# Uncertainty, Real Activity, and Risk Aversion during the Great Recession\*

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#### Abstract

We estimate a nonlinear VAR to quantify the responses of output, consumption, investment, and hours to a financial uncertainty shock in booms and busts in the post-WWII U.S. data. We find evidence of comovements both in expansions and in recessions, with stronger responses of all real activity indicators in the latter state. We interpret this state-dependent responses with a version of the Basu and Bundick (2017) model in which an uncertainty shock conceptually comparable to the one used in our VAR analysis generates comovements in real activity. A state-contingent estimation of this model conducted via Bayesian direct inference points to counter-cyclical risk aversion as the crucial ingredient to replicate the evidence produced with our nonlinear IVAR. An exercise focusing on the great recession suggests that the nonlinear DSGE model is able to replicate about 50% of the cumulative output loss in the 2009-2014 period, twice as much what the same model would predict if estimated conditional on a linear VAR.

Keywords: Uncertainty shock, comovement, nonlinear IVAR, nonlinear DSGE framework, minimum-distance estimation, great recession.

*JEL codes*: C22, E32, E52.

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#### 1 Introduction

The great recession has revamped the attention on the role played by fluctuations in uncertainty as a possible driver of the U.S. business cycle. This paper offers two contributions to the literature. First, we show that the effects of uncertainty shocks on a battery of real activity indicators are stronger in recessions by estimating an interacted vector autoregressive (IVAR) model with post-WWII U.S. quarterly data. A jump in uncertainty of the same size in the two phases of the business cycle is associated to a peak response of output in recessions 50% larger than in expansions, and to a response of investment and hours twice as large. Second, we estimate a version of the Basu and Bundick (2017) model with the Bayesian minimum-distance direct inference approach developed by Christiano et al. (2011) by considering as facts the impulse responses produced with our nonlinear VAR. This strategy, which allows us to estimate the nonlinear DSGE framework in a state-dependent fashion, is designed to identify the crucial parameter instabilities one needs to allow to replicate the IVAR asymmetric responses of real activity to an uncertainty shock. We find the estimated DSGE model to be able to replicate the documented facts in both states of the business cycle. While our strategy allow us to estimate a large vector of structural parameters, counter-cyclical risk aversion arises as the necessary and sufficient feature the model needs to possess to match the facts.

We then push our investigation further and scrutinize how the "risk-aversion only" story fares when it comes to replicating an extreme event such as the great recession. We do so by re-estimating the nonlinear DSGE framework conditional on the IVAR responses produced when focusing on the effects of an uncertainty shock in 2008Q4, which is the quarter associated to the highest realization of such shock in our sample. With respect to the estimates of the structural parameters one obtains when taking a linear VAR as the relevant auxiliary model, we find a combination of a high degree of risk aversion and a moderately inertial, weak policy response to inflation to be sufficient for replicating our nonlinear IVAR responses during the great recession. When using our estimated DSGE model to compute the contribution of an uncertainty shock to the loss of output during and after the great recession, we find that such a shock could have been responsible for as much as 40% of the total output loss in 2008-2014. If instead we calibrate the model by using the impulse responses of a linear VAR as a reference, the model suggests that just half of that loss can be attributed to uncertainty shocks. This result clearly speaks in favor of using correctly calibrated structural frameworks

for replicating the facts and, ultimately, for conducting policy analysis.

Our paper relates to different but interconnected strands of the literature. The identification of uncertainty shocks is achieved by focusing on financial uncertainty, which has recently been singled out as a possible driver of the US business cycle (Bloom (2009), Ludvigson, Ma, and Ng (2019), ?). Methodologically, we use a nonlinear IVAR model to establish novel facts regarding the different intensity of comovements along the business cycle. IVAR model have increasingly been exploited to study the nonlinear effects of macroeconomic shocks (Towbin and Weber (2013), Sá, Towbin, and Wieladek (2014), Aastveit, Natvik, and Sola (2017)). Caggiano, Castelnuovo, and Pellegrino (2017) employ IVARs to investigate the link between uncertainty shocks and the stance of systematic policy. With respect to them, we focus on the stance of the business cycle and document the nonlinear effects of uncertainty shocks in recessions and expansions. In computing our impulse responses, we follow Pellegrino (2017a,b) and Caggiano, Castelnuovo, and Pellegrino (2017) and allow both uncertainty and real activity - i.e., the elements composing the interaction terms in our nonlinear VAR - to endogenously evolve after an uncertainty shock. We do so because both real activity and uncertainty have been found to significantly respond to an uncertainty shock (Jurado, Ludvigson, and Ng (2015), Ludvigson, Ma, and Ng (2019)). Our IVAR-related findings, which point to more severe consequences for output, investment, consumption, and hours when uncertainty shocks hit in recessions, complement those documented with alternative nonlinear frameworks and related to unemployment (Caggiano, Castelnuovo, and Groshenny (2014), Caggiano, Castelnuovo, and Figueres (2017)) and industrial production and employment (Caggiano, Castelnuovo, and Nodari (2019)), or obtained with indicators correlated with the business cycle like financial stress (Alessandri and Mumtaz (2018)).

Turning to the DSGE-based part of the analysis, many nonlinear frameworks have recently been shown to be able to generate comovements in real activity in response to uncertainty shocks. Fernández-Villaverde, Guerrón-Quintana, Rubio-Ramírez, and Uribe (2011) and Born and Pfeifer (2014b) investigate the real effects of a second moment shock to the world real interest rate for Argentina, Brazil, Ecuador, and Venezuela. Turning to the U.S., Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2015) study the role of fiscal policy uncertainty; Born and Pfeifer (2014a) quantify the relevance of policy risk; Born and Pfeifer (2019) investigate the role of second moment shocks to technology and fiscal spending; Drautzburg, Fernández-Villaverde, and Guerrón-Quintana (2017) investigate the role of political distribution

risks; Basu and Bundick (2017) show that a demand uncertainty shock which can be interpreted as a negative financial uncertainty shock triggers a negative response of real activity indicators. Our decision of working with a version of the last framework is justified by the presence, in such model, of a formally precise definition of implied financial volatility which can be meaningfully matched to the one we employ in our IVAR analysis. We exploit this match to estimate the third-order approximation of the DSGE model via the Bayesian impulse response function matching proposed by Christiano, Trabandt, and Walentin (2011). This novel empirical strategy enables us to estimate the nonlinear DSGE model we work with in a state-dependent fashion, something we do in order to unveil instabilities at a structural level which are needed to track the dynamic response of real activity in booms and busts. Our approach, which is Bayesian and focuses on a subset of shocks (in our application, one shock only) and on their dynamic effects, represents an alternative to the GMM/SMM-based methodology recently developed by Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2017), which is frequentist and relies on moments simulated by considering all modeled shocks. The common characteristic of these two approaches is that they solve the nonlinear DSGE framework by appealing to perturbation, developed by Schmitt-Grohe and Uribe (2004) and Andreasen (2012). This solution method has been shown to be particularly efficient against alternatives - such as value function iteration or projection - by Caldara, Fernández-Villaverde, Rubio-Ramírez, and Yao (2012). Methodologically, the closest approach to ours is probably the one by Ruge-Murcia (2017), who estimates a small-scale thirdorder approximated DSGE model with an impulse-response matching procedure based on a class of nonlinear VAR models as an auxiliary model for the purpose of indirect inference via a classical minimum distance estimator. In doing so, he imposes the perturbation solution of the nonlinear DSGE model on the nonlinear VAR framework to approximate as closely as possible the DSGE-related policy functions. His approach, which is extremely neat, becomes unfortunately difficult to implement when one works with models with several states. Our novel estimation strategy easily accommodates large state spaces.

The paper develops as follows. Section 2 presents the non-linear VAR model em-

<sup>&</sup>lt;sup>1</sup>Castelnuovo and Pellegrino (2018) perform a similar exercise by working with Bayesian impulse response function matching in order to assess the performance of some medium-scale DSGE frameworks in presence of high/low uncertainty. Crucially, they work with linearized models, while this paper estimates a truly nonlinear DSGE framework in a state-dependent fashion. At least a third-order approximation of the model is required to study the effects of a time-varying uncertainty shock in a structural framework.

ployed and presents results on the business cycle-dependent consequences of uncertainty shocks from this relatively unrestricted framework. Section 3 briefly presents the Basu and Bundick (2011) model, describes the econometric strategy adopted to estimate the DSGE model and discusses the regime-dependent estimation results found with a focus on the sources of the different transmission mechanism of uncertainty shocks between recessions and expansions. Section 4 quantifies the performance of our empirical framework as regards the negative effects of the large uncertainty shock happened at the onset of the great recession. Section 5 concludes.

# 2 Uncertainty-driven comovements: Empirical evidence

#### 2.1 Nonlinear empirical methodology

The IVAR is a nonlinear framework which augments a standard linear VAR model with interaction terms to determine how the effects of a shock to a variable depend on the level of another variable. Following Pellegrino (2017a,b) and Caggiano, Castelnuovo, and Pellegrino (2017), we focus on a parsimonious IVAR to maximize the available degrees of freedom while capturing the nonlinearity of interest.

Our IVAR is the following:

$$\mathbf{Y}_{t} = \boldsymbol{\alpha} + \sum_{j=1}^{L} \mathbf{A}_{j} \mathbf{Y}_{t-j} + \left[ \sum_{j=1}^{L} \mathbf{c}_{j} \ln VXO_{t-j} \times \Delta \ln GDP_{t-j} \right] + \mathbf{u}_{t}$$
 (1)

where  $\mathbf{Y}_t$  is the  $(n \times 1)$  vector of the endogenous variables,  $\boldsymbol{\alpha}$  is the  $(n \times 1)$  vector of constant terms,  $\mathbf{A}_j$  are  $(n \times n)$  matrices of coefficients, and  $\mathbf{u}_t$  is the  $(n \times 1)$  vector of error terms whose variance-covariance (VCV) matrix is  $\boldsymbol{\Omega}$ . The interaction term in brackets makes an otherwise standard VAR a non-linear IVAR model. Per each lag, such interaction term includes a  $(n \times 1)$  vector of coefficients  $\mathbf{c}_j$ , a measure of uncertainty  $\ln VXO_t$ , and an indicator of the business cycle  $\Delta \ln GDP_{t-j} \equiv \ln GDP_{t-j} - \ln GDP_{t-j-1}$ , which is the quarter-on-quarter growth rate of real GDP. The interaction term  $\ln VXO_{t-j} \times \Delta \ln GDP_{t-j}$  enables us to capture the potentially state-contingent effects of a shock to  $\ln VXO_{t-j}$  (i.e., an uncertainty shock) conditional on the state of the business cycle, which is proxied by the growth rate of real GDP. Alternatives to IVAR frameworks - such as, e.g., regime switching frameworks or smooth transition VARs - are available to capture the nonlinear effects of macroeconomic shocks (for a

recent survey, see Teräsvirta (2018)). We prefer to employ the above formalized IVAR framework for three reasons. First, it closely resembles the approximated nonlinear policy functions of nonlinear DSGE frameworks we work with.<sup>2</sup> Second, it naturally focuses on nonlinearities related to the business cycle, which is the research question under scrutiny. Third, it does not feature nuisance parameters, which are often difficult to estimate in nonlinear frameworks.<sup>3</sup>

We model the vector  $\mathbf{Y}_t = [\ln VXO, \ln GDP, \ln C, \ln I, \ln H, \ln P, FFR]'$ , where VXO denotes the stock market S&P 100 implied volatility index, GDP per capita GDP, C per capita consumption, I per capita investment, H per capita hours worked, P the price level, and FFR the federal funds rate. Uncertainty shocks are identified via a Cholesky decomposition of the reduced-form VCV matrix  $\Omega$ , with the VXO ordered first. This assumption implies that the VXO does not contemporaneously respond to first moment shocks like, e.g., technology and preference shocks. Importantly, this assumption is in line with the predictions of Basu and Bundick's model. In fact, while being technically endogenous, the VXO in their model is almost exclusively explained by second moment preference shocks, i.e., uncertainty shocks. Non surprisingly, an exercise conducted by simulating data with the Basu and Bundick model and estimating a VAR framework on such simulated data with shocks identified as described above returns VAR impulse responses which replicate those produced by the DSGE model. Appendix B documents this result, which enables use to estimate the parameters of the Basu and Bundick model with a direct inference approach (explained below). Appendix B also documents the correlation between second moment preference shocks and the model consistent VXO in the Basu and Bundick model, which confirms that the latter is clearly driven by the former.

We estimate our IVAR model with four lags over the 1962Q3-2017Q2 sample. Given that the VXO is unavailable before 1986, we follow Bloom (2009) and splice it with the within-month volatility of S&P500 daily returns, which has displayed an extremely high correlation with the VXO since 1986. The sample includes the zero lower bound period

<sup>&</sup>lt;sup>2</sup>Such nonlinear policy functions typically feature different, higher order interaction terms. We focus on terms featuring uncertainty and the real GDP growth because of our interest in the nonlinear effects to uncertainty shocks along the business cycle. Simulations conducted with higher order terms deliver even stronger empirical results in favor of such nonlinear effects. Appendix A documents the state-dependent impulse responses obtained with a framework involving a higher number of terms.

<sup>&</sup>lt;sup>3</sup>Notice that IVARs featuring interactions terms resemble approximated Smooth Transition VAR frameworks (Teräsvirta, Tjøstheim, and Granger (2010)).

<sup>&</sup>lt;sup>4</sup>The vector closely resembles the one used by Basu and Bundick (2017) in their linear VAR analysis, which also features the presence of money. Adding money implies no changes in our empirical results. The definition and construction of the variables is exactly the same as in Basu and Bundick (2017).

experienced by the Federal Reserve during the period 2008Q4-2015Q4. Appendix C shows that our results are robust to the employment of the shadow rate constructed by Wu and Xia (2016).

A standard likelihood-ratio test favors our IVAR specification against the Basu and Bundick's (2017) linear VAR model (which is nested in our IVAR model in case of the overall exclusion of the interaction terms from model (1)). In particular, the LR test suggests a value for the test statistic  $\chi_{28}=60.16$ , which allows us to reject the null hypothesis of linearity at any conventional statistical level in favor of the alternative of our I-VAR model (p-value <<0.01).

The interaction term of our IVAR is treated as an endogenous object. We compute GIRFs à la Koop et al. (1996) to account for both the endogenous response of the growth rate of per capita GDP, i.e. our conditioning variable, to the uncertainty shock and the feedback this reaction can imply on the dynamics of the economy. Theoretically, the GIRF at horizon h of the vector  $\mathbf{Y}_t$  to a shock of size  $\delta$  computed conditional on an initial history  $\boldsymbol{\varpi}_{t-1} = \{\mathbf{Y}_{t-1}, ..., \mathbf{Y}_{t-L}\}$  is given by the following difference of conditional expectations:

$$GIRF_{Y,t}(h, \delta_t, \varpi_{t-1}) = E\left[\mathbf{Y}_{t+h} \mid \delta, \varpi_{t-1}\right] - E\left[\mathbf{Y}_{t+h} \mid \varpi_{t-1}\right].$$

Along with this history-conditional GIRF, thanks to which we can recover one empirical response for each quarter in our sample, one can also define some states so that to recover state-conditional GIRFs summarizing the average evidence for a given state. We will use this latter approach with the purpose of comparing IVAR-based responses with DSGE-based responses, something clarified later on. In computing our GIRFs, we follow Auerbach and Gorodnichenko (2012), Caggiano, Castelnuovo, and Groshenny (2014), and Caggiano, Castelnuovo, Colombo, and Nodari (2015) and focus on "extreme events", i.e., deep recessions (strong expansions) characterized by initial conditions associated to realizations of the real GDP growth rate belonging to the first (tenth) decile of the empirical distribution. We do so to minimize the risk of confounding recessions and expansions, a risk one runs when considering initial conditions which are too close to the sample mean of the conditioning variable (which is, the growth rate of real GDP) (Ferrara (2003)). The GIRF conditional on a given state is computed by simulating the system starting from the average initial condition across the histories linked to the state, i.e., starting from the unconditional mean of each state. Conditional on being in a regime, this method is consistent with the way responses are usually computed for a nonlinear DSGE model, i.e. by simulating the model starting from the ergodic mean.

Appendix D describes the algorithm used to compute the GIRFs.

#### 2.2 Empirical results

Figure 1 plots the generalized impulse responses computed with our IVAR approach. A few facts stand out. First, there is evidence of a negative response of all real activity indicators to an uncertainty shock in both states. Second, the response of real activity is larger in recessions than in expansions. To fix ideas about this point, Table 2 collects figures regarding the peak (i.e., maximum, in absolute value) responses produced with our nonlinear VAR. The peak response of output in recessions is about 50% larger than in expansions. The same holds as regards consumption, whose peak reaction is 36% larger in contractions, and even more so for investment and hours, whose peak responses is more than twice as large. Third, the larger strength of the response in recessions regards the entire path of the short-run response of real activity after an uncertainty shock, and not only its peak reaction. Fourth, the synchronization of real activity indicators in response to an uncertainty shock differs between the two phases of the business cycle, with investment and hours reacting - in relative terms with respect to output - about 40% and 50% more in recessions than in expansions. Finally, the response of the policy rate is negative and persistent in both states of the business cycle, while that of the price level is not, i.e., the price level persistently decreases in recessions but increases in expansions.

Are these responses actually different from a statistical standpoint? Figure 2 shows the one standard deviation confidence bands of differences between the reactions in recessions and in expansions.<sup>5</sup> As evident from the figure, the responses of output, investment, and hours are significantly larger in recessions, an evidence which offers statistical support to the comovement-related facts motivated above. The reaction of consumption is only borderline significant, with the mass of the distribution which however hints to a larger response in recessions. Finally, also the response of the price level and the nominal interest rate is found to be significantly different between the two

<sup>&</sup>lt;sup>5</sup>Per each variable, the figure is based on the distribution stemming from 1,000 differences between responses in recessions and responses in expansions. Such responses are generated from 1,000 samples obtained via the standard residual-based bootstrap, computing - per each sample - the corresponding state-conditional GIRFs in recessions and expansions, and taking the difference between the latter and the former. The 16th and 84th percentiles of each density are then reported. The construction of the test statistic takes into account the correlation between the estimated impulse responses. Given the equal size of the shock in the two states per each given draw, the differences take - by construction - an on-impact zero value.

states.6

Overall, these results point to an economically and significantly stronger response of real activity to an uncertainty shock. This fact calls for the use of a structural model, something we do in the next Section.

# 3 Uncertainty-driven comovements: A structural interpretation

#### 3.1 DSGE model: Description

The Basu and Bundick (2017) model extends an otherwise standard New Keynesian model to consider an ex-ante second moment shock in the preference shock process, which can be interpreted as a demand-side uncertainty shock. We offer a brief description of the model here focusing on the parts that are crucial for our study, and refer the reader to Basu and Bundick's (2017) paper for further details.

Households work, consume, and invest in equity shares and one-period risk-free bonds. Households are all similar and are indexed with  $j \in [0, 1]$ . They feature Epstein-Zin preferences over streams of consumption and leisure, formalized as follows:

$$V_t = \left[ \left( a_t \tilde{C}_t^{\eta} (1 - N_{t,})^{(1-\eta)} \right)^{(1-\sigma)/\theta_V} + \beta (E_t V_{t+1}^{1-\sigma})^{1/\theta_V} \right]^{\theta_V/(1-\sigma)}$$

where  $\tilde{C}_t = C_t - H_t$ ,  $C_t$  is consumption,  $H_t = bC_{t-1}$  captures external habit formation in consumption related to the level of aggregate consumption lagged one period,  $N_t$  is hours worked,  $\beta$  is the discount factor,  $\sigma$  measures the degree of risk aversion,  $\psi$  is the intertemporal elasticity of substitution,  $\theta_V \equiv (1 - \sigma)/(1 - \psi^{-1})^{-1}$  captures households' preferences for the resolution of uncertainty,  $\eta$  weights consumption and labor in households' happiness function, and  $a_t$  is a stochastic shifter influencing the relevance of today's realizations of consumption and labor vs. those expected to occur during the next period.

<sup>&</sup>lt;sup>6</sup>These results are robust to the use of a recursive-design wild bootstrap (based on a Rademacher distribution) which is robust to possible conditional heteroskedasticity of unknown form (see Goncalves and Kilian (2004) for the proposal, and Kilian (2009) and Mertens and Ravn (2014) as examples of applications).

<sup>&</sup>lt;sup>7</sup>Basu and Bundick (2017) use a different set of preferences featuring distributional weights on current and future utility in the Epstein Zin time aggregator which do not sum to one. de Groot, Richter, and Throckmorton (2018) show that these preferences imply an asymptote in the responses to an uncertainty shock with unit intertemporal elasticity of substitution. This paper employs the set of preferences proposed by Basu and Bundick (2018), which do not imply any asymptote.

The stochastic process followed by this preference shock is:

$$a_t = (1 - \rho_a)a + \rho_a a_{t-1} + \sigma_{t-1}^a \varepsilon_t^a$$
  
$$\sigma_t^a = (1 - \rho_{\sigma^a})\sigma^a + \rho_{\sigma^a}\sigma_{t-1}^a + \sigma^{\sigma^a}\varepsilon_t^{\sigma^a}$$

where  $\varepsilon_t^a$  is the first-moment preference shock, and  $\varepsilon_t^{\sigma^a}$  is a second-moment uncertainty shock to the preference process which loads the law of motion regulating the evolution of the time-varying second moment  $\sigma_t^a$  relative to the distribution of  $\varepsilon_t^a$ .<sup>8</sup> The original framework by Basu and Bundick (2017) is modified to allow for (external) habit formation in consumption. We do so to capture the hump-shaped response of consumption in the data (for another paper jointly modeling Epstein-Zin preferences and habits in consumption, see Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2017)).

Intermediate goods producers rent labor from households, pay wages, and produce intermediate goods in a monopolistically competitive framework. They own capital and choose its utilization rate, issue equity shares and one-period riskless bonds, and invest in physical capital to maximize the discounted stream of their profits. In doing so, they face quadratic costs of adjusting nominal prices à la Rotemberg (1982), capital adjustment costs à la Jermann (1998) and capital utilization costs influencing the capital depreciation rate. All intermediate firms have the same Cobb-Douglas production function, and are subject to a fixed cost of production and stationary technology shocks. Intermediate goods are packed by a representative final goods producer operating in a perfectly competitive market. The model is closed by assuming that the central bank follows a standard Taylor rule.

In this framework, an uncertainty shock propagates to the economy via precautionary savings and precautionary labor supply. The former effect reduces current consump-

<sup>&</sup>lt;sup>8</sup>Epstein-Zin preferences written in this way imply a direct impact of the uncertainty shock on the current utility level only. This way of writing such preferences is not uncommon, see, on top of Basu and Bundick (2017), Albuquerque, Eichenbaum, Luo, and Rebelo (2016), Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2017). A note recently circulated by de Groot, Richter, and Throckmorton (2017) shows that these preferences imply a response of real activity indicators to an uncertainty shock which is discontinuous over the set of values the intertemporal elasticity of substitution  $\psi$  can take. When  $\psi = 1$ , the reaction of real activity is not defined, while values of  $\psi$  below (above) one are consistent with a negative (positive) response of real activity. While acknowledging the intellectually stimulating point made by de Groot, Richter, and Throckmorton (2017), we stress here that our paper is an empirical contribution whose aim is that of replicating the negative response of real activity to an uncertainty shock with an estimated model. In this sense, we see no clash with the findings in de Groot, Richter, and Throckmorton (2017).

<sup>&</sup>lt;sup>9</sup>Given that adjustment costs are convex, this model does not imply a "wait-and-see" effect after an uncertainty shock. To solve the model, we use perturbation methods which require policy functions to be differentiable, a feature which is not possessed by threshold policy functions arising in presence of real option effects.

tion in response to an increase in uncertainty, while the latter increases labor supply, which drives real wages and firms' marginal costs down. Given that prices are sticky, the price mark up increases. Output, which is demand-driven in this model, falls due to the drop in consumption, and labor demand contracts driving hours down. Given the lower return on capital, investment falls too. Hence, in equilibrium, an increase in uncertainty causes a drop in all four real activity indicators, i.e., output, consumption, investment, and hours, which is what we observe in the data.

As anticipated above, the model features a well-defined implied financial volatility concept. This is due to the fact that intermediate firms issue equity shares on top of one-period riskless bonds. Leach equity share has a price  $P_t^E$  and pays dividends  $D_t^E$ , implying a one-period return  $R_{t+1}^E = \left(P_{t+1}^E + D_{t+1}^E\right)/P_t^E$ . The model-implied financial uncertainty index  $V_t^M$  is computed as the annualized expected volatility of equity returns, i.e.,  $V_t^M = 100 \cdot \sqrt{4 \cdot VAR_t \left(R_{t+1}^E\right)}$ , where  $VAR_t \left(R_{t+1}^E\right)$  is the quarterly conditional variance of the return on equity  $R_{t+1}^E$ . Equity returns are endogenous in the model, which makes  $V_t^M$  endogenous too. However, in this model  $V_t^M$  is almost entirely driven by second-moment preference shocks for a variety of plausible calibrations. This enables us to treat the uncertainty shock as a financial uncertainty shock proxied by  $V_t^M$ , which has a clear empirical counterpart and which enable us to exploit the facts established in the previous Section to estimate the model.

We work with a third-order approximation of the nonlinear framework, which we solve via perturbation techniques (Schmitt-Grohe and Uribe (2004)). The third order approximation of agents' decision rules feature an independent role for uncertainty, whose independent effect on the equilibrium values of the endogenous variables of the framework can therefore be studied (Andreasen (2012)). Perturbation represents an accurate and fast way to find a solution also working with frameworks featuring recursive preferences (Caldara, Fernández-Villaverde, Rubio-Ramírez, and Yao (2012)).

### 3.2 Minimum-distance estimation strategy

We estimate Basu and Bundick's (2017) model via an impulse response function-matching approach, i.e., we choose the values of the structural parameters of the DSGE model

 $<sup>^{10}</sup>$ Basu and Bundick (2017) assume that firms finance a share  $\nu$  of their capital stock each period with one-period riskless bonds. Given that the Modigliani-Miller theorem holds in their model, leverage does neither influence firms' value nor firms' optimal decisions. Firms' leverage only influences the first two unconditional moments of financial-related quantities (e.g., the average level and unconditional volatility of the model-implied VXO and the equity premium), but it does not influences impulse responses to an uncertainty shock.

which minimize a measure of the distance between our IVAR impulse responses, which are interpreted as "data", and the DSGE model-based ones. Following Christiano, Trabandt, and Walentin (2011), we employ a Bayesian approach via which we impose economically sensible prior densities on the structural parameters while asking the data to shape the posterior density of the estimated model. With respect to Christiano et al. (2011), who focus on a linearized DSGE framework and a linear VAR as auxiliary model, we estimate a nonlinear DSGE framework approximated at a third order with moments produced with an Interacted VAR.<sup>11</sup>

The state-dependent Bayesian minimum distance estimator works as follows. Denote by  $\widehat{\psi}^i$  the vector in which we stack the I-VAR estimated generalized impulse responses over a 20-quarter horizon to an uncertainty shock for each regime i=1,2 (i.e., the responses displayed in Figure 2).<sup>12</sup> When the number of observations per each regime  $n^i$  is large, standard asymptotic theory suggests that

$$\widehat{\boldsymbol{\psi}^{i}} \stackrel{a}{\sim} N(\boldsymbol{\psi}\left(\boldsymbol{\zeta}_{0}^{i}\right), \mathbf{V}^{i}(\boldsymbol{\zeta}_{0}^{i}, n^{i})), \text{ for } i = 1, 2$$
 (2)

where  $\zeta_0^i$  denotes the true vector of structural parameters that we estimate (i=1,2) and  $\psi(\zeta^i)$  denotes the model-implied mapping from a vector of parameters to the analog impulse responses in  $\widehat{\psi}^i$ .

As explained earlier, the IVAR GIRFs  $\widehat{\psi}^i$  conditional on a given state i are computed by iterating forward the system starting from the average initial condition across the histories linked to the state, i.e., starting from the unconditional mean of each state. Similarly, we compute the DSGE model-related responses per each given set of parameter values  $\psi(\zeta^i)$  by iterating forward the approximated solution of the DSGE model starting from the (state-specific in our case) stochastic steady state.<sup>13</sup> Both DSGE-based

<sup>&</sup>lt;sup>11</sup>One way of interpreting this exercise is to think of a regime-switching type of estimation in which we allow the parameters of the nonlinear DSGE model to be state-dependent. Bianchi and Melosi (2017) formally model policy-related uncertainty with a regime-switching approach which allows agent to formulate a prediction over future regime switches in an empirical framework where the DSGE model is a linearized framework within each state. A challenge for future research is how to conduct such an exercise with a nonlinear DSGE model like the one we work with.

<sup>&</sup>lt;sup>12</sup>For a paper proposing information criteria to select the responses that produce consistent estimates of the true but unknown structural parameters and those that are most informative about DSGE model parameters, see Hall, Inoue, Nason, and Rossi (2012).

<sup>&</sup>lt;sup>13</sup>Following Basu and Bundick (2017), we set the value of the exogenous processes to zero and iterate forward until the model converges to its stochastic steady state. Then, we hit the model with a one standard deviation uncertainty shock and compute impulse responses as the percent deviation between the stochastic path followed by the endogenous variables and their stochastic steady state. Given that no future shocks are considered, this way of computing GIRFs does not line up with Koop, Pesaran and Potter's (1996) algorithm. We do so to avoid simulating the model several times and

and VAR-based impulse responses are interpreted as percent deviations of variables induced by an uncertainty shock, with the exception - in our case - of the interest rate response which is measured in percentage points as implied by the VAR specification.

To compute the posterior density for  $\zeta^i$  given  $\widehat{\psi}^i$  using Bayes' rule, we first need to compute the likelihood of  $\widehat{\psi}^i$  conditional on  $\zeta^i$ . Given (2), the approximate likelihood of  $\widehat{\psi}^i$  as a function of  $\zeta^i$  reads as follows:

$$f(\widehat{\boldsymbol{\psi}^{i}}|\boldsymbol{\zeta}^{i}) = \left(\frac{1}{2\pi}\right)^{\frac{N^{i}}{2}} \left| \mathbf{V}^{i}(\boldsymbol{\zeta}_{0}^{i}, n^{i}) \right|^{-\frac{1}{2}} \times \exp\left[ -\frac{1}{2} \left( \widehat{\boldsymbol{\psi}^{i}} - \boldsymbol{\psi} \left( \boldsymbol{\zeta}^{i} \right) \right)^{'} \mathbf{V}^{i}(\boldsymbol{\zeta}_{0}^{i}, n^{i})^{-1} \left( \widehat{\boldsymbol{\psi}^{i}} - \boldsymbol{\psi} \left( \boldsymbol{\zeta}^{i} \right) \right) \right]$$

$$(3)$$

where  $N^i$  denotes the number of elements in  $\widehat{\psi}^i$  and  $\mathbf{V}^i(\zeta_0^i,n^i)$  is treated as a fixed value. We use a consistent estimator of  $\mathbf{V}^i$ . Because of small sample-related considerations, such estimator features only diagonal elements (see Christiano, Trabandt, and Walentin (2011) and Guerron-Quintana, Inoue, and Kilian (2017)).<sup>14</sup> In our case,  $\mathbf{V}^i$  is a regime-dependent diagonal matrix with the variances of the  $\widehat{\psi}^i$ 's along the main diagonal<sup>15</sup>. This choice is widely adopted in the literature and allows one to put more weight in replicating VAR-based responses with relatively smaller confidence bands. Treating eq. (3) as the likelihood function of  $\widehat{\psi}^i$ , it follows that the Bayesian posterior of  $\zeta^i$  conditional on  $\widehat{\psi}^i$  and  $\mathbf{V}^i$  is:

$$f(\boldsymbol{\zeta}^i|\widehat{\boldsymbol{\psi}^i}) = \frac{f(\widehat{\boldsymbol{\psi}^i}|\boldsymbol{\zeta}^i)p(\boldsymbol{\zeta}^i)}{f(\widehat{\boldsymbol{\psi}^i})},\tag{4}$$

then integrate across all simulations, a procedure which would be very time consuming, above all when combined with the MCMC algorithm we adopt for our Bayesian estimation. Basu and Bundick (2017) show that the differences between these two ways of computing GIRFs are negligible with a framework like theirs. We also verified that our IVAR GIRFs remained unchanged when future shocks are not taken into account, something which augments the comparability between IVAR and DSGE GIRFs. Analytical expressions for GIRFs produced with nonlinear models are available in Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2017).

<sup>14</sup>Guerron-Quintana, Inoue, and Kilian (2017) studie the asymptotic theory for VAR-based impulse response matching estimators of the structural parameters of linearized DSGE models when the number of impulse responses exceeds the number of linear VAR model parameters. The number of impulse responses in our analysis (140) is lower than the number of estimated coefficients of the VAR (251, constants excluded). We are aware of no contributions studying the asymptotic theory for this estimator when nonlinear frameworks are employed.

<sup>15</sup>Denoting by  $\hat{\mathbf{W}}^i$  the bootstrapped variance-covariance matrix of VAR-based impulse responses  $\hat{\boldsymbol{\psi}}^i$  for regime i, i.e.,  $\frac{1}{M}\sum_{j=1}^{M}(\psi_j^i-\bar{\psi}^i)(\psi_j^i-\bar{\psi}^i)'$  (where  $\psi_j^i$  denotes the realization of  $\hat{\boldsymbol{\psi}}^i$  in the  $j^{th}$  (out of M=1000) bootstrap replication and  $\bar{\boldsymbol{\psi}}^i$  denotes the mean of  $\psi_j^i$ ),  $\mathbf{V}^i$  is based on the diagonal of this matrix. Notice that  $\mathbf{V}^i$  contains the same variances that will be used to plot the confidence intervals for the I-VAR responses in next Section. This is the same approach used in Altig, Christiano, Eichenbaum, and Lindé (2011).

where  $p(\zeta^i)$  denotes the priors on  $\zeta^i$  and  $f(\widehat{\psi}^i)$  is the marginal density of  $\widehat{\psi}^i$ . The mode of the posterior distribution of  $\zeta^i$  is computed by maximizing the value of the numerator in 4. The posterior distribution of  $\zeta^i$  is computed using a standard Markov Chain Monte Carlo (MCMC) algorithm.

We estimate 8 structural parameters, i.e.  $\zeta^i = \left[ \rho_{\sigma^a}, \sigma, b, \phi_K, \phi_P, \rho_R, \rho_\pi, \rho_y \right]$ . These parameters are the persistence of the second moment preference shock  $\rho_{\sigma^a}$ , the household risk aversion parameter  $\sigma$ , the consumption habit formation parameter b, the parameter regulating investment adjustment costs  $\phi_K$ , the parameter regulating price adjustment costs  $\phi_P$ , and the parameters of the Taylor rule  $\rho_R, \rho_\pi, \rho_y$  which - respectively - capture the degree of interest rate smoothing and the systematic response to inflation and output growth. Our priors are described in columns 3-4 of Table 3. We calibrate our prior means with the parameters used in Basu and Bundick's (2017) analysis, and we use diffuse priors. For the habit formation parameter and the parameters of the Taylor rule, we use the same priors employed by Christiano et al. (2011). The remaining parameters of the model are calibrated as in Basu and Bundick (2017). We confine a discussion on the calibration of these parameters to Appendix E for the sake of brevity.

#### 3.3 Regime-specific estimation results

Our state-conditional model-based responses are reported in Figures 3 and 4 along with the IVAR-based bootstrapped confidence bands.<sup>17</sup> The model captures remarkably well the VAR dynamics in both regimes. Most of the DSGE impulse responses lie within the 68% confidence bands of the IVAR impulse responses. The model is able to replicate the stronger responses of real variables during contractions as well as the fact that their responses are longer-lived than responses in expansions. While working well for output, consumption, and investment, a note on the response of hours in recessions produced

<sup>&</sup>lt;sup>16</sup>Canova and Sala (2009) show that the use of priors can hide identification issues even in population when it comes to estimating linearized DSGE frameworks. Given that we use priors common to the two regimes we focus on, lack of identification would work against finding state-dependent parameter estimates. We anticipate that our results point to substantial differences in the parameter estimates between regimes. An exercise dealing with identification issues in the estimation of nonlinear DSGE frameworks is material for future research.

 $<sup>^{17}\</sup>text{Our}$  bootstrapped confidence bands are based over 1,000 bootstrapped realizations for the impulse responses, which are used to compute the bootstrapped estimate of the standard errors of the impulse response functions. As in Altig, Christiano, Eichenbaum, and Lindé (2011), the confidence bands are constructed by considering the point estimates of the impulse response  $\pm 1.64$  times the bootstrapped estimate of the standard errors.

by the DSGE model is warranted. The model is able to generate a persistent contractionary response in hours worked whose shape is similar to the one produced by the IVAR framework. However, it falls short in replicating the magnitude of the response, above all during recessions. This issue, which we share with Basu and Bundick (2017), may be due to various reasons. First, as pointed out by Basu and Bundick (2017), the dynamics of hours worked in the data during recessions is substantially influenced by low-productive types, which tend to quickly exit the labor market and whose dynamics drive hours worked on aggregate. By contrast, the model we work with features a representative agent. Hence, by construction, it is ill-suited to capture aggregate dynamics driven by heterogeneities in the labor market. Second, the model predicts an expansions in precautionary labor supply which is contrasted by the contraction of labor demand due to the weakened demand for goods. A strong effect on hours by the expansion in labor supply makes the life of the model hard when it comes to generating a contraction in the equilibrium level of hours. Third, the labor market model in this framework features no relevant rigidities. Leduc and Liu (2016) show that search frictions work in favor of magnifying the effects of uncertainty shocks on labor market indicators. Cacciatore and Ravenna (2018) show that combining matching frictions with an occasionally binding constraint on downward wage adjustment exhacerbates the negative effects of uncertainty shocks on the labor market. Our choice of working with a flexible labor market makes our results more directly comparable to those in Basu and Bundick (2017). Moreover, while falling short from a quantitative standpoint, the model is clearly able to generate comovements involving also hours worked in recessions.

The overall good performance of the model can also be appreciated by looking at Table 2, which compares the peak responses produced by the DSGE framework with the data. The model is clearly able to generate a relatively strong response of all real activity indicators in recessions with respect to expansions.

Turning to the nominal side, the performance of the model is admittedly less successful. The response of prices is, in general, not well captured by the DSGE model. The reason is that this model features an upward pricing bias (also present in other frameworks, see Born and Pfeifer (2014a), Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2015), and Mumtaz and Theodoridis (2016)). This upward pricing bias, which relates to the uncertainty over future profits faced by entrepreneurs when setting their prices, contrasts the price effects of the contraction in real activity. Consequently, the model response of the federal funds rate is also milder than in the data. However, the lack of adherence of the model with the facts does not prevent the DSGE

framework to replicate the generalized fall in real activity after an uncertainty shock in the two states of interest, which is what we focus on in this paper.

Table 3 (last two columns) collects the estimated parameters of the DSGE model for both regimes. In spite of sharing the same priors, some of the estimated parameters are clearly state-dependent. <sup>18</sup> In particular, household's risk aversion is estimated to be larger in recessions; prices are found to be stickier during recessions. On the other hand, the persistence of the second moment preference shock  $\rho_{\sigma^a}$  is estimated to be the same between states. This implies that the different effects of uncertainty shocks in model-based responses are fully due to a different propagation mechanism which is only explained by differences in deep structural parameters. The degree of habits in consumption b and the parameter regulating investment adjustment costs  $\phi_K$  are estimated to be basically the same in the two states. Given the difference between the prior means on the parameters and their posterior means, as well as the smaller posterior standard deviation with respect to prior standard deviations, this result does not seem to be driven by an identification issue. Finally, the estimated policy rule signals a mildly stronger response to inflation and output in expansions, where monetary policy is also found to be more inertial. However, the differences between estimated policy parameters appear to be small.

#### 3.4 Role of risk aversion

Going back to the parameter instabilities detected by our econometric exercise, one may wonder what the contribution of each single parameter is to the different responses produced by our DSGE model in recessions and expansions. We then check the impact of each parameter on the impulse responses produced by the DSGE model as follows. Conditional on the set of estimates in expansions, we replace the value of each parameter with the corresponding estimated value in recessions. To be sure, the way in which the exercise is designed is such that, if we replaced all estimated parameters contemporaneously, by construction we would replicate the impulse responses produced by the DSGE in recessions. Appendix H documents the outcome of this exercise. Among the unstable parameters of the model, the dominant role is played by risk aversion. In line of this finding, we re-estimate the DSGE model by allowing only the degree of risk aversion to be state-dependent, while the other parameters take the estimated values

<sup>&</sup>lt;sup>18</sup>Appendix F plots priors and posterior densities of the estimated parameters. While not being a sufficient condition for identification, it is interesting to notice that priors and posteriors are clearly different. Appendix G reports detailed convergence diagnostics for the MCMC estimation.

conditional on the impulse responses produced with the linear version of the VAR. The estimated parameters conditional on the linear VAR are presented in Table 4 (fourth column). When we allow only the risk aversion parameter to vary we get estimates of 150.36 and 83.90 for recessions and expansions, respectively. These figures, however, do not take into account the role that endogenous labor supply and habit formation play in affecting the coefficient of relative risk aversion. Swanson (2012) proposes closed-form expressions for risk aversion that take into account the role played by adjustments in household's labor supply. Swanson (2018) extends this analysis to the case of generalized recursive preferences, which include Epstein-Zin preferences. Following Swanson (2018), we compute the value of the relative risk aversion corresponding to our estimates of the parameter  $\sigma$  and conditional on the structure of the economy we deal with. We obtain a value of 85 for the coefficient of relative risk aversion in recessions, and 56 in expansions. These values are in the ballpark of the calibrated (75) and estimated (110) ones in Rudebusch and Swanson (2012). However, these levels of risk aversion are high with respect to those typically used in the macroeconomic literature. A possible reason is the lack of model uncertainty in our framework. Barillas, Hansen, and Sargent (2009) employ a max-min expected utility theory approach to show that models with high risk aversion in which rational agents are endowed with the knowledge of the true underlying structure of the economy can be reinterpreted as frameworks in which risk aversion is low but households have doubts about the model specification. Our model does not embed any doubts about the underlying economy by households. Therefore, it is likely to understate the true quantity of risk faced by households in the data, which is the reason why it requires high levels or risk aversion to match the VAR facts.

Figures 3 and 4 plots the impulse responses produced by the version of the DSGE model in which risk aversion is the only parameter free to adjust between states over the responses produced by allowing all parameters to adjust. Evidently, risk aversion, which is estimated to be countercyclical, does the job by itself.

Wrapping up, our empirical investigation reveals that a DSGE model estimated by matching facts produced with a linear VAR does a good job in replicating the impulse responses produced with a nonlinear VAR framework and related to recessions and expansions. However, such a good job occurs if risk aversion is actually allowed to change between states. In particular, risk aversion is estimated to be larger in recessions. Interestingly, a countercyclical risk aversion has recently been advocated by Cochrane (2017) as a feature macro-finance models should possess to match the data. Cohn, Engelmann, Fehr, and André Maréchal (2015) provide experimental evidence suggesting that finan-

cial market professionals are more risk averse during a financial bust than a boom. This evidence suggests that fear may play an important role in explaining countercyclical risk aversion. The same conclusion is reached by Guiso, Sapienza, and Zingales (2017), who provide experimental evidence in favor of a fear model in which agents experience higher risk aversion in periods of crisis. Kim (2014) estimates a consumption-based capital asset pricing model with time-varying risk aversion based on the Epstein–Zin recursive utility, and finds strong support for the countercyclicality of the risk aversion parameter. Our paper lines up with these contributions by identifying countercyclical risk aversion as crucial to replicate the asymmetric response of real activity to uncertainty shocks along the business cycle. As conjectured by Cohn, Engelmann, Fehr, and André Maréchal (2015), we find that risk aversion amplifies economic dynamics in response to a shock.<sup>19</sup>

The ability of the estimated model to replicate the empirical facts produced with our nonlinear VAR depends exclusively from the instability in the risk aversion parameter. In fact, the role played by initial conditions in the nonlinear DSGE model we work with is empirically negligible. Appendix I shows this by computing the DSGE-based (state-) conditional GIRFs as defined in Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2017). Conditional GIRFs relative to uncertainty shocks are found to be quantitatively insensitive to different initial conditions. Hence, to replicate the different real effects of uncertainty shocks, the estimated DSGE model has to rely on parameter instability. The message in this paper is that instability in risk aversion is empirically found to be necessary and sufficient for the DSGE model to replicate the facts.

## 4 The great recession

So far, our analysis has shown that a nonlinear DSGE model estimated with an auxiliary nonlinear VAR framework goes a long way in replicating the asymmetric reaction of real activity indicators to an uncertainty shock. In performing this exercise, the reference facts we considered regarded the different responses of real activity in recessions and expansions in general. However, the recent great recession is clearly a rare event which

<sup>&</sup>lt;sup>19</sup>Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2017) show that, in a model featuring a portfolio allocation problem related to short- and long-term bonds plus a systematic response of the central bank to the term spread, uncertainty shocks to households' preferences generate moments consistent with the data even in presence of moderate values of risk aversion. The moments studied by Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2017) are, however, unconditional moments, i.e., they are not state-specific.

hardly falls under the "standard recessions" category. A way to lend support to this statement is to exploit the flexibility of our nonlinear VAR, which allows us to move from an analysis between states to an analysis across dates. This is possible because of the dependence of impulse responses to initial conditions in nonlinear models (Koop, Pesaran, and Potter (1996)). We then work out the temporal evolution of the peak response of GDP, consumption, investment and hours worked over a five-year horizon to an equally sized uncertainty shock by associating to each quarter an uncertainty shock occurring in each initial quarter of our sample.<sup>20</sup>

Figure 5 displays the outcome of this exercise. The peak responses are much higher (in absolute terms) in recessions, a finding in line with the empirical facts documented in Section 2. Moreover, the peaks point to a particularly strong response of real activity in three recessions, i.e., 1974-1975, 1981-1982, and 2007-2009. Clearly, the great recession is an outlier, i.e., all four real activity indicators display their maximum responses (in absolute terms) during that extreme event. When compared to the peaks predicted by a linear VAR model, the nonlinear framework returns figures regarding the great recession which are between two and three times larger.

Can our structural DSGE model replicate the drop in real activity occurred during the great recession? While to replicate the average effects of uncertainty shocks in recession an adjustment of the calibration of the risk aversion parameter may be enough, for this particular recession some other characteristic of the model may also need to adjust. In fact, the beginning of the great recession called for a dramatic cut of the policy rate, which moved from 5.25% in July 2008 to basically zero in just five months. Hence, one would think that also the Taylor rule parameters should adjust to have the model replicate the spectacular contraction experienced by the U.S. economy. To investigate this issue, we then focus on 2008Q4, the quarter associated to the largest realization of the uncertainty shock according to our IVAR model, and estimate our DSGE model conditional on the IVAR responses for this quarter.

Table 4 (last three columns) collects the estimates relative to different versions of the estimated model, i.e., one which just allows risk aversion to change; one which allows risk aversion and the policy parameters to adjust; and one which allows all parameters to adjust. The information reported in the table confirms that changes in the estimated values of risk aversion and the policy parameters are needed to replicate the great recession.

<sup>&</sup>lt;sup>20</sup>The figure with the cumulative responses of real activity over a five-year horizon delivers the very same qualitative message.

Equipped with this estimated framework, we conduct an exercise which aims at quantifying the contribution of uncertainty shocks for the cumulative output loss recorded by the U.S. economy during and after the great recession. To do so, we follow Basu and Bundick (2017) and take as an external reference the CBO output gap, which is a "detrended" measure of output.<sup>21</sup> The though experiment goes as follows. Assume the economy to be at its stochastic steady state before the advent of the large uncertainty shock in 2008Q4. Then, the real world is hit by the large uncertainty shock in 2008Q4, as well as a variety of other shocks. We produce the response of output with our estimated DSGE model in which risk aversion and the Taylor rule parameters are free to adjust. To appreciate the role of nonlinearities for the calibration of the DSGE model, we also produce the response of output to an equally sized uncertainty shock with a version of the DSGE model estimated conditional on the impulse responses produced with a linear VAR. We consider responses to a 4.5 standard deviation uncertainty shock, which is the one estimated by our VAR for the 2008Q4. We then contrast the predictions of these differently calibrated DSGE frameworks with the the cumulative detrended output loss (with detrended output normalized to zero in 2008Q3) in 2008Q4-2014Q2, which is the period during which detrended output recorded negative realizations.

Figure 6 compares the responses produced with our favorite DSGE models, i.e., the one estimated on the nonlinear VAR, a version of the DSGE model estimated conditional on a linear VAR, and the evolution of detrended output (normalized to zero in 2008Q3, i.e., before the shock hits). The cumulative loss of output in the period considered here is equal to 53% (with respect to the trend). Our nonlinear DSGE model estimates an uncertainty shock in 2008Q4 to be associated to a cumulative output loss equal to 24%, which is almost 1/2 of the total loss. This is twice as much the loss predicted by a DSGE model estimated conditional on a linear VAR, which is, 13%. Allowing for the risk aversion only parameter to adjust would still imply a good prediction of the peak response of output after the shock. However, it would also imply a too quick return of predicted output to the steady state, which would lead to an underestimation of the output loss caused by the uncertainty shock.

<sup>&</sup>lt;sup>21</sup>The evolution of the CBO output gap is quantitatively very similar to that of the Hodrick-Prescott filtered log real GDP (smoothing weight: 1,600).

### 5 Conclusion

This paper employs a nonlinear VAR framework to document that financial uncertainty shocks exert a stronger effect on real activity in recessions. A nonlinear structural DSGE model is fitted to the data relative to booms and busts to interpret this evidence. The estimation of the DSGE model is conducted by working with a Bayesian direct inference approach, which is applied in a novel manner to a nonlinear DSGE structure. Counter-cyclical risk aversion is identified as the key element that enables the DSGE model to replicate the empirical facts. Focusing on the great recession, we show that a combination of high risk aversion and weak, low-inertial response to inflation is sufficient for the structural model to replicate the response of real activity in 2008Q4 produced by the nonlinear VAR. When estimated targeting such response, the model assigns about 50% of the output loss materialized in 2008-2014 to a big financial uncertainty shock occurred at the end of 2008. The same DSGE model estimated by targeting the impulse responses produced by a linear VAR is shown to substantially downplay the role played by such shock.

This paper offers solid support to Federal Reserve's former Chairman Alan Greenspan view on macroeconomic modeling: "[...] it is apparent that a prominent shortcoming of our structural models is that, for ease in parameter estimation, not only are economic responses presumed fixed through time, but they are generally assumed to be linear. An assumption of linearity may be adequate for estimating average relationships, but few expect that an economy will respond linearly to every aberration." Our results stress the importance of using nonlinear frameworks for correctly quantifying the effects of uncertainty shocks - and, more generally, macroeconomic shocks - for the U.S. business cycle. Nonlinear VARs can fruitfully be used to establish facts which serve as a reference to build and evaluate structural frameworks used to conduct policy analysis. Correctly calibrating such frameworks is of paramount importance to conduct informative historical and policy analysis.

Our paper shows that a state-of-the-art nonlinear DSGE model is able to explain the state-dependent response of real activity to an uncertainty shock via the instability of the estimated risk aversion parameter. We see this result as informative for the construction of theoretical models featuring endogenous mechanisms able to replicate the nonlinear effects of uncertainty shocks.

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	Peak re	esponses rela	ative to s	trong expansions
	Deep c	ontractions	Stro	ng expansions
Variable	VAR	$\overline{\text{DSGE}}$	VAR	DSGE
Y	1.49	1.47	1	1
$\mathbf{C}$	1.32	1.49	1	1
I	2.15	1.48	1	1
Н	2.19	1.49	1	1
	P	eak response	es relativ	e to output
	Deep c	ontractions	Stro	ng expansions
Variable	VAR	DSGE	VAR	DSGE
Y	1	1	1	1
$\mathbf{C}$	0.55	0.58	0.62	0.58
I	3.33	2.29	2.29	2.29
H	1.41	0.64	0.96	0.63

Table 1: IVAR responses: Relative moments. Sample: 1962Q3-2017Q2. VAR estimated with four lags.

		$ ho_{ m I}$	Priors	Post	Posteriors
				Deep contractions	Strong Expansions
		Distribution	Mean, std.dev.	Mean, std.dev.	Mean, std.dev.
Parameter	Parameter Interpretation	[spunoq]		[5%  and  95%]	[5%  and  95%]
$ ho_{\sigma^a}$	Pers. unc, shock	Beta	0.77, 0.10	<b>0.67</b> , 0.02	<b>0.64</b> , 0.02
		[0,1]		[0.64, 0.70]	[0.60, 0.69]
Ο	HHs' risk av.	Gamma	100, 60	<b>313.05</b> , 42.07	<b>204.30</b> , 43.45
		[0,8]		[241.61, 379.90]	[132.14, 274.36]
q	Habit form.	Beta	0.75, 0.15	0.75, 0.03	<b>0.67</b> , 0.07
		[0,1]		[0.69, 0.81]	[0.55, 0.79]
$\phi_K$	Inv. adj. costs	Gamma	3.92, 2	1.55, 0.23	<b>1.20</b> , 0.26
		$[0,\infty]$		[1.14, 1.94]	[0.65, 1.73]
$\phi_P$	Pr. adj. costs	Normal	240, 40	<b>290.74</b> , 35.42	<b>225.07</b> , 41.98
		$[0,\infty]$		[230.89, 347.92]	[159.58, 293.56]
$ ho_{\pi}$	TR par., infl.	Gamma	1.5, 0.25	<b>1.09</b> , 0.01	<b>1.06</b> , 0.01
		[1.01, 4]		[1.07, 1.11]	[1.04, 1.08]
$ ho_y$	TR par., out. gr.	Gamma	0.20, 0.15	<b>0.14</b> , 0.02	<b>0.11</b> , 0.02
s		[0,2]		[0.11, 0.18]	$[\ 0.07,\ 0.15]$

Table 2: Regime-dependent estimated parameter values. Posteriors computed via MCMC with a random walk metropolis algorithm. 60 000 draws, 20 percent for burn-in. Acceptance rates: 27 percent for times of deep contractions,  $26\ \mathrm{percent}$  for times of strong contractions.

		Priors		Posteriors	
			Linear VAR	Great Recession	ecession
			All param.	$\sigma$ and $\rho_{\sigma^a}$	All param.
Parameter	Interpretation	D(mean,std) Mean,std	Mean,std	Mode,std	Mode,std
$ ho_{\sigma^a}$	Unc.shock,pers.	B(0.77,0.10)	<b>0.66</b> , 0.02	<b>0.70</b> , 0.01	<b>0.70</b> , 0.01
$\sigma$	Risk aversion	G(100,60)	<b>283.82</b> , 41.58	<b>372.64</b> , 15.43	<b>404.46</b> , 39.16
q	Habits	B(0.75,15)	<b>0.73</b> , 0.04	ı	<b>0.77</b> , 0.02
$\phi_K$	Inv. adj. costs	G(3.92,2)	<b>1.49</b> , 0.24	I	<b>1.60</b> , 0.18
$\phi_P$	Price adj. costs	N(240,60)	<b>269.06</b> , 36.43	I	<b>326.33</b> , 31.52
$ ho_{\pi}$	TR par., inflat.	G(1.5,0.15)	<b>1.08</b> , 0.01	I	<b>1.09</b> , 0.01
$\rho_y$	TR par., out. gr.	G(0.2,0.15)	<b>0.14</b> , 0.02	1	<b>0.15</b> , 0.02

Table 3: DSGE model: Average evidence vs. Great recession. Values estimated conditional on both the linear VAR impulse responses and on the IVAR impulse responses in 2008Q4.

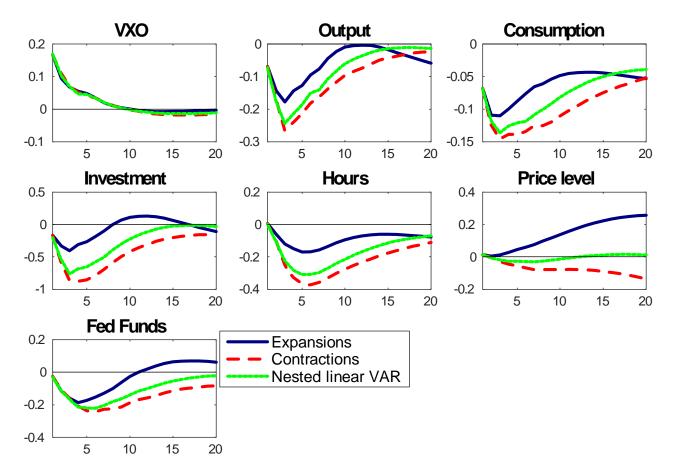


Figure 1: **IVAR** impulse responses to an uncertainty shock in recessions and **expansions.** Red line: Deep recessions. Blue line: Strong expansions. Green line: Responses associated to the nested linear VAR. Sample: 1962Q3-2017Q2. VAR estimated with four lags.

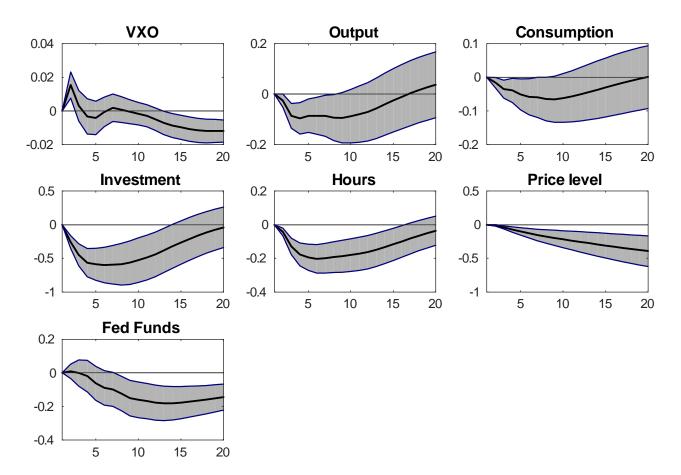


Figure 2: Differences of the IVAR impulse responses to an uncertainty shock: Recessions vs. expansions. Solid black lines: difference between the point estimated state-conditional GIRFs in recessions and expansions (taking the difference between the latter and the former). Grey areas: 68 percent confidence bands of the difference (from its distribution constructed with 2,000 bootstrap draws). Sample: 1962Q3-2017Q2. VAR estimated with four lags.

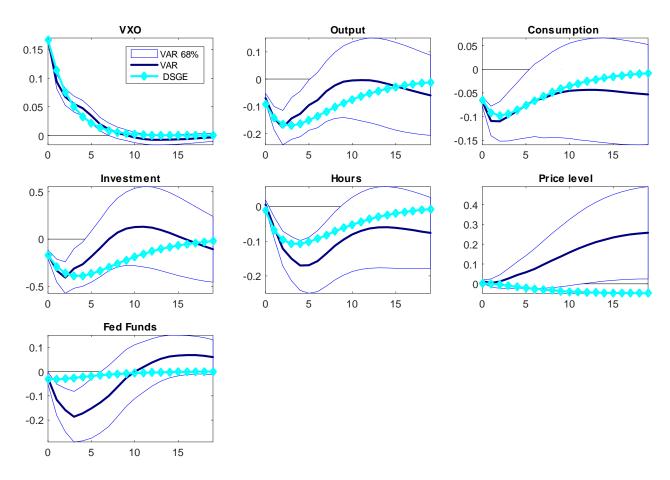


Figure 3: **DSGE vs. IVAR impulse responses to an uncertainty shock: Expansions.** Solid lines with squares: Responses of the DSGE model estimated by allowing only risk aversion to adjust between recessions and expansions. Solid lines with diamonds: Responses of the DSGE model estimated by allowing all parameters to adjust between recessions and expansions. Areas identified by blue lines: 68% confidence interval produced with the IVAR. Sample: 1962Q3-2017Q2. VAR estimated with four lags.

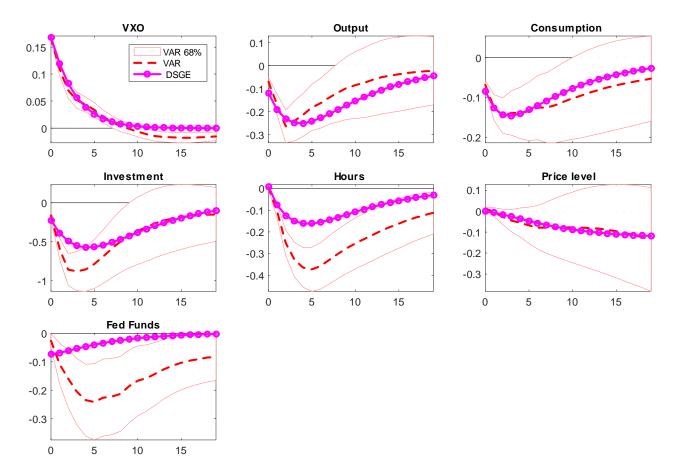


Figure 4: **DSGE vs. IVAR impulse responses to an uncertainty shock: Recessions.** Solid lines with squares: Responses of the DSGE model estimated by allowing only risk aversion to adjust between recessions and expansions. Solid lines with diamonds: Responses of the DSGE model estimated by allowing all parameters to adjust between recessions and expansions. Areas identified by blue lines: 68% confidence interval produced with the IVAR. Sample: 1962Q3-2017Q2. VAR estimated with four lags.

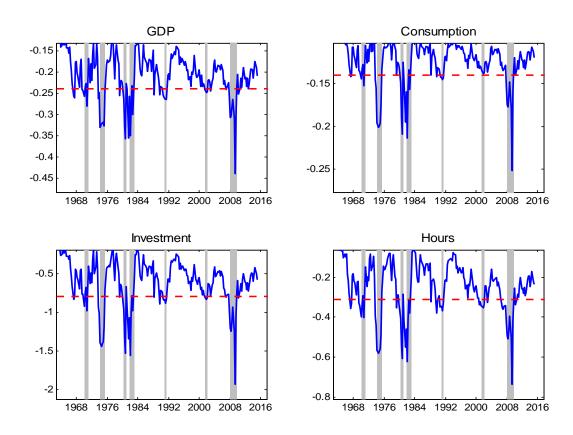


Figure 5: **IVAR time-varying impulse responses to an uncertainty shock.** Blue lines: Peak responses over a five-year horizon. Red lines: Peak responses as computed by a linear VAR. Sample: 1962Q3-2017Q2. VARs estimated with four lags.

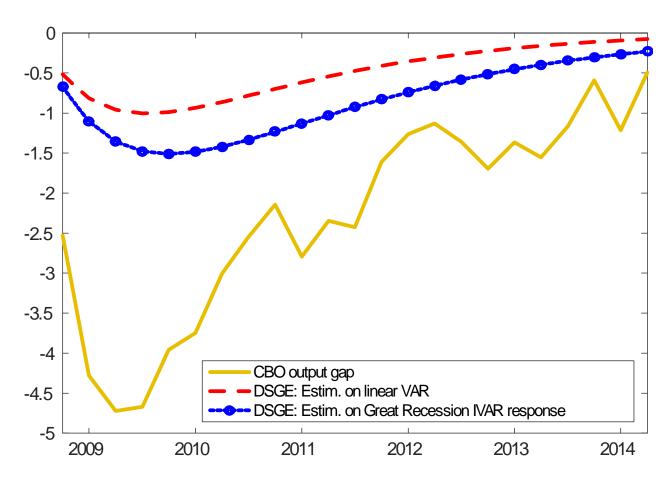


Figure 6: **Great Recession: Contribution of uncertainty shocks.** Output gap normalized to zero in 2008Q3. DSGE: Estim on Great Recession IVAR (only sigma) and (only sigma and rho\_a): Non-estimated parameters calibrated at the value found when estimating the model with a linear VAR.

# **Appendix**

#### A: Parsimonious vs. extended IVAR

The IVAR model employed in the paper is a parsimonious version of a more sophisticated IVAR which we estimated to check the robustness of our results. Thinking of the third-order approximation of the DSGE model we work with, it is natural to extent our baseline IVAR framework to add extra interaction terms involving quadratic terms as follows:

$$\mathbf{Y}_{t} = \boldsymbol{\alpha} + \sum_{j=1}^{L} \mathbf{A}_{j} \mathbf{Y}_{t-j} + \begin{bmatrix} \sum_{j=1}^{L} \mathbf{c}_{j} \ln VXO_{t-j} \times \Delta \ln GDP_{t-j} \\ + \sum_{j=1}^{L} \mathbf{c}_{j} (\ln VXO_{t-j})^{2} \times \Delta \ln GDP_{t-j} \\ + \sum_{j=1}^{L} \mathbf{c}_{j} \ln VXO_{t-j} \times (\Delta \ln GDP_{t-j})^{2} \end{bmatrix} + \mathbf{u}_{t}$$

Cubic terms  $((\ln VXO_{t-j})^3, (\Delta \ln GDP_{t-j})^3)$  are omitted to minimize the risk of explosiveness.

Figure A1 contrasts the impulse responses obtained with our baseline model with those produced with the enriched framework. If anything, the reactions produced by this framework speak even more clearly in favor of nonlinearities in the data.

#### B: Match between VAR and DSGE model

The Basu and Bundick (2017) has a structure which is de facto consistent with the assumptions undertaken in Section 2 which regard the identification of uncertainty shocks. In our recursively identified nonlinear VAR model the VXO is ordered first and hence it is assumed that, while uncertainty shocks can contemporaneously affect all variables in the VAR, the VXO cannot be contemporaneously affected by other shocks. The Basu and Bundick (2017) model features an endogenous measure of financial uncertainty (a model-consistent VXO) which responds to three shocks, i.e., a first-moment technology shock, a first-moment preference shock, and a second-moment preference shock, this last one being the uncertainty shock. Conditional on Basu and Bundick's (2017) calibration, however, the uncertainty shock and the model-consistent VXO move hand-in-hand, i.e., the VXO reacts very little to shocks other than the uncertainty one.

A Monte Carlo exercise with artificial data simulated with the Basu and Bundick (2017) framework confirms this statement. We conduct a population analysis and simulate a sample of 50,000 observations with the model calibrated as in Basu and Bundick

(2017). <sup>22</sup> We then estimate a linear VAR and produce impulse responses to an uncertainty shock identified with a Cholesky decomposition of the reduced-form variance-covariance matrix. In the VAR, the VXO is ordered first. We focus on a population analysis and on a linear VAR to make sure that our result is not driven by any small-sample issue or fancy nonlinear reduced-form framework.

Figure A2 documents the performance of the Cholesky-VAR in replicating the DSGE-model consistent impulse responses. The ability of the VAR to correctly capture the responses of the DSGE model is impressive. This result justifies the use of a direct impulse-response function matching approach when we estimate the DSGE framework. Figure A3 adds evidence on the "quasi-exogeneity" of the model consistent VXO process by plotting the volatility of the preference shock against that of the VXO. The two series clearly comove, and their correlation is equal to 0.95.

### C: GIRFs in presence of the shadow rate

Figure A4 shows that our results are practically the same if the Wu and Xia (2016) shadow rate is used in place of the federal funds rate for the period of zero lower bound. As shown by the authors the shadow rate, and its meaningful variations, can be used to proxy unconventional monetary policy at the zero lower bound.

# D: Computation of the Generalized Impulse Response Functions

The algorithm for the computation of the Generalized Impulse Response Functions follows the steps suggested by Koop, Pesaran, and Potter (1996), and it is designed to simulate the effects of an orthogonal structural shock as in Kilian and Vigfusson (2011). The idea is to compute the empirical counterpart of the theoretical  $GIRF_{\mathbf{y}}(h, \delta, \boldsymbol{\omega}_{t-1})$  of the vector of endogenous variables  $\mathbf{y}_t$ , h periods ahead, for a given initial condition  $\boldsymbol{\omega}_{t-1} = \{\mathbf{y}_{t-1}, ..., \mathbf{y}_{t-k}\}$ , k is the number of VAR lags, and  $\delta$  is the structural shock hitting at time t. Following Koop, Pesaran, and Potter (1996), such GIRF can be expressed as follows:

$$GIRF_{\mathbf{y}}(h, \delta, \boldsymbol{\omega}_{t-1}) = E[\mathbf{y}_{t+h} | \delta, \boldsymbol{\omega}_{t-1}] - E[\mathbf{y}_{t+h} | \boldsymbol{\omega}_{t-1}]$$

<sup>&</sup>lt;sup>22</sup>Notice that nothwistanding the fact that we use a DSGE model with three shock to simulate data on which to estimate a seven variables VAR model, we do not suffer from stochastic singularity given that the DSGE model at the basis of the simulation is nonlinearized, and hence there is no linear combination of variables that is perfectly collinear to others.

where  $E[\cdot]$  is the expectation operator, and h = 0, 1, ..., H indicates the horizons from 0 to H for which the computation of the GIRF is performed.

Given our model (1), we compute our GIRFs as follows:

- 1. we pick an initial condition  $\omega_{t-1}$ . Notice that, given that uncertainty and the policy rate are modeled in the VAR, such set includes the values of the interaction terms  $(\ln VXO \times \Delta \ln GDP)_{t-j}$ , j=1,...,k;
- 2. conditional on  $\omega_{t-1}$  and the structure of the model (1), we simulate the path  $[\mathbf{y}_{t+h} | \omega_{t-1}]^r$ , h = [0, 1, ..., 19] (which is, realizations up to 20-step ahead) by loading our VAR with a sequence of randomly extracted (with repetition) residuals  $\widetilde{\mathbf{u}}_{t+h}^r \sim d(0, \widehat{\Omega})$ , h = 0, 1, ..., H, where  $\widehat{\Omega}$  is the estimated VCV matrix,  $d(\cdot)$  is the empirical distribution of the residuals, and r indicates the particular sequence of residuals extracted;
- 3. conditional on  $\omega_{t-1}$  and the structure of the model (1), we simulate the path  $[\mathbf{y}_{t+h} | \delta, \boldsymbol{\omega}_{t-1}]^r$ , h = [0, 1, ..., 19] by loading our VAR with a perturbation of the randomly extracted residuals  $\widetilde{\mathbf{u}}_{t+h}^r \sim d(0, \widehat{\Omega})$  obtained in step 2. In particular, we Cholesky-decompose  $\widehat{\Omega} = \widehat{\mathbf{C}}\widehat{\mathbf{C}}'$ , where  $\widehat{\mathbf{C}}$  is a lower-triangular matrix. Hence, we recover the orthogonalized elements (shocks)  $\widetilde{\boldsymbol{\varepsilon}}_t^r = \widehat{\mathbf{C}}^{-1}\widetilde{\mathbf{u}}_t^r$ . We then add a quantity  $\delta > 0$  to the  $\widetilde{\boldsymbol{\varepsilon}}_{unc,t}^r$ , where  $\widetilde{\boldsymbol{\varepsilon}}_{unc,t}^r$  is the scalar stochastic element loading the uncertainty equation in the VAR. This enable us to obtain  $\widetilde{\boldsymbol{\varepsilon}}_t^r$ , which is the vector of perturbed orthogonalized elements embedding  $\widetilde{\boldsymbol{\varepsilon}}_{unc,t}^r$ . We then move from perturbed shocks to perturbed residuals as follows:  $\widetilde{\mathbf{u}}_t^r = \widehat{\mathbf{C}}\widetilde{\boldsymbol{\varepsilon}}_t^r$ . These are the perturbed residuals that we use to simulate  $[\mathbf{y}_{t+h} | \delta, \boldsymbol{\omega}_{t-1}]^r$ ;
- 4. we compute the difference between paths for each simulated variable at each simulated horizon  $[\mathbf{y}_{t+h} | \delta, \boldsymbol{\omega}_{t-1}]^r [\mathbf{y}_{t+h} | \boldsymbol{\omega}_{t-1}]^r$ , h = [0, 1, ..., 19];
- 5. we repeat steps 2-4 a number of times equal to R = 500. We then store the horizon-wise average realization across repetitions r. In doing so, we obtain a consistent estimate of the GIRF per each given initial quarter of our sample, i.e., an history-dependent GIRF,  $\widehat{GIRF}_{\mathbf{y}}(h, \delta_t, \boldsymbol{\omega}_{t-1}) = \widehat{E}[\mathbf{y}_{t+h} | \delta, \boldsymbol{\omega}_{t-1}] \widehat{E}[\mathbf{y}_{t+h} | \boldsymbol{\omega}_{t-1}],$  h = [0, 1, ..., 19]. If a given initial condition  $\boldsymbol{\omega}_{t-1}$  leads to an explosive response (namely if this is explosive for most of the R sequences of residuals  $\widetilde{\mathbf{u}}_{t+h}^r$ , in the sense that the response of the shocked variable diverges instead than reverting to

- zero), then such initial condition is discarded (i.e., they are not considered for the computation of state-dependent GIRFs in step 6);<sup>23</sup>
- 6. in order to produce our state-dependent GIRFs for recessions and expansions, we first split previous initial conditions into two subsets of interest. To do so, an initial condition  $\varpi_{t-1} = \{\mathbf{Y}_{t-1}, ..., \mathbf{Y}_{t-L}\}$  is classified to belong to the "deep contractions" state if  $\Delta \ln GDP_{t-1}$  is in the bottom decile of the quarter-on-quarter GDP growth rate empirical distribution and to the "strong expansions" state if  $\Delta \ln GDP_{t-1}$  is in the top decile of the quarter-on-quarter GDP growth rate empirical distribution. Out of these two sets of initial conditions we take the within-set average to obtain, for each state, the average initial condition across the histories linked to the state, i.e.  $\bar{\varpi}_{t-1}^{rec}$  and  $\bar{\varpi}_{t-1}^{exp}$ , which work as a sort of unconditional mean for each state. Then, to produce our state-dependent GIRFs for recessions and expansions,  $\widehat{GIRF}_{Y,t}(\delta_t, \bar{\varpi}_{t-1}^{rec})$  and  $\widehat{GIRF}_{Y,t}(\delta_t, \bar{\varpi}_{t-1}^{exp})$ , we adopt the same steps 1-5 above for  $\bar{\varpi}_{t-1}^{rec}$  and  $\bar{\varpi}_{t-1}^{exp}$  as initial conditions.
- 7. confidence bands surrounding the point estimates obtained in step 6 are computed via a bootstrap procedure. In particular, we simulate S=1,000 samples of size equivalent to the one of actual data. Then, per each dataset, we i) estimate our nonlinear VAR model; ii) implement step  $6.^{24}$  In implementing this procedure the initial conditions and VCV matrix used for our computations now depend on the particular dataset s used, i.e.,  $\omega_{t-1}^s$  and  $\Omega_t^s$ . Confidence bands are the constructed by considering the 84th and 16th percentiles of the resulting distribution of state-conditional GIRFs.

#### E: Model calibration

Some parameters of the model are calibrated as in Basu and Bundick (2017) for comparability reasons. Table A1 collects all the calibrated parameters. We do not estimate these parameters for several reasons. We follow a long tradition in macroeconomics and calibrate the capital's share in production  $\alpha$ , the household discount factor  $\beta$  and the steady state depreciation rate  $\delta$  to values that are standard in the literature. The first-order utilization parameter  $\delta_1$  and the consumption weight in the period utility function

<sup>&</sup>lt;sup>23</sup>This never happens for our responses estimated on actual data. We verified that it happens quite rarely as regards our bootstrapped responses.

<sup>&</sup>lt;sup>24</sup>The bootstrap used is similar to the one used by Christiano, Eichenbaum, and Evans (1999) (see their footnote 23). The code discards the explosive artificial draws to be sure that exactly 1,000 draws are used. In our simulations, this happens a negligible fraction of times.

 $\eta$  cannot be estimated, because the first is determined endogenously by a steady state relationship (involving  $\delta$  and  $\beta$ ) and the second is fixed in order to imply a Frisch elasticity equal to 2. The steady state inflation rate  $\Pi$  cannot be estimated by a impulse response functions matching procedure that focuses on out-of-steady state dynamics, i.e., deviations from the (stochastic) steady state. The firm leverage parameter  $\nu$  does not influence impulse responses in the absence of financial frictions and hence is not identified. As regards the parameters of the stochastic shock processes, we calibrate the volatility of the second moment preference shock  $\sigma_{\sigma^a}$  by appealing to the estimated responses of our nested linear VAR model. The parameters governing the processes of the preference and technological shocks, i.e.  $\rho^a$ ,  $\sigma^a$ ,  $\rho^Z$  and  $\sigma^Z$  are calibrated by borrowing values from Basu and Bundick (2017). In spite of our focus on the effects of the uncertainty shocks, we calibrate also these parameters because these stochastic processes can in principle influence (even on-impact) the response of the model-consistent VXO to an uncertainty shock. We also do not estimate the second-order utilization parameter  $\delta_2$ , the elasticity of substitution between intermediate goods  $\theta_{\mu}$ , and the IES  $\psi$  to not further increase the computational burden of the estimation procedure.

## F: Priors vs. posterior densities in the DSGE model estimation

Figure A5 displays the prior and posterior densities of our estimated parameters. The evidence points to the information in our sample as able to shift and modify the prior densities.

## G: DSGE model estimation convergence diagnostics

Table A2 shows the results of the Geweke (1992)-convergence diagnostics test that compares the means of the first 20% retained draws with that of the last 50%. As indicated by the p-values of the  $\chi^2$ -test for the equality of the means, all MCMC chains converge to their stationary distribution. Figures A7 and A8 show the corresponding MCMC chains and the evolution of their means over time.

## H: Counterfactuals to identify relevant parameter instabilities

We conduct counterfactual exercises per each version of our DSGE model to identify the relevant parameters affecting the impulse responses of the variables of our interest to an uncertainty shock. As regards our analysis of recessions and expansions, we check the impact of each parameter on the impulse responses produced by the DSGE model as follows. Conditional on the set of estimates in expansions, we replace the value of each parameter with the corresponding estimated value in recessions. To be sure, the way in which the exercise is designed is such that, if we replaced all estimated parameters contemporaneously, by construction we would replicate the impulse responses produced by the DSGE in recessions. Figures A8 and A9 display the outcome of this exercise. Bottom line: The parameter which leads to a substantial change of the impulse responses is clearly the risk aversion parameter.

#### I: DSGE model state-conditional GIRFs

This Section investigates whether the initial conditions in the nonlinear DSGE model we employ play a role for the dynamics of the system after an uncertainty shock. Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2017) show that the initial values of the states are potentially very important for the effects of the macroeconomic shocks they study. The computation of the GIRFs in our paper follows Basu and Bundick (2017) and do not properly take into account the role of initial conditions. Hence, this possible omitted factor could be behind the evidence of countercyclical risk aversion we find.<sup>25</sup> It is therefore important to provide a check on the relevance of initial conditions in the model we work with.

Figure A10 compares DSGE-related unconditional GIRFs computed at the ergodic mean of the states with those computed in a state-conditional manner, i.e., by taking initial conditions corresponding to deep contractions and strong expansions.<sup>26</sup> To control for the role of parameter instability, all responses displayed in Figure 10 are computed conditional on the estimates we obtained with the facts established by the linear VAR. To ease comparison, we also plot our baseline responses à la Basu and Bundick (2017) based on the same parameters values. The two states/regimes of deep contractions and strong expansions are defined consistently with the definition adopted for the GIRFs computed with our IVAR model.<sup>27</sup>

Two comments are in order. First, the Basu and Bundick (2017) way of comput-

<sup>&</sup>lt;sup>25</sup>As explained in the main text, we compute responses in the model starting from the regime-specific stochastic steady state implied by the estimated set of parameters.

<sup>&</sup>lt;sup>26</sup>Unconditional and conditional GIRFs are computed based on the replication codes of Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2017). Consistently with their definition, these responses have to be interpreted as deviations from the (deterministic) steady state of the model.

<sup>&</sup>lt;sup>27</sup>Consistently with Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2017), we split the initial values of the state variables between the two regimes on the basis of the first and last deciles of the distribution of the GDP growth rate obtained from a simulated sample path. 500 draws in each regimes are selected.

ing responses produces results very similar to the ones produced by the Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2017) method.<sup>28</sup> Second, the initial conditions in the DSGE model do not materially influence the computed GIRFs to an uncertainty shock. The evidence documented in Figure 10 points to the role of initial conditions for the computation of the state-consistent GIRFs as negligible.<sup>29</sup>

<sup>&</sup>lt;sup>28</sup>One difference is observed in the response of investment. This appears due to a different computation of GIRFs between the two approaches. The adoption of the Basu and Bundick method in our work is justified by two reasons. First, this choice enhance comparability with their empirical results. Second, Basu and Bundick (2017) show that their methodology produces impulse responses that are very similar to the unconditional *simulation-based* GIRFs à la Koop, Pesaran and Potter (1996).

<sup>&</sup>lt;sup>29</sup>Consistently with Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2017), we find that initial conditions are particularly relevant for first moments shocks, like the preference shock in our model (results available upon request). Our intuition is that initial conditions are less relevant for the propagation of second moments shocks because such shocks propagate only via a third-order approximated part of the solution.

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Par.	Description	Value	Source
$\sigma_{\sigma^a}$	volatility of the uncertainty shock	0.005	linear VAR
$\rho^a$	persistence of the preference shock	0.94	BB (2017)
$\sigma^a$	volatility of the preference shock	0.003	BB (2017)
$\rho_Z$	persistence of the technology shock	0.99	BB (2017)
$\sigma^Z$	volatility of the technology shock	0.001	BB (2017)
$\sigma$	capital's share in production	0.333	BB (2017)
β	household discount factor	0.994	BB (2017)
$\delta$	steady state depreciation rate	0.025	BB (2017)
$\delta_1$	first-order utilization parameter	0.03	BB (2017)
$\mu$	consumption weight in the period utility function	0.35	BB (2017)
П	steady state inflation rate	1.005	BB (2017)
$\mathcal{V}$	firm leverage parameter	0.0	BB (2017)
$\delta_2$	second-order utilization parameter	0.0003	BB (2017)
$\theta_{\mu}$	elasticity of subst. between intermediate goods	0.9	BB (2017)
$\varphi$	intertemporal elasticity of substitution	0.95	BB (2017)

Table A1: DSGE model: Calibrated parameters. BB (2017) stands for Basu and Bundick (2017).

Par.	Interpretation	4% taper	8% taper	15% taper	4% taper	8% taper	15% taper
		De	ep contracti	ons	Str	ong expansi	ons
$ ho_{\sigma^a}$	Pers. unc. shock	0.081	0.066	0.029	0.113	0.105	0.076
Ь	Risk aversion	0.760	0.731	0.682	0.408	0.387	0.400
9	Habit form.	0.168	0.190	0.136	0.764	0.755	0.757
$\phi_K$	Inv. adj. costs	0.266	0.267	0.269	0.460	0.456	0.405
$\phi_P$	Pr. adj. costs	0.081	0.099	0.112	0.176	0.184	0.175
$\rho_R$	TR par., smooth.	0.735	0.747	0.753	0.416	0.400	0.388
$ ho_{\pi}$	TR par., infl.	0.012	0.013	0.004	0.101	0.117	0.123
$\rho_y$	TR par., out. gr.	0.749	0.747	0.745	0.241	0.269	0.274

Table A2: **DSGE model estimation:** Geweke (1992) Convergence Diagnostics. Numbers are p-values of the Chi-squared test for equality of means of the first 20draws (after the first 12000 draws are discarded as burn-in).

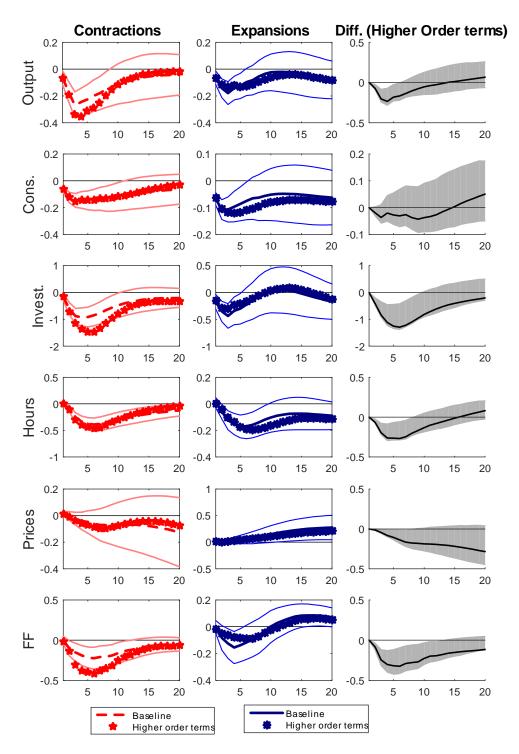


Figure A1: **IVAR impulse responses: Role of higher order terms.** Solid lines in the first and second columns: Impulse responses and 68% confidence bands produced with the baseline, parsimonious IVAR. Lines with stars (first and second columns): Impulse responses produced with the expanded IVAR featuring extra-interaction terms. Densities of the differences between recessions and expansions (68% bands) plotted in the third column.

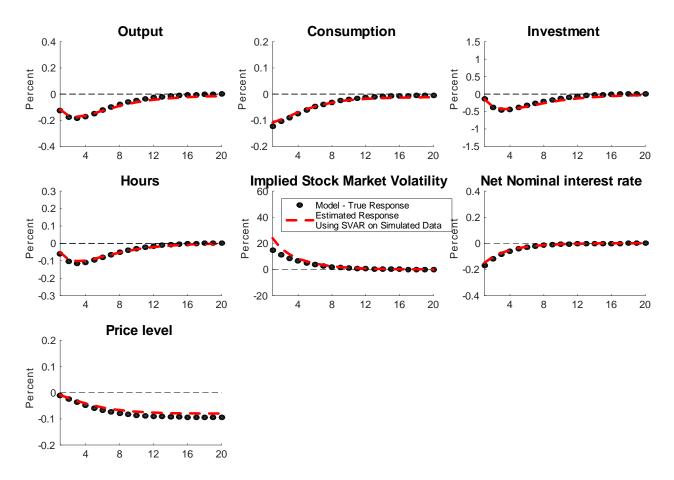


Figure A2: Monte Carlo simulation: DSGE model vs. VAR responses to an uncertainty shock. Calibration of the DSGE model as in Basu and Bundick (2017). Size of the simulated sample: 50,000 observations. Uncertainty shock in the VAR framework identified by assuming a recursive structure of the economic system with the VXO ordered first.

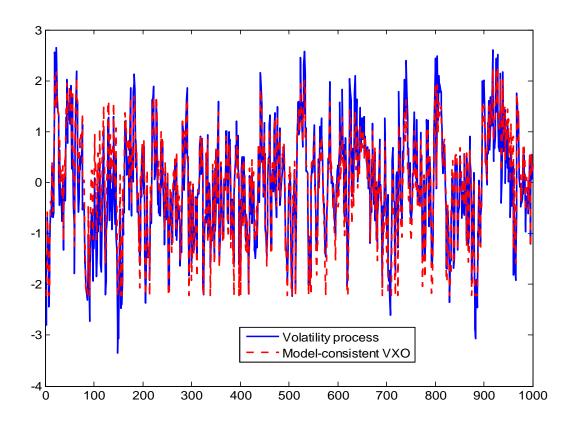


Figure A3: **DSGE-consistent processes: Volatility vs. VXO**. Series produced with the Basu and Bundick (2017) model. Simulated series: 100,000 observations, 99,000 used as a burn in. Both series are standardized to ease readibility. Correlation coefficient: 0.95.

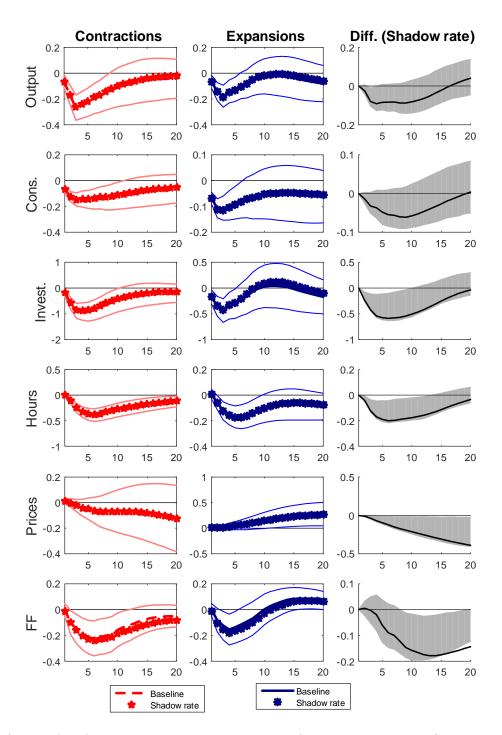


Figure A4: **IVAR impulse responses: check with shadow rate.** Solid lines in the first and second columns: Impulse responses and 68% confidence bands produced with the baseline IVAR with the FFR on all the sample. Lines with stars (first and second columns): Impulse responses produced with the IVAR featuring the Wu and Xia (2016) shadow rate for the period of ZLB. Densities of the differences between recessions and expansions (68% bands) plotted in the third column.

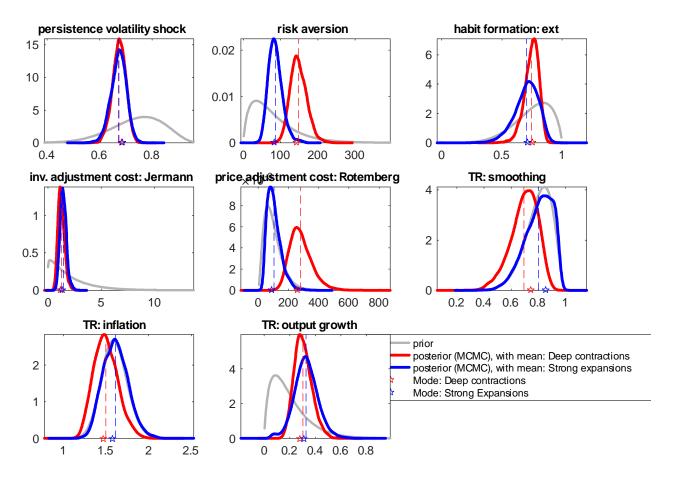


Figure A5: Estimated parameters: Priors vs. posterior densities. Priors in table 3 of the paper. Posteriors computed via MCMC with a random walk metropolis algorithm. 60 000 draws, 20 percent for burn-in. Acceptance rates: 27 percent for times of deep contractions, 26 percent for times of strong contractions.

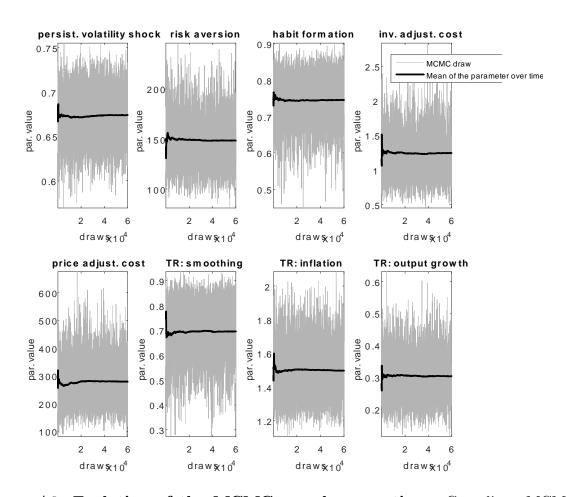


Figure A6: Evolution of the MCMC sampler over time. Grey line: MCMC evolution for a particular parameter (60,000 draws). Black line: Expanding-window mean of of the chain over time.

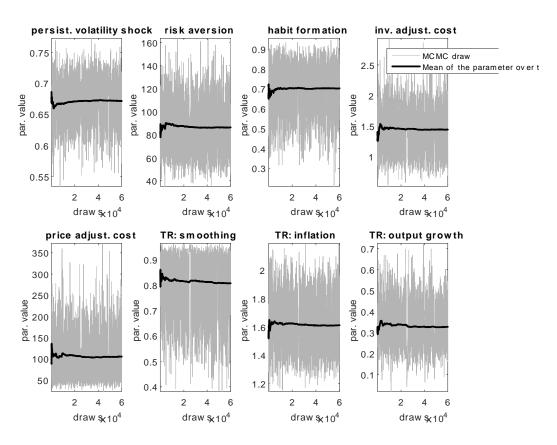


Figure A7: Evolution of the MCMC sampler over time. Grey line: MCMC evolution for a particular parameter (60,000 draws). Black line: Expanding-window mean of of the chain over time.

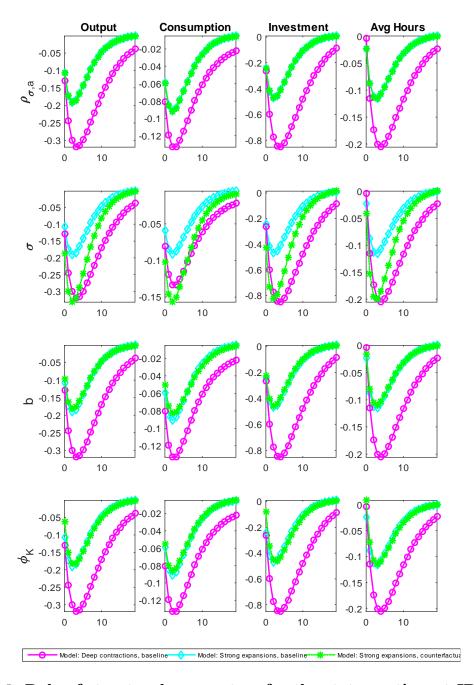


Figure A8: Role of structural parameters for the state-contingent IRFs produced by the DSGE model: First set of parameters.

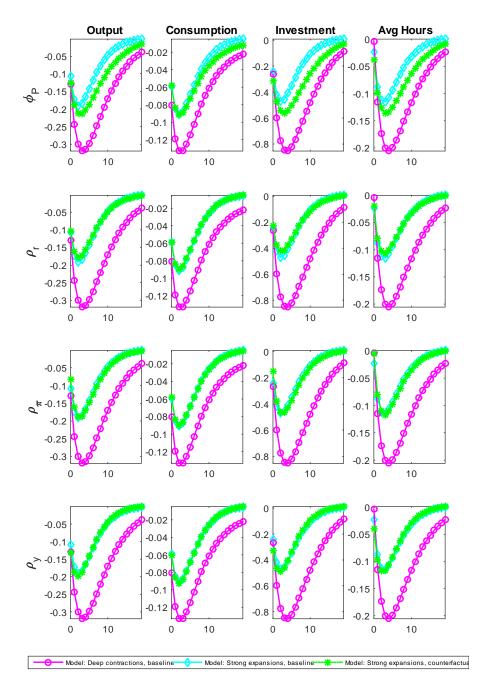


Figure A9: Role of structural parameters for the state-contingent IRFs produced by the DSGE model: Second set of parameters.

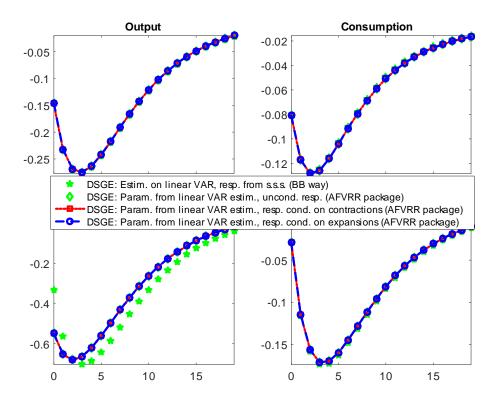


Figure A10: Role of initial conditions for DSGE impulse responses: unconditional GIRFs versus conditional GIRFs à la Andreasen, Fernández-Villaverde, and Rubio-Ramírez (AFVRR, 2017). Green stars: response of the DSGE estimated on the linear VAR model with computation à la Basu and Bundick (2017). Green diamonds: unconditional response of the DSGE for the same set of parameters values with computation à la AFVRR (2017). Red squares: response of the DSGE conditional on deep contractions for the same set of parameters values with computation à la AFVRR (2017). Blue circles: response of the DSGE conditional on strong expansions for the same set of parameters values with computation à la AFVRR (2017).