Animal Spirits, Heterogeneous Expectations and the Duration of Crises *

Tiziana Assenza[a,b] William A. Brock[c] Cars H. Hommes[b]

[a] *ITEMQ, Catholic University of Milan,*

[b] *CeNDEF, University of Amsterdam,* [c] *University of Wisconsin*

First Draft: May, 2009

This Draft: July, 2011

Abstract

to be done...

JEL codes:

Keywords:

Acknowledgment:

*This is preliminary version of the paper. Please do not quote without the authors' permission.

1 Introduction

In a recent book Akerlof and Shiller (2009) have stressed the importance of "Animal Spirits" (confidence being one of them) in the origin and propagation of a financial crisis, of the subsequent recession and of the "exit" process from the recession. They concentrate on recent advances in behavioral economics in order to identify different types of "animal spirits." However their approach to "confidence" (one of their major "animal spirits") contains a major weakness: "confidence" (whatever this means) shares with "financial factors" (whatever this means) the fate of being difficult to conceptualize, model, and measure. The present paper is essentially an attempt to build a dynamic equilibrium model of agents' confidence. We illustrate, for example, how a sudden collapse of confidence may, on one hand, accelerate and amplify the downturn after a negative shock, and, on the other hand, slow down the recovery. The truly core ingredient of our model is the crucial role we assign to expectations' heterogeneity and, especially how the dynamics of that heterogeneity feeds into the dynamics of wages and the dynamics of contracting terms that the lending side of the economy imposes on the borrowing side of the economy in dynamic equilibrium.

After all, it is almost a commonplace that the behavior of a variable in the aggregate - i.e. at the macroeconomic level - does not necessarily correspond to the behavior of the same variable as decided at the microeconomic level by a "representative" individual: "Any meaningful model of the macroeconomy must analyze not only the characteristics of the individuals but also the structure of their interactions." (Colander et al., (????), p.237).

In the last decade bounded rationality and adaptive learning (see Evans and Honkapohja, 2001 for an extensive discussion) have played an increasingly important role in macroeconomics. But also in this case the representative agent assumption is still the workhorse of contemporary models. Moreover most of the times the learning process ends with the discovery of the "true model" of the economy, thus confirming rational expectations ex post. There are only few exceptions in the literature that take heterogeneity seriously in expectations formation. Some recent examples of macro models with heterogeneous expectations include Brock and de Fontnouvelle (2000), Evans and Honkapohja (2003, 2006), Branch and McGough (2009), Berardi (2007), Assenza and Berardi (2009), Assenza et al. (2009) and Brazier et al. (2008).

Alp Simsek (April 7, 2011) and Jose Scheinkman and W. Xiong (2003) have stressed the role of overly optimistic (over confident) believers in driving bubble like phenomena. We are attempting to integrate this type of thought into an overlapping generations macro economic model with heterogeneous believers that is still tractable enough that we may capture the impact on the dynamics of wages and the dynamics of the contract terms on loans. We abstract from the complexity of the real world contract terms for a loan by using a one dimensional rate variable that we call the "contract rate". The reader should think of this is a "risk spread" over a short term interest rate, e.g. the 90 day T-bill rate in the U.S. More will be said about this later.

The literature has established the persistence of heterogeneity in expectations and their non negligible effects in the transmission mechanism as a robust empirical regularity. Branch (2004), Santoro and Pfajfar (2006) recently provided empirical evidence in support of heterogeneous expectations using survey data on inflation expectations, while Hommes, Sonnemans, Tuinstra, and van de Velden (2005) and Adam (2007) found evidence for heterogeneity in learning-to-forecast laboratory experiments with human subjects.

We discipline our selection of expectational schemes studied here by our laboratory experiments in expectations dynamics as well as a review of the literature on heterogeneous expectations.

We believe that modeling exercises like ours which emphasize the power of expectations alone to drive dynamics have potential use in policy design. For example simply attempting set monetary policy as a linear function of the inflation rate and the output gap may not be enough in a world where optimistic believers are using leverage to drive asset prices above their fundamentals and,hence, exposing the economy to the fragility documented by Reinhardt and Rogoff (2008). Survey data on the pattern of expectations of asset traders and data on leverage patterns of traders might be useful in designing a modified "Taylor type" rule that might improve upon received Taylor type rules provided that it was transparent to the actors in the economy. Of course this is speculation on our part but the simulations we present below of our model's dynamics under different expectational schemes hint at the desirability of "expectational" interventions as well as more conventional policy design.

To put it another way, we start from this heterogeneity in expectation formation processes to suggest alternative policy interventions in a depressed economy, to inject confidence - above and beyond liquidity - in the economy. For example, if a policy maker could credibly underpin expectations of a beneficial change in regime, i.e. a sort of "trust injection" process rather like Sargent's (1982, page 89) example of the end of four great inflations. He says (Sargent (1982, page 89)), "The essential measures that ended hyperinflation...were, first, the creation of an independent central bank that was legally committed to refuse the government's demand for additional uncsecured credit and, second, a simulataneous alteration in the fiscal policy regime.". In our context, this was a direct manipulation of the distribution of heterogeneous expectations of the four economies under study by Sargent.

In order to model the process of heterogeneous expectations, we borrow from a recent paper by Brock and Manski (2008, 2011), (B&M hereafter) the description and conceptualization of ambiguity and pessimism in a credit market economy. In particular B&M take into account the existence in credit markets of an informational problem due to partial knowledge of loan repayments, i.e. lenders do not know a priori whether a borrower will totally repay his debt or only part of it, or, in the worst case scenario, he will not repay at all. In B&M lenders must build a model of borrower behavior which they are unable to completely specify due to lack of knowledge. We assume that lenders lack rational expectations in forming expectations about the future share of loans that will be paid back. We replace rational expectations with other types of expectational schemes which will be explained in detail below. Therefore in this setting pessimism and/or ambiguity embedded in different expectational schemes will play an important role when the credit market experiences an unexpected negative shock.

The present paper is closely linked to the "confidence" "Animal Spirits" of Akerlof and Shiller (2009), because we introduce a measure of confidence, represented by the lender's expectation about the borrower's probability of success (i.e. the event in which he will repay the debt). In fact we can interpret the probability of success as a measure of optimism about the share of borrowers that will be solvent. In other words the higher the expectation of the probability of success the higher the lender's confidence that tomorrow the borrower will reimburse the loan (and vice versa). We view our paper as moving a step ahead introducing the role of the dynamics of heterogeneous expectations and building an explicit dynamic model of part of Akerlof and Shiller's conceptual framework and putting explicit dynamics into the B&M rather static model. This enables us to study the way in which heterogeneity affects the path towards recovery after a negative shock to the economy. In particular we find that a snap collapse of confidence, due to an unanticipated negative shock, in the presence of heterogeneous agents, may cause a downturn and may keep the economy in a recession phase for a longer period than in the case in which we consider a representative agent. To put it another way we show how different dynamic expectational schemes on "confidence" impact the dynamics of wages and contract terms in our model. While we emphasize the problems caused by excessively pessimistic beliefs and/or the present of ambiguity and the aversion caused by it, we could just as easily use our model to study the opposite case of problems caused by excessively optimistic beliefs. Turn now to the details of the modelling.

2 The model

In this section we describe the basic ingredients of our framework. We will consider a market for loanable funds that is populated by households/lenders and firms/borrowers.

The households' sector, which also represents the supply side of the market for loanable funds, is built by means of an overlapping generations framework in which each agent when young consumes ($c_{t,t}$) and saves (s_t) earnings from work, with wages $w_{p,t}$ and an endowment (ω_y). Savings are invested either in a safe asset or in a risky asset. When old the agent consumes an endowment (ω_o) and the average return on investments.

The demand side of the market for loanable funds in our economy is represented by firms that borrow a certain amount of capital (x_t) for production and remunerate work after paying back their debt. The remuneration from work is used by household to consume and to save. Savings will be employed to extend loans to the firms' sector.

2.1 Households

The supply side of our economy is described by means of a two-period overlapping generations structure. We assume that the young agent in t has preferences defined over consumption when young $c_{t,t}$ and when old $c_{t,t+1}$. For the sake of convenience, we assume a logarithmic utility function. The objective function therefore is

$$u_t = \ln c_{t,t} + \ln c_{t,t+1} , \tag{2.1}$$

When young, the agent works and earns a real wage $w_{p,t}$ (i.e. wages from the productive sector), and receives an (exogenous) endowment ω_y . He invests his savings s_t partly in a safe asset, which yields a known fixed return ρ at $t+1$, and partly in a risky asset whose rate of return λ_{t+1} in period $t+1$ is uncertain. Investment in the risky asset can be conceived of as employment of resources ("capital") in the productive sector, whose output is uncertain. The expectations by the young formed at date t on the return of the risky asset at date $t+1$ are denoted by λ^e_{t+1}.[1] The share of savings invested in the risky asset in t is denoted by δ_t.

When old, the agent retires and receives an (exogenous) endowment ω_o (at the beginning of old age) and the return on asset investments. The budget constraint of the agent when young and when old respectively, therefore, are

$$c_{t,t} \leq w_t - s_t \,, \tag{2.2}$$

$$c_{t,t+1} \leq \omega_o + s_t[(1-\delta_t)\rho + \delta_t\lambda^e_{t+1}] \,, \tag{2.3}$$

where $w_t = w_{p,t} + \omega_y$, with $w_{p,t}$ labour income and ω_y endowment of the young. The decision problem of the young is to "optimize" (2.1) subject to (2.2) and (2.3). At date t the young agent decides real savings s_t and allocates a fraction δ_t to the "risky" asset which he anticipates to produce a real amount $s_t\delta_t\lambda^e_{t+1}$ available for consumption in $t+1$. Therefore $s_t\delta_t\lambda^e_{t+1}$ can be interpreted as expected production obtained employing $s_t\delta_t$ in the productive sector. It follows that λ^e_{t+1} can be interpreted as the expected "*average productivity of capital*" in this context.

The amount $s_t(1-\delta_t)$ allocated at date t to the safe asset is known by the young at date t to produce $s_t(1-\delta_t)\rho$ available for consumption in period $t+1$. The expression in brackets in (2.3) i.e.,

$$\mu^e_{t+1} =: (1-\delta_t)\rho + \delta_t\lambda^e_{t+1}, \tag{2.4}$$

will be denoted as the expected *average return on investment*. Substituting the constraints into the objective function one ends up with the following maximization problem

$$\max_{s_t} \;\; \ln(w_t - s_t) + \ln(\omega_0 + s_t\mu^e_{t+1}) \tag{2.5}$$

[1] Superscripts on symbols denote expectations (e) or actual (a) realized values

The FOC gives the following expression for savings

$$s_t = \frac{1}{2}\left(w_t - \frac{\omega_o}{\mu^e_{t,t+1}}\right). \tag{2.6}$$

Assuming, for the sake of simplicity, zero endowment when old i.e., $\omega_o = 0$, the FOC simplifies to

$$s_t = \frac{w_t}{2}. \tag{2.7}$$

Note that (2.7) says that, conditional on w_t, the demand for investment is perfectly inelastic w.r.t. known and unknown returns on assets next period.[2]

2.2 Firms' demand for loanable funds

Following Brock and Manski (2008) we assume that borrowers get into debt in order to finance productive investments. Moreover, if returns on investments turn out to be too low, they may not be able to pay back. Therefore, we introduce a (time varying) probability of success, p_t and a probability of bankruptcy $1 - p_t$.

Given the assumptions above firms choose the amount of capital x_t, borrowed from the lending side of the economy, at time t solving the maximization problem:

$$\max_{x_t} \ \{p_t(g(x_t) - r_t x_t) + (1 - p_t)(-r_t x_t)\} = \max_{x_t} \ \{p_t g(x_t) - r_t x_t\}, \tag{2.8}$$

where $r_t = 1 + r_{0,t}$ is the contract rate (i.e. "*rental rate*" on capital) and $g(x_t)$ is the production function, assumed to be strictly concave with decreasing returns to scale[3].

The maximization problem yields the following FOC:

$$p_t g'(x_t) = r_t \Longrightarrow x_t = x(r_t; p_t) = g'^{-1}\left(\frac{r_t}{p_t}\right). \tag{2.9}$$

[2]We will enrich the dynamics by tying the wage $w_{p,t}$ to investment in the productive sector in the previous period. E.g. if all savings are "buried underneath the mattress" last period, w_t this period is zero.

[3]More precisely, we assume $g'(x_t) > 0$, $g''(x_t) < 0$ with right hand and left hand Inada conditions i.e., $g(0) = 0$, $g'(0) = \infty$, $g'(\infty) = 0$

Given the features of the production function $g(x_t)$, (2.9) represents a decreasing relation between the amount of capital at period t and the rental rate on capital in the same period therefore it defines the *demand* for capital in this setting.

We can define the returns to the "other factor" (i.e. labor) besides factor x as a function of the amount of factor x hired. In other words what is left over after overheads and capital are paid goes to other factors and the bulk of other factors are types of labor. Hence wages from the productive sector at time t, $w_{p,t}$, in our economy can be defined as:

$$w_{p,t} := p_{t-1}g(x_{t-1}) - r_{t-1}x_{t-1}. \tag{2.10}$$

Substituting eq. (2.9) we get

$$w_{p,t} := p_{t-1}g(x_{t-1}) - p_{t-1}g'(x_{t-1})x_{t-1}. \tag{2.11}$$

In the case of a Cobb Douglas production function $g(x_t) = x_t^{\alpha}$, where $0 < \alpha < 1$ represents the capital's share, (2.9) and (2.10) specialize to the *demand function* and wages

$$x_t = x(r_t; p_t) = \left(\frac{r_t}{p_t\alpha}\right)^{\frac{1}{\alpha-1}}, \tag{2.12}$$

$$w_{p,t} = p_{t-1}(1-\alpha)x_{t-1}^{\alpha}. \tag{2.13}$$

Substituting the demand for capital x_t from (2.12) into (2.13) we get the labor income in the case of Cobb Douglas production function

$$w_{p,t} = \eta(p_{t-1})^{\frac{1}{1-\alpha}} r_{t-1}^{\frac{\alpha}{\alpha-1}}, \tag{2.14}$$

where $\eta = \alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha)$. Since lenders get zero under bankruptcy and consequently wages for bankrupt firms are zero it follows that (2.14) represents wages paid by successful firms at time t.

For later use it will also be useful to define the inverse demand function as

$$r_t = r(x_t; p_t) = \alpha p_t x_t^{\alpha-1}. \tag{2.15}$$

3 Equilibrium

In this section we will compute the equilibrium of our economy. As in Brock and Mansky (2008) we indicate with $x_j(r_t)$ the j-th borrower's loan demand at an contract rate r_t. Hence for a "sample" of J firms the lender's expected *loan return* is given by

$$\lambda_{t+1}^e(r_t) = \frac{\frac{1}{J}\sum_{j=1}^{J} \min\{\imath(j \in S_t)g(x_{j,t}), r_t x_{j,t}\}}{\frac{1}{J}\sum_{j=1}^{J} x_{j,t}}, \tag{3.1}$$

where $\imath(j \in S_t)$ is the indicator function which is unity if firm j is successful at date t and is zero otherwise. Moreover the numerator represents aggregate repayment and the denominator aggregate loan demand.

We assume success is independently distributed across firms at each date t. Therefore, firm j chooses $x_{j,t}$ to satisfy:

$$x_{j,t} = \max_{x_{j,t}}\{p_{j,t} g(x_{j,t}) - r_t x_{j,t}\} \tag{3.2}$$

provided that the maximized quantity is nonnegative, otherwise firm j shuts down and does not operate in period t. In this case it chooses $x_{j,t} = 0$.

Assume that the probability of success is the same for all firms at date t, i.e. $p_{j,t} \equiv p_t$, for all j. Then each firm solves the same maximization problem and the optimal solution is the same for all firms. Apply the Law of Large Numbers to Eq. (3.1) to obtain the "population" loan return function:

$$\lambda_{t+1}^e(r_t) = p_{t+1}^e r_t. \tag{3.3}$$

Moreover the *no arbitrage condition* is such that the return on the risky asset equals the return on the risk free investment i.e., $\lambda_t = \rho$. It follows that the no arbitrage value of the contract rate (r_t^*) is given by the following relation

$$r_t^* \;=\; \frac{\rho}{p_{t+1}^e}. \tag{3.4}$$

At this stage we have all the necessary ingredients to compute the equilibrium of our model. Let us define

$$\delta_t^*(r_t) := \bar{\imath}[r_t > \frac{\rho}{p_{t+1}^e}] := \bar{\imath}[r_t > r_t^*], \tag{3.5}$$

where the upper bar over the indicator function means that it is the set $[0, 1]$ when the $>$ is replaced by $=$. Hence we can define the *loan supply correspondence*, when old age endowment ω_o is zero, by

$$S_t(r_t) := \frac{w_t}{2}\bar{\imath}[r_t > r_t^* = \frac{\rho}{p_{t+1}^e}] \tag{3.6}$$

Note that it is the belief $p_{1,t+1}^e$formed at date t about the probability of success in $t+1$ that determines the loan supply at time t.

The demand for capital an the equilibrium value for the contract rate $(\bar{r}_t)$ are determined by market clearing, i.e.

$$x(r_t; p_t) = S_t(r_t). \tag{3.7}$$

Since the supply correspondence is a (time varying) step function, there are two possibilities for the equilibria, points A and B, as illustrated in Figure 1.

The first possibility (point A) is given by

$$r_A^* \;=\; \frac{\rho}{p_{t+1}^e} \tag{3.8}$$

$$x_A^* \;=\; x(r_A^*; p_t) = \left(\frac{\rho}{\alpha p_t p_{t+1}^e}\right)^{\frac{1}{\alpha-1}}, \tag{3.9}$$

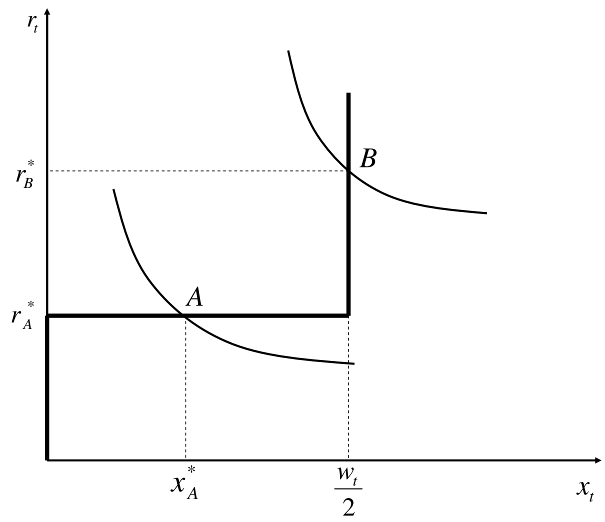

Figure 1: Points A and B represents the two possible configurations of the equilibrium depending on the features of the demand curve

and it arises when $x(r_A^*; p_t) < w_t/2$, where $x(\cdot)$ is the demand function (2.12).

The other possibility (point B) is given by

$$r_B^* = r(x_B^*; p_t) = \alpha p_t \left\{ \frac{1}{2} \left[\omega_y + (p_{t-1})^{\frac{1}{1-\alpha}} r_{t-1}^{\frac{\alpha}{\alpha-1}} \eta \right] \right\}^{(\alpha-1)} \tag{3.10}$$

$$x_B^* = \frac{w_t}{2} = \frac{1}{2} \left[\omega_y + (p_{t-1})^{\frac{1}{1-\alpha}} r_{t-1}^{\frac{\alpha}{\alpha-1}} \eta \right] \tag{3.11}$$

and it arises when $x(r_A^*; p_t) > w_t/2$.

It is important to note the crucial role played by expectations on the firms' probability of success (p_{t+1}^e). In fact, given the return on the risk free asset, the higher the probability of success the lower will be the non arbitrage contract rate (r_t^*) and, consequently, the higher will the the demand for capital (x_t^*). On the other hand, a low expected probability of success p_{t+1}^e causes the contract equilibrium rate r_A^* to rise sharply and marks a crisis.

4 Homogeneous beliefs

So far we have not assumed any specific features about expectations of the probability of success. Before investigating the role of heterogeneous expectations, in this section we consider a number of benchmark specifications of the lender's forecasting rules in the simple case of a representative agent, i.e. we will consider the case of homogeneous beliefs. In particular, we consider a number of cases departing from the standard rational expectations view allowing for bounded rationality.

We also have to specify an exogenous stochastic probability process. We focus on the simple case of an AR(1) probability process given by

$$p^e_{t+1} = \mu + a(p_t - \mu) + \epsilon_t, \tag{4.1}$$

where μ is the long run average, a is the first order autocorrelation coefficient and ϵ_t is an IID random variable drawn from a normal distribution. Throughout the paper we fix $\mu = 0.95$, $a = 0.8$ and $\sigma = 0.01$, so that there is quite some persistence in the probability of success or default.

4.1 Naive expectations

In the case of naive expectations, the forecast of the probability of success at period $t+1$ is given by last period's observation, i.e.,

$$p^e_{t+1} = p_t. \tag{4.2}$$

Figure 2 (top left panel) illustrates time series of the realized probability p_t, the naive forecast, together with the equilibrium contract rate r_t. Clearly the naive forecast lags realized probability and the contract rate spikes when the probability of success hits its lowest value, or equivalently when the probability of default hits its highest value in period 12. The dynamics of the contract rates is characterized by mean reversion to its long run equilibrium value $\bar{r} = \rho/\mu$, where μ is the long

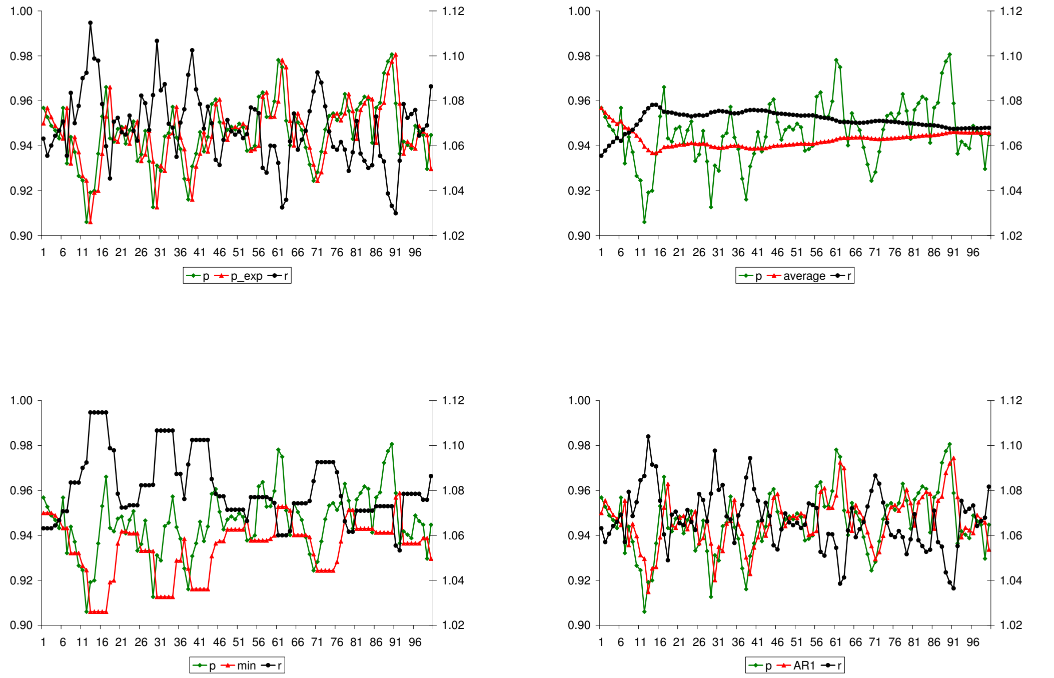

Figure 2: Four homogeneous expectations benchmarks. Blue line: contract rate, red line: realized probability of success, purple line: expected probability of success. Top left: naive expectations; Top right: average expectations; Bottom left: minimum expectations, and Bottom right: rational, AR1 expectations.

run mean of the AR(1) stochastic probability process.

4.2 Average beliefs

In the case of average expectations, the forecast of the probability of success is given by the sample average of past observation, i.e.,

$$p^e_{t+1} = \sum_{i=0}^{t} p_i. \tag{4.3}$$

Figure 2 (top right panel) illustrates time series of the realized probability p_t, the average forecast, together with the equilibrium contract rate r_t. The average forecast adjust slowly following realized probability and decreases gradually in the first 15 periods, when when the probability of success hits low values, in periods 10-14. Thereafter, the sample average forecast slowly decreases converging to the long run sample mean μ of the probability process. As a result, the contract rate initially increases reaching its maximum around period 15 and thereafter slowly mean reverts to its long run equilibrium level $\bar{r} = \rho/\mu$.

4.3 Minimum beliefs

In the case of minimum expectations, the forecast of the probability of success is given by its minimum realization in the last T periods, i.e.,

$$p^e_{t+1} = \min\{p_{t+1-T}, p_{t-T}, \cdots, p_{t-1}, p_t\}; \tag{4.4}$$

as a typical example we choose $T = 5$. Figure 2 (bottom left panel) illustrates time series of the realized probability p_t, the minimum forecast, together with the equilibrium contract rate r_t. The minimum forecast adjust according to the worst case probability and decreases until its lowest value in periods 12 to stay there for 10 periods, after the probability of success hits its lowest value, in period 11. As a result, the contract rate increases and hits its highest value in period 13 to stay there for 10 periods. Under minimum, pessimistic beliefs after each low bottom of the probability of success the contract rate spikes at high values and stays there for $T = 10$ periods. Hence, pessimistic minimum expectations causes crises to be more persistent.

4.4 Rational expectations

Now consider the case of rational expectations, where lenders are assumed to know the stochastic probability process to be an AR(1) process and have perfect knowledge about its parameters. The rational forecast of the probability of success at period $t + 1$ is given by

$$p^e_{t+1} = \mu + a(p_t - \mu). \tag{4.5}$$

Figure 2 (bottom right panel) illustrates time series of the realized probability p_t, the rational AR(1) forecast, together with the equilibrium contract rate r_t. The rational forecast closely tracks realized probability and the contract rate spikes when the probability of success hits its lowest value, or equivalently when the probability of default hits its highest value. Interest rate dynamics under rational expectations is in fact similar to the case of naive expectations. The only difference is that the peaks are less extreme, because the rational AR(1) forecast correctly predicts mean reversion after an extreme observation, while naive expectations uses the worst case. Under rational expectations, the dynamics of the contract rates is characterized by mean reversion to its long run equilibrium value $\bar{r} = \rho/\mu$, with the same speed as the probability process.

5 Heterogeneous beliefs

In this section we will extend our framework in order to take into account heterogeneity in agents' beliefs. In particular we will follow Brock and Hommes (1997) to model heterogeneous expectations by a discrete choice model.

5.1 Heterogeneous expectations

Assume there are J types of lenders in our economy at date t. At date t, type j's forecast for period $t+1$ of the return on the risky asset is given by

$$\lambda^e_{j,t+1} = p^e_{j,t+1} r_t. \tag{5.1}$$

Hence, each forecasting rule is determined by its forecast $p_{j,t+1}$ of the probability of success. Agents can choose between J different forecasting rules. The key idea of the switching model is that agents choose based upon the relative past performance of the forecasting rules. Let $U_{j,t}$ be a weighted average of past squared forecasting errors of the returns, that is,

$$U_{j,t} = r_t^2 \big(p_t - p^e_{j,t}\big)^2 + \eta\, U_{j,t-1}\,, \tag{5.2}$$

and let $u_{j,t}$ be the relative past squared forecasting errors of the returns, that is,

$$u_{j,t} = U_{j,t}/U_t^{tot}, \qquad U_t^{tot} = \sum_{j=1}^{J} U_{j,t}. \tag{5.3}$$

The fraction of the expectations rule j is updated according to a *discrete choice model with asynchronous updating* (Hommes et al., 2005; Diks and van der Weide, 2005)

$$n_{h,t} = \delta\, n_{h,t-1} + (1-\delta)\, \frac{e^{-\beta\, u_{h,t-1}}}{z_{t-1}}\,, \tag{5.4}$$

where $z_{t-1} = \sum_{j=1}^{J} \exp(-\beta\, u_{h,t-1})$ is a normalization factor. The asynchronous updating parameter $0 \leq \delta \leq 1$ reflects inertia in the choice of the heuristics. In the extreme case $\delta = 1$, the initial impacts of the rules never change, no matter what their past performance was. At the other extreme, $\delta = 0$, we have the special case of synchronous updating, where all agents switch to better strategies in each period. In general, in each period only a fraction $1-\delta$ of the heuristic's weight is updated according to the *discrete choice model.* The parameter $\beta \geq 0$ represents the intensity of choice measuring how sensitive predictor choice is to differences in heuristics' performance. In the extreme case $\beta = 0$, the relative weights of heuristics are not updated; at the other extreme $\beta = +\infty$, agents a fractions $1-\delta$ of agents switch to the best predictor.

5.2 Heterogeneous market equilibrium

Under heterogeneous expectations, we define total supply of loans at date t as

$$S_t(r_t) = \frac{w_t}{2} \sum_{j=1}^{J} n_{j,t} \quad \bar{\imath}[\lambda^e_{j,t+1}(r_t) > \rho] \tag{5.5}$$

Recalling Eq. (3.3) we have

$$S_t(r_t) = \frac{w_t}{2} \sum_{j=1}^{J} n_{j,t} \quad \bar{\imath}[p^e_{j,t+1} r_t > \rho], \tag{5.6}$$

where $p^e_{j,t+1}$ represents expectations of type j about the probability of success and $n_{j,t}$ represents the fraction of agents of type j at time t.

Figure 3 illustrates market equilibrium in the case of two types of agents. Recall that, in the homogeneous case, the loan supply correspondence (3.6) is a step function (see Figure 1), with the loan supply switching from 0 to $w_t/2$ at the critical threshold $r^* = \rho/p^e_{t+1}$ determined by the expected probability of success. In the case with two types of expectations, $p^e_{1,t+1}$ and $p^e_{2,t+1}$ the loan supply correspondence is a 2-step function with critical threshold levels at $r^*_1 = \rho/p^e_{1,t+1}$, where the loan supply switches from 0 to $n_{1t}w_t/2$, and at $r^*_2 = \rho/p^e_{2,t+1}$, where the loan supply switches from $n_{1t}w_t/2$ to $w_t/2$.

5.3 Two type examples

In this section we study two examples. In the first average expectations competes against minimum expectations. In the second example rational expectations, using the correct AR1 forecasting rule for the probability process, competes against minimum expectations. For the exogenous AR1 stochastic time series of the probability of success, we use the same realizations as in the previous section.

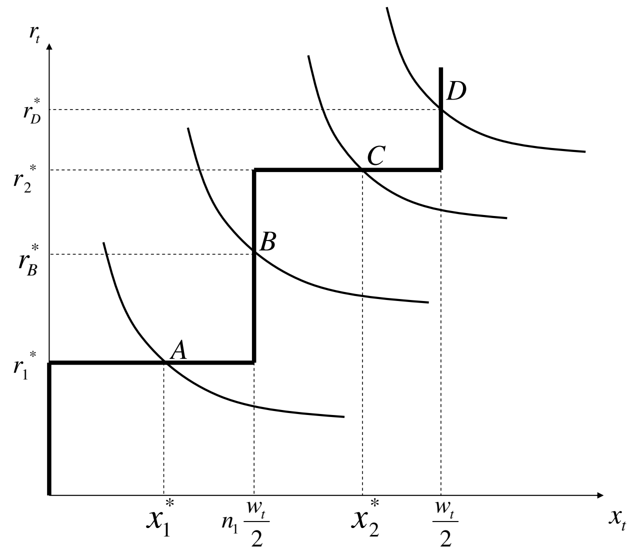

Figure 3: Four possible loan market equilibrium points in the case of 2 types, depending on the supply and demand curves. The figure illustrates the case $p_{1,t+1} > p_{2,t+1}$. The loan supply function is a 2-step function with critical threshold levels at $\bar{r} = \rho/p^e_{1,t+1}$, where the loan supply switches from 0 to $n_{1t}w_t/2$, and at $\bar{r} = \rho/p^e_{2,t+1}$, where the loan supply switches from $n_{1t}w_t/2$ to $w_t/2$.

5.4 Average versus minimum beliefs

Figure 4 (top panel) shows time series of the probability of success and the average and minimum forecasts as well as time series of the contract rate together with the fraction of minimum beliers. The contract rate switches between persistent phases of high contract rates, when pessimistic expectations dominate, and phases of intermediate contract rates (around 6-8%), when average expectations dominate.

5.5 Rational versus minimum beliefs

Figure 4 illustrates the case of rational versus minimum beliefs. Rational agents know the probability generating process and therefore use the optimal AR1 forecasting rule. The contract rate switches between persistent phases of high contract rates, when pessimistic expectations dominate, and phases of low contract rates (around 3-4%), when rational expectations dominate.

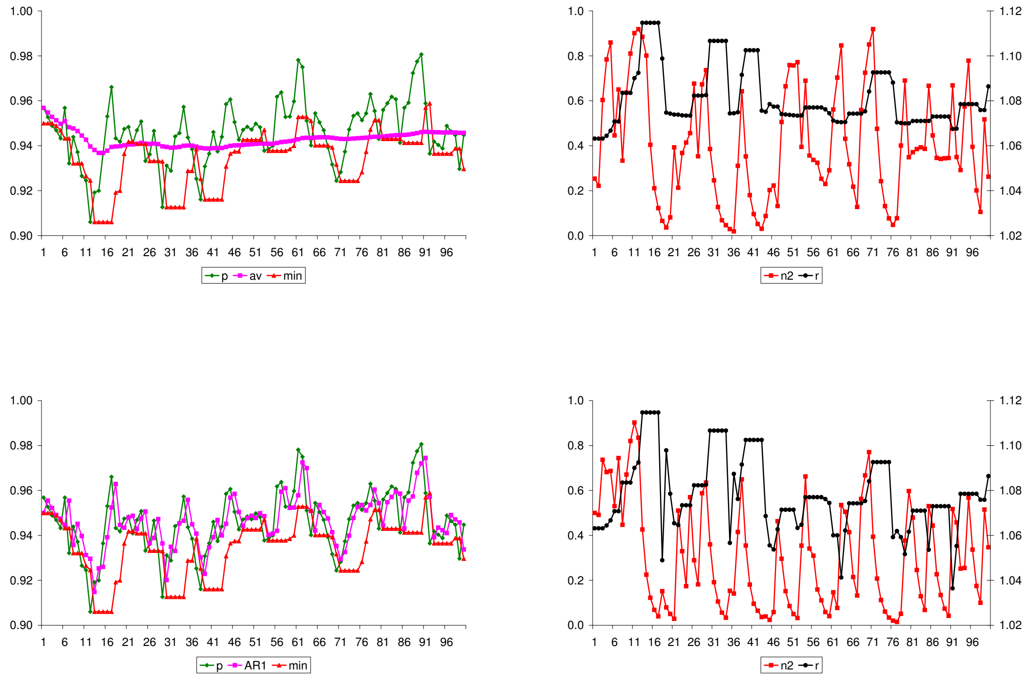

Figure 4: Simulations for two 2-type examples. Average vs minimum expectations (top panel) and rational (AR1) versus minimum. Left panels: green line: realized probability of success, purple line: average expectations resp. AR1 expectations, red line: minimum expectations; Right panels: black line: contract rate; red line; fraction of minimum expectations

Persistent phases of high contract rate occur when the majority of agents switches to minimum expectations. Average expectations drive down the contract rate somewhat, after the probability recovers. Rational expectations more accurately track the true probability process and lead to low contract rate as soon as the true probability attains relatively high values. In a heterogeneous world with pessimistic expectations, rational agents however can not avoid that the length of the crises increases due to pessimistic expectations.

APPENDIX

REFERENCES

Alp Simsek, April 7, 2011, "Belief disagreements and collateral constraints," Department of Economics, Harvard University.

Scheinkman, Jose' and W. Xiong, 2003, "Overconfidence and speculative bubbles," JOURNAL OF POLITICAL ECONOMY, 111, 1183-1219.