from time S the economy is characterized by a price level P^* such that prices for $t \geq S$, $P_t = P^*(1+\gamma)^{t-S}$. As before consumption $C_t = C(P^*)$ and $\hat{\theta}_t = \hat{\theta}(P^*)$.

Prices before time S are defined recursively. Given a price P_t^* at time $t \leq S$, the price level at time $t-1$ is

$$P_{t-1}^* = \Phi_{t-1}^{-1}(R_t\Gamma_t(P_t^*)), \tag{A85}$$

where

$$\Phi_t(P_t) = \frac{u'(C(P_t, B_t)) - \int_{\hat{\theta}(P_t)}^{\infty} \theta v'(B_t/P_t) dF(\theta)}{P_t F(\hat{\theta}(P_t))} \tag{A86}$$

and the function

$$\Gamma_t(P_{t+1}) = \beta \frac{u'(C(P_{t+1}, B_{t+1})}{P_{t+1}} \tag{A87}$$

and Φ is strictly monotone and therefore invertible. Note that these are actual prices P and not detrended prices $\tilde{P}$ as the economy is not in steady state before time S.

This recursive definition yields a price sequence

$$(P_0^*, P_1^*, \ldots, P_t^*, \ldots, P_S = P^*, P_{S+1} = P^*(1+\gamma), P_{S+2} = P^*(1+\gamma)^2, \ldots,) \tag{A88}$$

I now show that this the unique equilibrium sequence of prices. Suppose now to the contrary that there is another equilibrium price sequence $\hat{P}$ and let k be the first time when the price levels differ $\hat{P}_k \neq P_k^*$ (and $\hat{P}_m = P_m^*$ for $m < k$). The uniqueness of a steady-state price level implies that $k < S$ since from period S onwards, the equilibrium price equals $P^*(1+\gamma)^{t-S}$).

The price at time k, $\hat{P}_k$, uniquely pins down the full price sequence recursively,

$$\hat{P}_{m+1} = \Gamma_{m+1}^{-1}(\Phi_m(\hat{P}_m)/R_{m+1}), \tag{A89}$$

for all $m \geq k$. Since both Γ_t and Φ_t are strictly decreasing functions, the concatenation $\Gamma_{l+1}^{-1}(\Phi_l(P_l^*)/R_{l+1})$ is strictly increasing. This implies that if $\hat{P}_k > P_k^*$, then $\hat{P}_m > P_m^*$ for all $m \geq k$. In particular

$\hat{P}_S > P_S^* = P^*$, which is not an equilibrium price.

Similarly, if $\hat{P}_k < P_k^*$, then $\hat{P}_m < P_m^*$ for all $m \geq k$. In particular $\hat{P}_S < P_S^* = P^*$, which is not an equilibrium price either.

Together this implies that $\hat{P}_k \neq P_k^*$ is proven wrong by contradiction, implying that $\hat{P}_t = P_t^*$ for all t is the only equilibrium price sequence.

Proof of Proposition 7 The proof proceeds by backwards induction. In period $S-1$, the detrended price level $\tilde{P}_{S-1}$ solves

$$\Phi(\tilde{P}_{S-1}) = \frac{R_S}{1+\gamma}\Gamma(P^*) \geq \frac{R}{1+\gamma}\Gamma(P^*) = \Phi(P^*), \tag{A90}$$

which implies that $\tilde{P}_{S-1} \leq P^*$ with strict inequality if $R_S < R$.

Now assume that $\tilde{P}_{t+1} \leq \tilde{P}_{t+2} \leq \ldots$. The detrended price level $\tilde{P}_t$ solves

$$\Phi(\tilde{P}_t) = \frac{R_{t+1}}{1+\gamma}\Gamma(\tilde{P}_{t+1}) \geq \frac{R_{t+2}}{1+\gamma}\Gamma(\tilde{P}_{t+1}) \geq \frac{R_{t+2}}{1+\gamma}\Gamma(\tilde{P}_{t+2}) = \Phi(\tilde{P}_{t+1}), \tag{A91}$$

which implies that $\tilde{P}_t \leq \tilde{P}_{t+1}$ with strict inequality if $R_{t+1} < R_{t+2}$. The inequality uses that Γ is strictly decreasing and $\tilde{P}_{t+1} \leq \tilde{P}_{t+2}$.

This proves the statement for detrended prices by induction. This immediately implies

$$\frac{P_{t+1}}{P_t} \geq (1+\gamma) \tag{A92}$$

for non-detrended prices since the trend is $(1+\gamma)$.

Proof of Proposition 8 The proof proceeds by backwards induction. In period $S-1$, the detrended price levels $\tilde{P}^a_{S-1}$ and $\tilde{P}^b_{S-1}$ solve

$$\Phi(\tilde{P}^a_{S-1}) = \frac{R^a_S}{1+\gamma}\Gamma(P^*) \geq \frac{R^b_S}{1+\gamma}\Gamma(P^*) = \Phi(\tilde{P}^b_{S-1}). \tag{A93}$$

which implies that $\tilde{P}^a_{S-1} \leq \tilde{P}^b_{S-1}$ with strict inequality if $R^a_S > R^b_S$ since Φ is decreasing.

Now assume that $\tilde{P}^a_{t+1} \leq \tilde{P}^b_{t+1}$. Then

$$\Phi(\tilde{P}^a_t) = \frac{R^a_{t+1}}{1+\gamma}\Gamma(\tilde{P}^a_{t+1}) \geq \frac{R^b_{t+1}}{1+\gamma}\Gamma(\tilde{P}^a_{t+1}) \geq \frac{R^b_{t+1}}{1+\gamma}\Gamma(\tilde{P}^b_{t+1}) = \Phi(\tilde{P}^b_t), \tag{A94}$$

implying that $\tilde{P}^a_t \leq \tilde{P}^b_t$ with strict inequality if $R^a_{t+1} > R^b_{t+1}$.

Proof of Proposition 9

Define

$$\Phi(\tilde{P}_t, G) = \frac{u'(C(\tilde{P}_t, G)) - \int_{\hat{\theta}(\tilde{P}_t,G)}^{\infty} \theta v'(B/\tilde{P}_t) dF(\theta)}{\tilde{P}_t F(\hat{\theta}(\tilde{P}_t))} \tag{A95}$$

$$\Gamma(\tilde{P}_{t+1}, G) = \beta \frac{u'(C(\tilde{P}_{t+1}), G)}{\tilde{P}_{t+1}}, \tag{A96}$$

where both functions are strictly decreasing in $\tilde{P}$ and strictly increasing in G.

The proof again proceeds by backwards induction. In period $S-1$, the detrended price level $\tilde{P}_{S-1}$ solves

$$\Phi(\tilde{P}_{S-1}, \hat{G}) = \frac{R}{1+\gamma} \Gamma(P^*, G) = \Phi(P^*, G) \leq \Phi(P^*, \hat{G}), \tag{A97}$$

which implies that $\tilde{P}_{S-1} \geq P^*$ with strict inequality if $\hat{G} > G$ since Φ is decreasing in $\tilde{P}$.

Now assume that $\tilde{P}_{t+1} \geq \tilde{P}_{t+2} \geq \ldots$. The detrended price level $\tilde{P}_t$ solves

$$\Phi(\tilde{P}_t, \hat{G}) = \frac{R}{1+\gamma} \Gamma(\tilde{P}_{t+1}, \hat{G}) \leq \frac{R}{1+\gamma} \Gamma(\tilde{P}_{t+2}, \hat{G}) = \Phi(\tilde{P}_{t+1}, \hat{G}), \tag{A98}$$

which implies that $\tilde{P}_t \geq \tilde{P}_{t+1}$. The same arguments show that a more expansive fiscal policy leads to bigger price increases.

Proof of Proposition 10 The same arguments as in the proofs of Propositions 7 and 8 apply here.

Proof of Proposition 11 Define

$$\Phi(\tilde{P}_t, A) = \frac{u'(C(\tilde{P}_t, A)) - \int_{\hat{\theta}(\tilde{P}_t,A)}^{\infty} \theta v'(B/\tilde{P}_t) dF(\theta)}{\tilde{P}_t F(\hat{\theta}(\tilde{P}_t))} \tag{A99}$$

$$\Gamma(\tilde{P}_{t+1}, A) = \beta \frac{u'(C(\tilde{P}_{t+1}), A)}{\tilde{P}_{t+1}}, \tag{A100}$$

where both functions are strictly decreasing in $\tilde{P}$ and A.

The proof again proceeds by backwards induction. In period $S-1$, the detrended price level $\tilde{P}_{S-1}$ solves

$$\Phi(\tilde{P}_{S-1}, \hat{A}) = \frac{R}{1+\gamma} \Gamma(P^*, A) = \Phi(P^*, A) \geq \Phi(P^*, \hat{A}), \tag{A101}$$

which implies that $\tilde{P}_{S-1} \leq P^*$ with strict inequality if $\hat{A} > A$ since Φ is strictly decreasing in $\tilde{P}$.

Now assume that $\tilde{P}_{t+1} \leq \tilde{P}_{t+2} \leq \ldots$. The detrended price level $\tilde{P}_t$ solves

$$\Phi(\tilde{P}_t, \hat{A}_t) = \frac{R}{1+\gamma}\Gamma(\tilde{P}_{t+1}, \hat{A}_{t+1}) \geq \frac{R}{1+\gamma}\Gamma(\tilde{P}_{t+2}, \hat{A}_{t+2}) = \Phi(\tilde{P}_{t+1}, \hat{A}_{t+1}) \geq \Phi(\tilde{P}_{t+1}, \hat{A}_t), \quad \text{(A102)}$$

which implies that $\tilde{P}_t \leq \tilde{P}_{t+1}$.

Proof of Proposition 4 Using assumption (30), assuming that prices are sticky or that there is some fractional backing imply that there is an upper bound on prices. As a result the price space is compact and standard fixpoint arguments imply existence since all functions characterizing the equilibrium are continuous.

Proof of Proposition 5 I consider the local dynamics of prices around the steady state price level P^*. The local dynamics of prices P_t and P_{t+1} are described through

$$\Phi(P_t) = \Gamma(P_{t+1}), \quad \text{(A103)}$$

for functions Φ and Γ evaluated at the steady-state,

$$\Phi(P_t) = \frac{u'(C(P_t)) - \int_{\hat{\theta}_t(P_t)}^{\infty} \theta v'\left(\frac{\bar{B}}{\bar{P}}\left(\frac{P_t}{\bar{P}}\right)^{\rho_p^B - 1}\left(\frac{A_{ss}}{\bar{A}}\right)^{\rho_A^B}\right) dF(\theta)}{P_t F(\hat{\theta}(P_t))(1+\bar{i})\left(\frac{P_t}{\bar{P}}\right)^{\rho_p^i}\left(\frac{A_{ss}}{\bar{A}}\right)^{\rho_A^i}}, \quad \text{(A104)}$$

$$\Gamma_t(P_{t+1}) = \beta \frac{u'(C(P_{t+1}))}{P_{t+1}} \quad \text{(A105)}$$

and $C(P_t) = A_{ss} - \frac{\bar{G}}{\bar{P}}\left(\frac{P_t}{\bar{P}}\right)^{\rho_p^G - 1}\left(\frac{A_{ss}}{\bar{A}}\right)^{\rho_z^G} - E_\theta c_t(\theta)$.

The price $P_{t+1}(P_t)$ satisfies $\Phi(P_t) = \Gamma(P_{t+1}(P_t))$ and therefore showing $\frac{\partial P_{t+1}}{\partial P_t} > 1$ is equivalent to

$$\left.\frac{\frac{\partial \Phi(P_t)}{\partial P_t}}{\frac{\partial \Gamma(P_{t+1})}{\partial P_{t+1}}}\right|_{P_t = P_{t+1} = P^*} > 1. \quad \text{(A106)}$$

$$\begin{aligned}
& \frac{\partial \Phi(P_t)}{\partial P_t}\bigg|_{P_t=P^*} - \frac{\partial \Gamma(P_{t+1})}{\partial P_{t+1}}\bigg|_{P_{t+1}=P^*} \\
= & \frac{-1}{(P^*)^2 F(\hat{\theta}(P^*))(1+i_{ss})}\Big[u'(C(P^*)) - \int_{\hat{\theta}(P^*)}^{\infty} \theta v'(B_{ss}/P^*) dF(\theta) - (1+i_{ss})F(\hat{\theta}(P^*))\beta u'(C(P^*))\Big] \\
+ & \frac{u''(C(P^*))C'(P^*)}{P^* F(\hat{\theta}(P^*))(1+i_{ss})}[1-(1+i_{ss})F(\hat{\theta}(P^*))\beta] \\
+ & \frac{\int_{\hat{\theta}(P^*)}^{\infty} \theta(1-\rho_p^B)\frac{\bar{B}}{\bar{P}^2}\big(\frac{P^*}{\bar{P}}\big)^{\rho_p^B-2}\big(\frac{A_{ss}}{\bar{A}}\big)^{\rho_A^B} v''(B_{ss}/P^*) dF(\theta)}{P^* F(\hat{\theta}(P^*))(1+i_{ss})} \\
+ & \hat{\theta}'(P^*)\frac{f(\hat{\theta}(P^*))}{F(\hat{\theta}(P^*))^2(1+i_{ss})}\left[\hat{\theta}(P^*)v'(B_{ss}/P^*)F(\hat{\theta}(P^*)) - u'(C(P^*)) + \int_{\hat{\theta}(P^*)}^{\infty} \theta v'(B_{ss}/P^*) dF(\theta)\right] \\
- & \rho_p^i \frac{\Phi(P^*)}{P^*} \\
< & \ 0,
\end{aligned}$$

since the first term is zero at the steady state (the term in square brackets is the FOC and thus zero), the second term is negative since $[1-1+i_{ss})F(\hat{\theta}(P^*))\beta] > 0$, $u'' < 0$ and $C'(P^*) > 0$ (see below), the third term is negative since $v'' < 0$ and $(1-\rho_p^B) > 0$, the fourth term was shown above to be negative (equation (A80)) and the last term is negative since $\rho_p^i, \Phi(P) > 0$.

Since both derivatives are negative this implies that (A106) holds and thus

$$\frac{\partial P_{t+1}}{\partial P_t} > 1. \tag{A107}$$

To see that C is increasing in P, take derivatives of the fixpoint equation

$$C(P) = A - \frac{\bar{G}}{\bar{P}}\big(\frac{P}{\bar{P}}\big)^{\rho_p^G-1}\big(\frac{A_{ss}}{\bar{A}}\big)^{\rho_z^G} - (1-F(\hat{\theta}(P)))\big(\frac{\bar{B}}{\bar{P}}\big(\frac{P}{\bar{P}}\big)^{\rho_p^B-1}\big(\frac{A_{ss}}{\bar{A}}\big)^{\rho_A^B}\big) - \int_{\underline{\theta}}^{\hat{\theta}(P)} (v')^{-1}\big(\frac{u'(C(P))}{\theta}\big) dF(\theta).$$

w.r.t P,

$$\begin{aligned}
C'(P) \ = \ & (1-\rho_p^G)\frac{\bar{G}}{\bar{P}^2}\big(\frac{P}{\bar{P}}\big)^{\rho_p^G-2}\big(\frac{A_{ss}}{\bar{A}}\big)^{\rho_z^G} + (1-F(\hat{\theta}(P)))(1-\rho_p^B)\frac{\bar{B}}{\bar{P}^2}\big(\frac{P}{\bar{P}}\big)^{\rho_p^B-2}\big(\frac{A_{ss}}{\bar{A}}\big)^{\rho_A^B}) \\
- & [\int_{\underline{\theta}}^{\hat{\theta}(P)}\big(\frac{u''(C(P))}{\theta v''((v')^{-1}(\frac{u'(C(P))}{\theta}))}\big) dF(\theta)]C'(P) + \hat{\theta}'(P)\underbrace{[(\frac{B_{ss}}{P})f(\hat{\theta}(P)) - (v')^{-1}(\frac{u'(C(P))}{\theta})f(\hat{\theta}(P))]}_{=0},
\end{aligned}$$

so that

$$C'(P) \ = \ \frac{(1-\rho_p^G)\frac{\bar{G}}{\bar{P}^2}\big(\frac{P}{\bar{P}}\big)^{\rho_p^G-2}\big(\frac{A_{ss}}{\bar{A}}\big)^{\rho_z^G} + (1-F(\hat{\theta}(P)))(1-\rho_p^B)\frac{\bar{B}}{\bar{P}^2}\big(\frac{P}{\bar{P}}\big)^{\rho_p^B-2}\big(\frac{A_{ss}}{\bar{A}}\big)^{\rho_A^B})}{1+[\int_{\underline{\theta}}^{\hat{\theta}(P)}\big(\frac{u''(C(P))}{\theta v''((v')^{-1}(\frac{u'(C(P))}{\theta}))}\big) dF(\theta)]} > 0,$$

since both $u'' < 0$ and $v'' < 0$ and $(1-\rho_p^G) > 0, (1-\rho_p^B) > 0$. Having established that $C(P)$ is increasing in P immediately implies that

$$\hat{\theta}(P) = \frac{u'(C(P))}{v'(\frac{\bar{B}}{\bar{P}}(\frac{P}{\bar{P}})^{\rho_p^B-1}(\frac{A_{ss}}{\bar{A}})^{\rho_A^B})} \tag{A108}$$

is decreasing in P since the numerator is decreasing in P and the denominator is increasing in P.
Proof of Proposition 6 It is sufficient to show that

$$\begin{aligned} & \frac{u''(C(P^*))C'(P^*)}{P^*F(\hat{\theta}(P^*))(1+i_{ss})}[1-(1+i_{ss})F(\hat{\theta}(P^*))\beta] \\ + & \frac{\int_{\hat{\theta}(P^*)}^{\infty}\theta(1-\rho_p^B)\frac{\bar{B}}{\bar{P}^2}(\frac{P^*}{\bar{P}})^{\rho_p^B-2}(\frac{A_{ss}}{\bar{A}})^{\rho_A^B})v''(B_{ss}/P^*)dF(\theta)}{P^*F(\hat{\theta}(P^*))(1+i_{ss})} \\ - & \rho_p^i\frac{\Phi(P^*)}{P^*} < 0, \end{aligned}$$

since the term involving $\hat{\theta}'(P^*)$ is zero due to the assumption that $f(\hat{\theta}(P^*)) = 0$.

Equivalently I show that

$$\begin{aligned} & u''(C(P^*))C'(P^*)[1-(1+i_{ss})F(\hat{\theta}(P^*))\beta] - \rho_p^i u'(C(P^*))P^* \\ + & \int_{\hat{\theta}(P^*)}^{\infty}\theta(1-\rho_p^B)\frac{B_{ss}}{(P^*)^2}v''(B_{ss}/P^*) - \rho_p^i v'(B_{ss}/P^*)/P^* \\ < & \ 0. \end{aligned}$$

The second term is negative since

$$\begin{aligned} & \int_{\hat{\theta}(P^*)}^{\infty}\theta(1-\rho_p^B)\frac{B_{ss}}{(P^*)^2}v''(B_{ss}/P^*) - \rho_p^i v'(B_{ss}/P^*)/P^* \\ = & \ (\frac{B_{ss}}{P^*})^{-\sigma-1}\frac{B_{ss}}{(P^*)^2})\eta(-\sigma(1-\rho_p^B) - \rho_p^i) \\ < & \ 0. \end{aligned}$$

If $C'(P^*) \geq 0$ then the first term is clearly not positive. If $C'(P^*) < 0$ then

$$\begin{aligned} & u''(C(P^*))C'(P^*)[1-(1+i_{ss})F(\hat{\theta}(P^*))\beta] - \rho_p^i u'(C(P^*))P^* \\ = & \ C(P^*)^{-\sigma}/P^*\Big(-\rho_p^i - \sigma\frac{C'(P^*)P^*}{C(P^*)}[1-(1+i_{ss})F(\hat{\theta}(P^*))\beta]\Big) \\ < & \ C(P^*)^{-\sigma}/P^*\Big(-\rho_p^i - \sigma\min\{(1-\rho_p^B),(1-\rho_p^G)\}\frac{\frac{G_{ss}}{AP_{ss}}+\frac{B_{ss}}{AP_{ss}}}{1-\frac{G_{ss}}{AP_{ss}}-\frac{B_{ss}}{AP_{ss}}}\Big) \\ < & \ 0, \end{aligned}$$

where I have used that

$$
\begin{aligned}
C'(P^*)P^* &> \min\{(1-\rho_p^B),(1-\rho_p^G)\}\big(\frac{G_{ss}}{AP_{ss}}+\frac{B_{ss}}{AP_{ss}}\big), \\
C(P^*) &< A-\frac{G_{ss}}{AP_{ss}}-\frac{B_{ss}}{AP_{ss}}, \\
1 &\geq [1-(1+i_{ss})F(\hat{\theta}(P^*))\beta].
\end{aligned}
$$