

Optimal Monetary Policy with $r^* < 0$

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Motivation

- Widespread consensus on a substantial decline in the *average* natural rate of interest $r^* \equiv \mathbb{E}\{r_t^n\}$
- With a ZLB constraint: increased incidence of binding ZLB episodes, greater macro instability
- Existing literature: optimal monetary policy under a ZLB constraint and $r^* > 0$. Normal times: $r_t^n > 0 \Rightarrow i_t > 0$, successful stabilization of inflation and the output gap. Occasional episodes with $r_t^n < 0 \Rightarrow i_t = 0$, macro instability. Key role for forward guidance.
- *This paper*: optimal monetary policy under a ZLB constraint with $r^* < 0$. "New normal": $r_t^n < 0$. Occasional episodes with $r_t^n > 0$. Summers' "secular stagnation" speech.

What does the optimal monetary policy look like in that environment?

What are its implications for macro outcomes?

Outline

- The optimal monetary policy problem
 - Equilibrium under the optimal policy: The case of a constant natural rate
 - Equilibrium under the optimal policy: Fluctuations in response to natural rate shocks.
 - Implementation: optimal interest rate rule
- byproduct*: sufficient condition for determinacy in models with endogenous regime switches
- Concluding remarks

Related Literature

- Optimal monetary policy under the ZLB: Eggertsson and Woodford (2003), Jung, Teranishi and Watanabe (2005), Adam and Billi (2006), Nakov (2008), who analyze the problem of optimal policy under commitment in the basic New Keynesian model with a ZLB constraint.
- Optimal choice of an inflation target, conditional on a given interest rate rule: Coibion et al. (2012), Bernanke et al. (2019), and Andrade et al. (2020, 2021).
- Equilibrium determinacy in regime-switching models: Davig and Leeper (2007), Farmer et al. (2009) and Barthélemy and Marx (2017, 2019).

The Optimal Monetary Policy Problem

$$\min \frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \vartheta y_t^2)$$

subject to

$$\pi_t = \beta \mathbb{E}_t \{ \pi_{t+1} \} + \kappa y_t \quad (1)$$

$$y_t = \mathbb{E}_t \{ y_{t+1} \} - \frac{1}{\sigma} (i_t - \mathbb{E}_t \{ \pi_{t+1} \} - r_t^n) \quad (2)$$

$$i_t \geq 0 \quad (3)$$

$$r_t^n = r^* + z_t \quad (4)$$

all for $t = 0, 1, 2, \dots$ where $z_t \sim AR(1)$ and

$$r^* < 0$$

A Brief Detour: A Microfounded NK Model with $r^* < 0$

- Based on the NK-OLG model in Galí (2021)
- Consumers: constant "life" and "activity" survival rates (γ, v) . Objective function for consumer born in period s :

$$\mathbb{E}_s \sum_{t=s}^{\infty} (\beta\gamma)^{t-s} Z_t \log C_{t|s}$$

where $\beta \equiv \exp\{-\rho\}$ and $z_t \equiv \log Z_t \sim AR(1)$

- Firms: attached to founder, hence survival rate γv . Calvo pricing.
- Steady state:

$$r^* = \rho + \log v$$

- Condition for $r^* < 0$

$$v < \beta$$

- Linearized equilibrium conditions:

$$\pi_t = \beta\gamma\mathbb{E}_t\{\pi_{t+1}\} + \kappa y_t$$

$$y_t = \mathbb{E}_t\{y_{t+1}\} - (i_t - \mathbb{E}_t\{\pi_{t+1}\} - r_t^n)$$

with $r_t^n = r^* + (1 - \rho_z)z_t$

The Optimal Monetary Policy Problem

$$\min \frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \vartheta y_t^2)$$

subject to

$$\pi_t = \beta \mathbb{E}_t \{ \pi_{t+1} \} + \kappa y_t \quad (5)$$

$$y_t = \mathbb{E}_t \{ y_{t+1} \} - \frac{1}{\sigma} (i_t - \mathbb{E}_t \{ \pi_{t+1} \} - r_t^n) \quad (6)$$

$$i_t \geq 0 \quad (7)$$

$$r_t^n = r^* + z_t \quad (8)$$

all for $t = 0, 1, 2, \dots$ where $z_t \sim AR(1)$ and

$$r^* < 0$$

The Optimal Monetary Policy Problem

- Optimality conditions:

$$\pi_t = \tilde{\zeta}_{1,t} - \tilde{\zeta}_{1,t-1} + \beta^{-1}\tilde{\zeta}_{2,t-1} \quad (9)$$

$$\vartheta y_t = -\kappa\tilde{\zeta}_{1,t} - \sigma\tilde{\zeta}_{2,t} + \sigma\beta^{-1}\tilde{\zeta}_{2,t-1} \quad (10)$$

$$\tilde{\zeta}_{2,t} \geq 0 \quad (11)$$

$$\tilde{\zeta}_{2,t} [r_t^n + \mathbb{E}_t\{\pi_{t+1}\} + \sigma(\mathbb{E}_t\{y_{t+1}\} - y_t)] = 0 \quad (12)$$

with initial conditions $\tilde{\zeta}_{1,-1} = \tilde{\zeta}_{2,-1} = 0$.

Optimal Policy: The Case of a Constant Natural Rate

- Assumption

$$r_t^n = r^* < 0$$

- Steady State

$$\pi = \beta^{-1} \bar{\zeta}_2 \geq 0$$

$$\theta y = -\kappa \bar{\zeta}_1 + \sigma(\beta^{-1} - 1) \bar{\zeta}_2$$

$$\bar{\zeta}_2 \geq 0 \quad ; \quad r^* + \pi \geq 0$$

$$\bar{\zeta}_2(r^* + \pi) = 0$$

$$\Rightarrow \pi \geq -r^* > 0$$

$$\Rightarrow \bar{\zeta}_2 > 0 \quad \Rightarrow \quad i = 0 \quad \Rightarrow \quad \pi = -r^*$$

Optimal Policy: The Case of a Constant Natural Rate

- Transitional dynamics

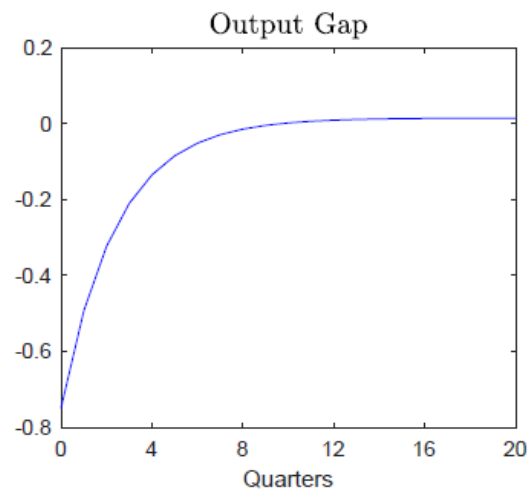
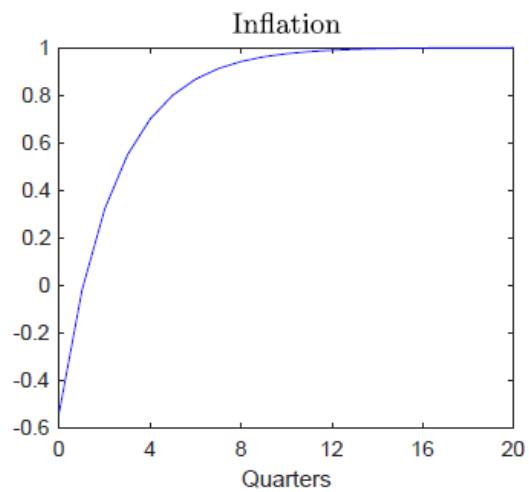
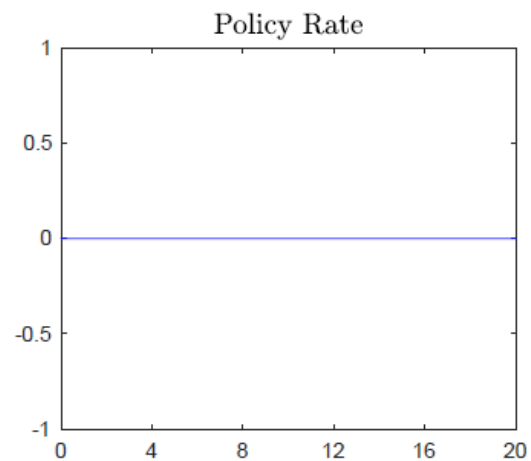
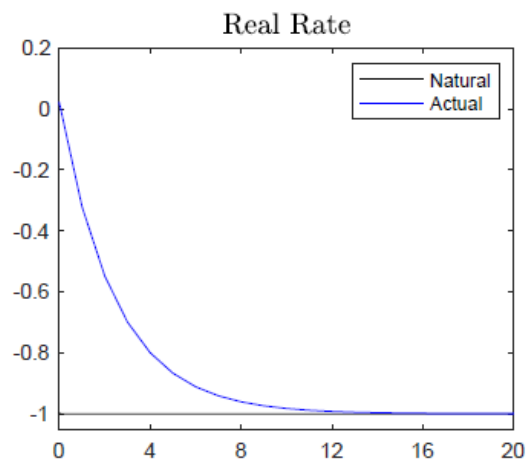
$$\begin{aligned}\hat{\pi}_t &= \beta\hat{\pi}_{t+1} + \kappa\hat{y}_t \\ \hat{\pi}_t &= \hat{\xi}_{1,t} - \hat{\xi}_{1,t-1} + \beta^{-1}\hat{\xi}_{2,t-1} \\ \vartheta\hat{y}_t &= -\kappa\hat{\xi}_{1,t} - \sigma\hat{\xi}_{2,t} + \sigma\beta^{-1}\hat{\xi}_{2,t-1} \\ \hat{\xi}_{2,t} + \xi_2 &\geq 0 \\ \hat{\pi}_{t+1} + \sigma(\hat{y}_{t+1} - \hat{y}_t) &\geq 0 \\ (\hat{\xi}_{2,t} + \xi_2) [\hat{\pi}_{t+1} + \sigma(\hat{y}_{t+1} - \hat{y}_t)] &= 0\end{aligned}$$

for $t = 0, 1, 2, \dots$ with initial conditions $\hat{\xi}_{1,-1} = -\xi_1$ and $\hat{\xi}_{2,-1} = -\xi_2$ and such that $\lim_{t \rightarrow \infty} \hat{x}_t = 0$ for $\hat{x}_t \in \{\hat{\pi}_t, \hat{y}_t, \hat{\xi}_{1,t}, \hat{\xi}_{2,t}\}$

- Simulations for a calibrated economy

$$\begin{aligned}\sigma &= 1, \beta = 0.99, \kappa = 0.1717, \vartheta = 0.0191 \text{ (Galí (2015))} \\ r &= -0.0025\end{aligned}$$

Optimal Policy: Transitional Dynamics



Optimal Policy: Fluctuations in Response to Natural Rate Shocks

- Stochastic equilibrium

$$\hat{\pi}_t = \beta \mathbb{E}_t \{ \hat{\pi}_{t+1} \} + \kappa \hat{y}_t$$

$$\hat{\pi}_t = \hat{\xi}_{1,t} - \hat{\xi}_{1,t-1} + \beta^{-1} \hat{\xi}_{2,t-1}$$

$$\vartheta \hat{y}_t = -\kappa \hat{\xi}_{1,t} - \hat{\xi}_{2,t} + \beta^{-1} \hat{\xi}_{2,t-1}$$

$$\hat{\xi}_{2,t} + \xi_2 \geq 0$$

$$\sigma(\mathbb{E}_t \{ \hat{y}_{t+1} \} - \hat{y}_t) + \mathbb{E}_t \{ \hat{\pi}_{t+1} \} + z_t \geq 0$$

$$[\hat{\xi}_{2,t} + \xi_2][\sigma(\mathbb{E}_t \{ \hat{y}_{t+1} \} - \hat{y}_t) + \mathbb{E}_t \{ \hat{\pi}_{t+1} \} + z_t] = 0$$

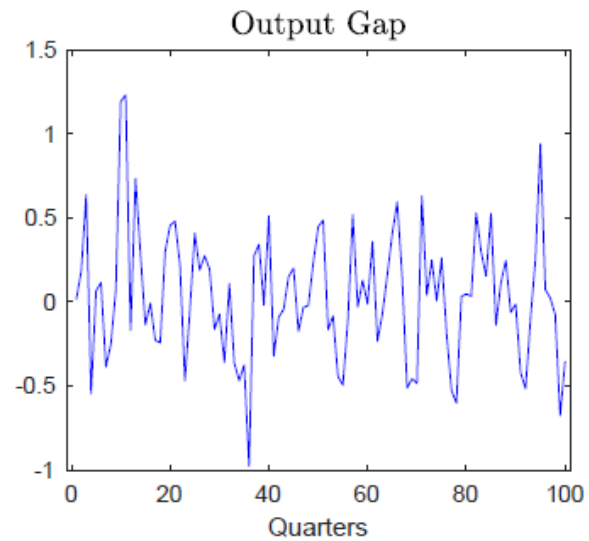
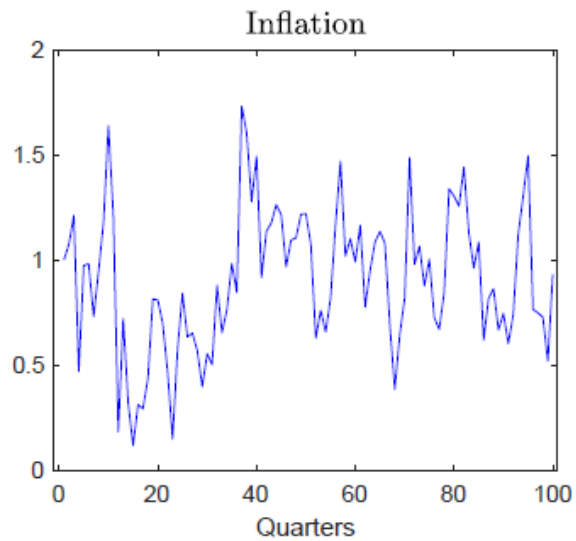
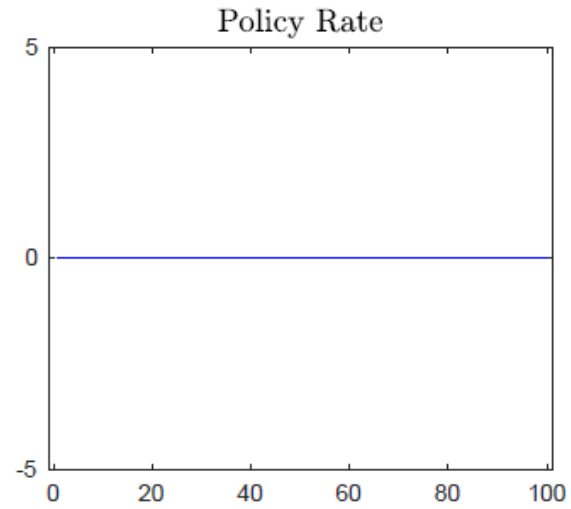
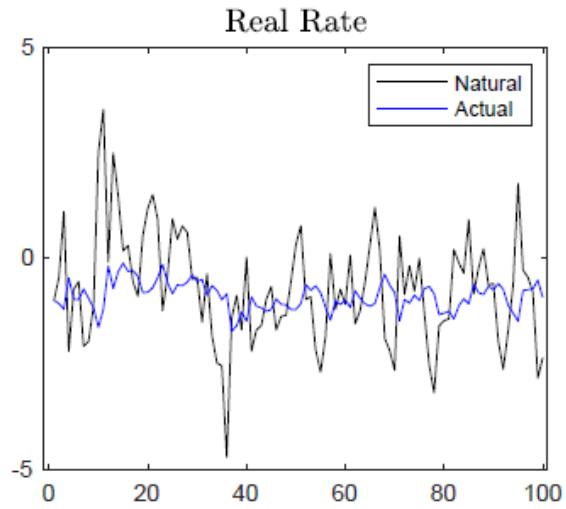
for $t = 0, 1, 2, \dots$ with initial conditions given by $\hat{\xi}_{1,-1} = 0$ and $\hat{\xi}_{2,-1} = 0$.

- Simulations

$$(\rho_z, \sigma_z) = (0.5, 0.0025)$$

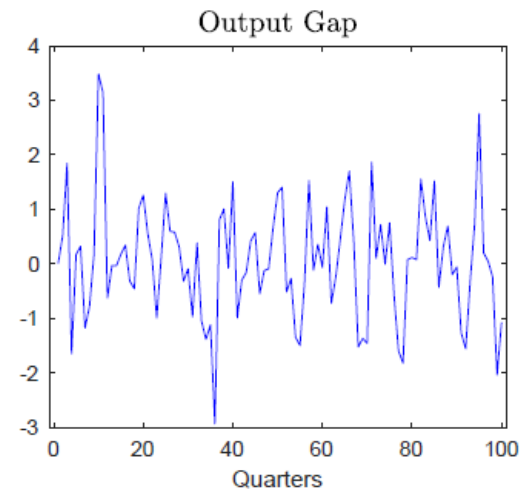
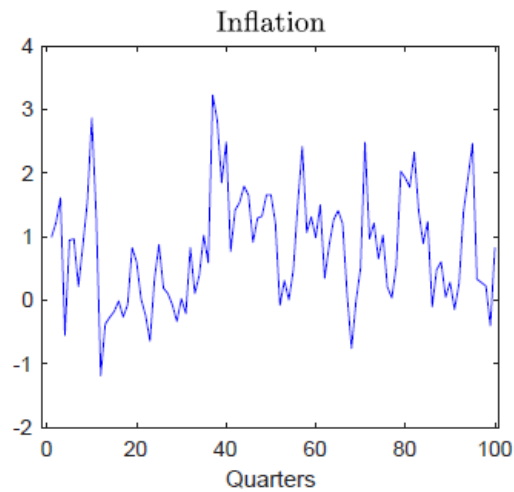
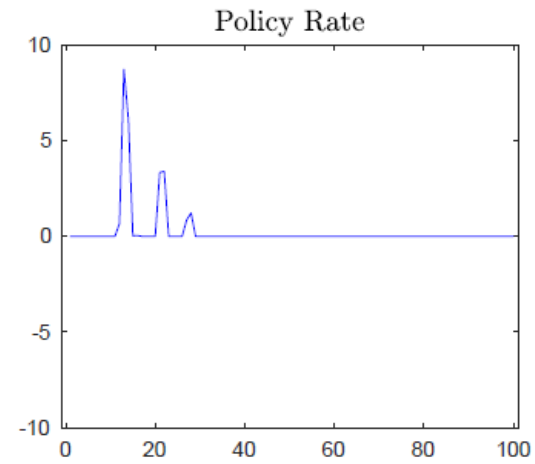
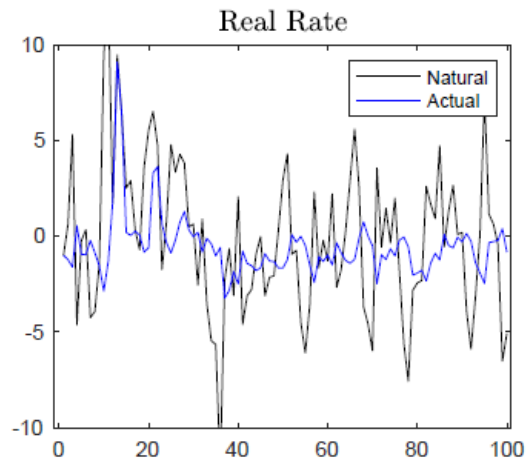
Optimal Policy: Fluctuations around the Steady State

Baseline Calibration



Optimal Policy: Fluctuations around the Steady State

High Volatility Calibration



Optimal Policy: Implementation

- Equilibrium outcomes under optimal policy: (i_t^*, y_t^*, π_t^*)
- Non-Policy block, in deviations from optimal path:

$$\tilde{\pi}_t = \beta \mathbb{E}_t \{ \tilde{\pi}_{t+1} \} + \kappa \tilde{y}_t \quad (13)$$

$$\tilde{y}_t = \mathbb{E}_t \{ \tilde{y}_{t+1} \} - \frac{1}{\sigma} (\tilde{i}_t - \mathbb{E}_t \{ \tilde{\pi}_{t+1} \}) \quad (14)$$

where $\tilde{x}_t \equiv x_t - x_t^*$ for $x \in \{\pi, y, i\}$

- *Wanted*: policy rule that guarantees $(\tilde{\pi}_t, \tilde{y}_t, \tilde{i}_t)$ is the only equilibrium.
- Candidate rule:

$$i_t = i_t^* \Leftrightarrow \tilde{i}_t = 0$$

for all t . Combined with non-policy block \Rightarrow multiplicity of solutions in addition to (i_t^*, y_t^*, π_t^*)

Optimal Policy: Implementation

- Proposed rule:

$$i_t = i_t^* + \phi_{\pi,t} |\tilde{\pi}_t| + \phi_{y,t} |\tilde{y}_t| \quad (15)$$

- Piecewise linear representation:

$$i_t = \begin{cases} i_t^* + \phi_{\pi} \tilde{\pi}_t + \phi_y \tilde{y}_t & \text{if } \tilde{\pi}_t \geq 0 \text{ and } \tilde{y}_t \geq 0 \\ i_t^* - \phi_{\pi} \tilde{\pi}_t - \phi_y \tilde{y}_t & \text{if } \tilde{\pi}_t < 0 \text{ and } \tilde{y}_t < 0 \\ i_t^* + \phi_{\pi} \tilde{\pi}_t - \phi_y \tilde{y}_t & \text{if } \tilde{\pi}_t \geq 0 \text{ and } \tilde{y}_t < 0 \\ i_t^* - \phi_{\pi} \tilde{\pi}_t + \phi_y \tilde{y}_t & \text{if } \tilde{\pi}_t < 0 \text{ and } \tilde{y}_t \geq 0 \end{cases} \quad (16)$$

Optimal Policy: Implementation

- Regime switching representation

$$\begin{bmatrix} \tilde{y}_t \\ \tilde{\pi}_t \end{bmatrix} = \mathbf{A}_t \begin{bmatrix} \mathbb{E}_t\{\tilde{y}_{t+1}\} \\ \mathbb{E}_t\{\tilde{\pi}_{t+1}\} \end{bmatrix}$$

where

$$\mathbf{A}_t = \begin{cases} \frac{1}{\sigma + \phi_y + \kappa \phi_\pi} \begin{bmatrix} \sigma & 1 - \beta \phi_\pi \\ \sigma \kappa & \kappa + \beta(\sigma + \phi_y) \end{bmatrix} & \equiv \mathbf{A}^{(1)} & \text{if } \tilde{\pi}_t \geq 0 \text{ and } \tilde{y}_t \geq 0 \\ \frac{1}{\sigma - \phi_y - \kappa \phi_\pi} \begin{bmatrix} \sigma & 1 + \beta \phi_\pi \\ \sigma \kappa & \kappa + \beta(\sigma - \phi_y) \end{bmatrix} & \equiv \mathbf{A}^{(2)} & \text{if } \tilde{\pi}_t < 0 \text{ and } \tilde{y}_t < 0 \\ \frac{1}{\sigma - \phi_y + \kappa \phi_\pi} \begin{bmatrix} \sigma & 1 - \beta \phi_\pi \\ \sigma \kappa & \kappa + \beta(\sigma - \phi_y) \end{bmatrix} & \equiv \mathbf{A}^{(3)} & \text{if } \tilde{\pi}_t \geq 0 \text{ and } \tilde{y}_t < 0 \\ \frac{1}{\sigma + \phi_y - \kappa \phi_\pi} \begin{bmatrix} \sigma & 1 + \beta \phi_\pi \\ \sigma \kappa & \kappa + \beta(\sigma + \phi_y) \end{bmatrix} & \equiv \mathbf{A}^{(4)} & \text{if } \tilde{\pi}_t < 0 \text{ and } \tilde{y}_t \geq 0 \end{cases}$$

Equilibrium Determinacy in (Possibly Endogenous) Regime Switching Models

- A benchmark regime switching model

$$\mathbf{x}_t = \mathbf{A}_t \mathbb{E}_t \{ \mathbf{x}_{t+1} \} \quad (17)$$

where \mathbf{x}_t is an $(n \times 1)$ vector of non-predetermined variables and \mathbf{A}_t is an $(n \times n)$ non-singular matrix. Assume $\mathbf{A}_t \in \mathcal{A} \equiv \{ \mathbf{A}^{(1)}, \mathbf{A}^{(2)}, \dots, \mathbf{A}^{(Q)} \}$.

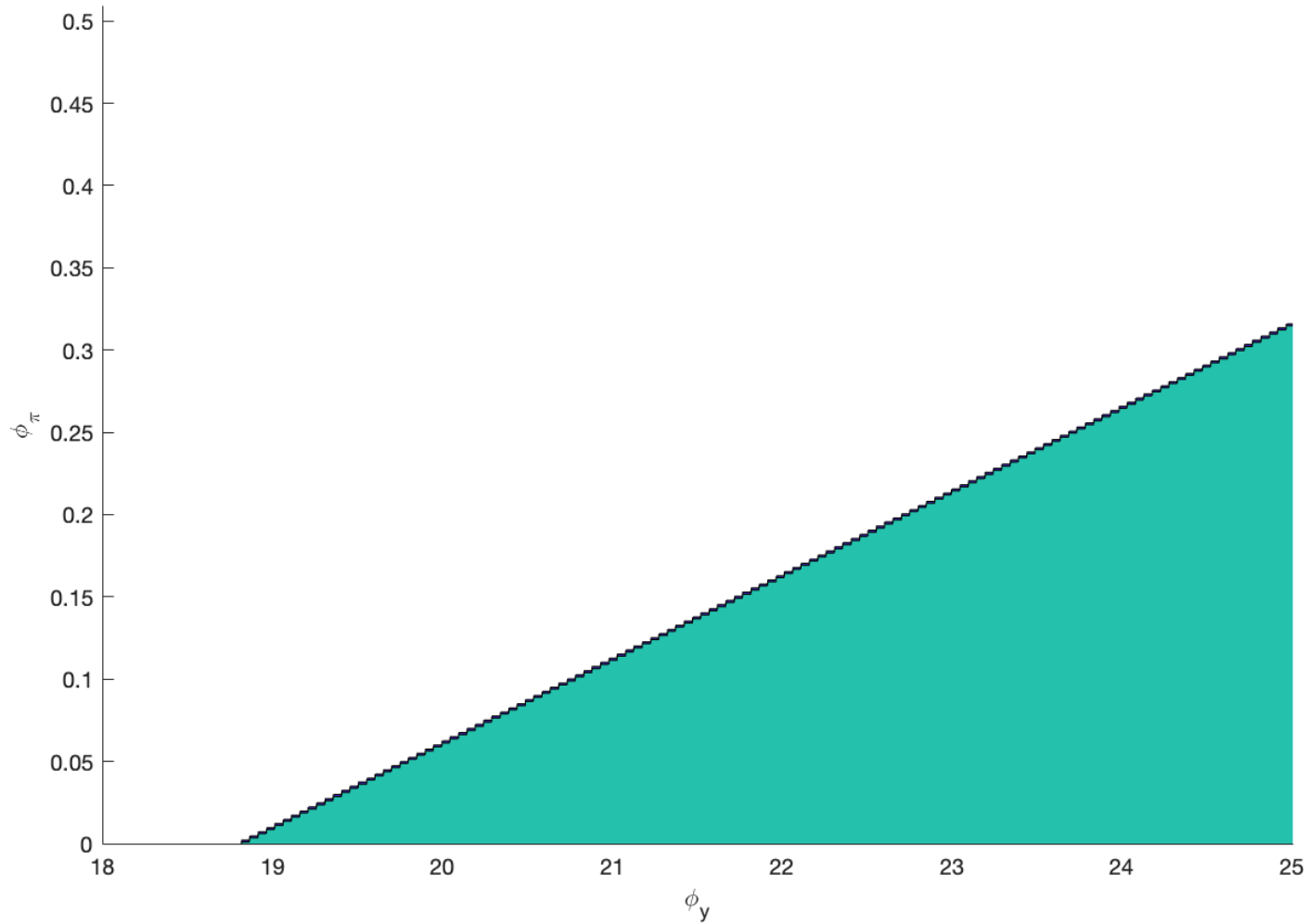
- Goal: establish *sufficient* conditions on \mathcal{A} that guarantee that $\mathbf{x}_t = 0$ all t is the only bounded solution to (17), i.e. $\lim_{T \rightarrow +\infty} \mathbb{E}_t \{ \|\mathbf{x}_{t+T}\| \} > M \|\mathbf{x}_t\|$ for any $M > 0$ and $\mathbf{x}_t \neq 0$, and where $\|\cdot\|$ is the usual L^2 norm.
- Define $\|\mathbf{A}^{(q)}\| \equiv \max_{\mathbf{x}} \|\mathbf{A}^{(q)} \mathbf{x}\|$ subject to $\|\mathbf{x}\| = 1$. In addition, $\alpha \equiv \max \{ \|\mathbf{A}^{(1)}\|, \|\mathbf{A}^{(2)}\|, \dots, \|\mathbf{A}^{(Q)}\| \} > 0$.

Theorem [*sufficient condition for determinacy*]: If $\alpha < 1$, then $\mathbf{x}_t = 0$ for all t is the only bounded solution to (17)

Optimal Policy: Implementation

- Determinacy region
- Discussion: time inconsistency, credibility

Implementation: Determinacy Region for Simple Rule



Concluding remarks

- Optimal monetary policy with a ZLB constraint and $r^* < 0$.
- The optimal policy aims to approach *gradually* a steady state with positive average inflation and a binding ZLB.
- Around that steady state, inflation and the output gap display (second-best) fluctuations in response to shocks. Those fluctuations coexist with a nominal rate that remains at its ZLB most (or all) of the time.
- The central bank can implement the optimal policy as a (locally) unique equilibrium by means of an appropriate state-contingent rule.
- In order to establish that result, we derive a sufficient condition for local determinacy in a general model with endogenous regime switches, a finding that may be of interest beyond the problem studied in the present paper.

Implementation: Determinacy Regions

